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QUASIPARTICLE-PHONON NUCLEAR MODEL.

1. Basic Assumptions

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Квазичастично-фононная ядерная модель. 1. Основные положения

Изложены основные положения квазичастично-фононной модели сложных ядер. Обсужден выбор модельного гамильтониана в виде среднего поля и остаточных сил. Представлены фононное описание и взаимодействие квазичастиц с фононами. Получены системы основных уравнений и их приближенные решения. Приближение выбрано так, чтобы получить наиболее точное описание не всей волновой функции, а только ее малоквазичастичных компонент. Изложен метод силовых функций, играющий решающую роль в практической реализации квазичастично-фононной модели для описания ряда свойств сложных ядер. Определена область применения квазичастично-фононной модели ядра: малоквазичастичные компоненты волновых функций при низких, промежуточных и высоких энергиях возбуждения, усредненные в некоторых энергетических интервалах.

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Quasiparticle-Phonon Nuclear Model. 1. Basic Assumptions

The general assumptions of the quasiparticle-phonon model of complex nuclei are given. The choice of the model Hamiltonian as an average field and residual forces is discussed. The phonon description and quasiparticle-phonon interaction are presented. The system of basic equations and their approximate solutions are obtained. The approximation is chosen so as to obtain the most correct description of few-quasiparticle components rather than of the whole wave function. The method of strength functions is presented, which plays a decisive role in practical realization of the quasiparticle-phonon model for the description of some properties of complex nuclei. The range of applicability of the quasiparticle-phonon nuclear model is determined as few-quasiparticle components of the wave functions at low, intermediate and high excitation energies averaged in a certain energy interval.

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I n t r o d u c t i o n

A quasiparticle-phonon model which is the basis for a unique description of few-quasiparticle components of the wave functions of complex nuclei at low, intermediate and high excitation energies is constructed within the semimicroscopic nuclear theory. The quasiparticle-phonon nuclear model develops from the description of the low-lying nuclear states as quasiparticle[1] and one-phonon states[2] by generalizing the phonons and quasiparticle-phonon interaction[3]. The quasiparticle-phonon interaction is important for the calculation of the energies and wave functions of nonrotational states in odd-A nuclei[4-6]. The methods of description of the low-lying states of atomic nuclei have been generalized and applied to the study of the state structure at intermediate and high excitation energies[7]. The analysis of ref.[8] has shown that the excited states of atomic nuclei can uniquely be described.

The quasiparticle-phonon model is based on the following assumptions[7,9-11]:

1) The two-quasiparticle and vibrational states are considered to be the one-phonon states.

2) The coupling of single-particle and collective motions is described as the quasiparticle-phonon interaction.

3) The main approximation is chosen so as to obtain the most correct description of few-quasiparticle components rather than of the whole wave function.

The wave functions of highly excited states of complex nuclei comprise several million of components. It is very difficult to find the wave function of each state. This is demonstrated for light nuclei in ref.[12] where the matrices of very high order have been diagonalized when calculating the energies and wave functions. The investigations within the approach based on the operator form of the wave function[13-15] have shown that such characteristics of highly excited states as the photoexcitation total cross sections, spectroscopic factors of the one-nucleon transfer reactions, neutron strength functions, partial radiative strength functions for direct transitions to the low-lying states and others are determined by the few-quasiparticle components of their wave functions. The problem is essentially simplified if only few quasiparticle components of the wave functions are to be well described in a certain energy interval. In this case one should use different types of the strength functions.

At present within the quasiparticle-phonon model the fragmentation (distribution of strength) of one-quasiparticle, one-phonon and quasiparticle plus phonon states over many nuclear levels can be calculated. This makes it possible to study many nuclear processes and properties of complex nuclei in a wide range of excitation energies.

In this review we present the general assumptions of the quasiparticle-phonon model of complex (middle and heavy) nuclei. For definiteness the formulae are given for deformed nuclei, though the judgment of the given material concerns also spherical nuclei. The main results of calculation of the properties of spherical and deformed nuclei will be given elsewhere.

1. The Hartree-Fock-Bogolubov Variational Principle and the Semimicroscopic Description

The Hartree-Fock-Bogolubov Variational Principle is among the basic and widely used methods of solution of the many-body problem. Most of the equations which are solved in the nuclear theory are particular cases of the basic equations obtained within this method. The Hartree-Fock-Bogolubov Method allows one to express higher correlation functions in terms of the lower ones. As a result the equation of motion can be written in a closed form. The main steps formulated in this section allow one to trace the procedure of solving the many-body nuclear problem.

The Hamiltonian of the system is

$$H = \sum_{f f'} T'(f, f') a_f^+ a_{f'} - \frac{1}{4} \sum_{f_1, f_2, f_2', f_1'} G(f_1, f_2; f_2', f_1') a_{f_1}^+ a_{f_2}^+ a_{f_2'} a_{f_1'} , \tag{1}$$

where f is the set of quantum numbers which describe the state of a nucleon. The creation and absorption operators of a nucleon a_f^+ , a_f fulfill the usual commutation relations

$$a_{f_1}^+ a_{f_2} + a_{f_2} a_{f_1}^+ = \delta_{f_1, f_2} ,$$

$$a_{f_1} a_{f_2} + a_{f_2} a_{f_1} = 0 .$$

Further,

$$T'(f, f') = T(f, f') - \lambda \delta_{f, f'}$$

and λ denotes the chemical potential.

We introduce the correlation function

$$\phi(f_1, f_2) = \langle | a_{f_1} a_{f_2} | \rangle \tag{2}$$

and the density function

$$\rho(f_1, f_2) = \langle | a_{f_1}^+ a_{f_2} | \rangle . \tag{3}$$

Here $$\phi(f_1, f_2) = -\phi(f_2, f_1),$$
$$\rho^*(f_1, f_2) = \rho(f_2, f_1) \quad ,$$
the expectation value is taken over an arbitrary state $|\rangle$.

Let us consider the amplitude a_f in the Heisenberg representation where $a_f(t)$ depends explicitly on time. Now we introduce the functions $\rho_t(f_1, f_2)$, $\phi_t(f_1, f_2)$ which depend on time. From the equation of motion it follows that

$$i\frac{\partial \rho_t(f_1, f_2)}{\partial t} = \langle |[a^+_{f_1}(t)a_{f_2}(t), H]|\rangle , \tag{4}$$

$$i\frac{\partial \phi_t(f_1, f_2)}{\partial t} = \langle |[a_{f_1}(t)a_{f_2}(t), H]|\rangle . \tag{5}$$

Then we should write down the equations for $\langle|a^+_{f_1}(t)a^+_{f_2}(t)a_{f_2'}(t)a_{f_1'}(t)|\rangle$ and $\langle|a^+_{f_1}(t)a_{f_2}(t)a_{f_2'}(t)a_{f_1'}(t)|\rangle$ and express them in terms of the distribution function of higher order and so on. One can pass to the closed system of approximate equations by means of an approximation expressing higher correlation functions in terms of the lower ones.

In the nuclear theory the Hartree-Fock-Bogolubov method is used to obtain approximate equations by expressing higher correlation functions in terms of the lower ones. For instance, the function $\langle|a^+_{f_1'}a^+_{f_2'}a_{f_1}a_{f_2}|\rangle$ can be expressed in terms of the functions ρ and ϕ as follows:

$$\langle|a^+_{f_1'}a^+_{f_2}a_{f_1}a_{f_2}|\rangle = \rho(f_1', f_2)\rho(f_2', f_1) - \\ -\rho(f_1', f_1)\rho(f_2', f_2) + \phi^*(f_2', f_1')\phi(f_1, f_2), \tag{6}$$

in the Hartree approximation the r.h.s. containing only the first term, V.A.Fock has added the second term which takes into account antisymmetrization and N.N.Bogolubov has introduced the third term which allows the description of the superconductig pairing correlations. If one uses an approximation of the type (6) then equations (4),(5) become closed. These equations can symbolically be written as folows:

$$i\,\frac{\partial \rho_t(f_1,f_2)}{} = \mathcal{B}(f_1,f_2)\,, \tag{7}$$

$$i\,\frac{\partial \Phi_t(f_1,f_2)}{} = \mathcal{U}(f_1,f_2)\,. \tag{8}$$

In the stationary case they are

$$\mathcal{B}(f_1,f_2) = 0, \tag{7'}$$

$$\mathcal{U}(f_1,f_2) = 0\,. \tag{8'}$$

The explicit form of these functions is given in ref.[16/]. The development of the Hartree-Fock-Bogolubov method became possible when N.N.Bogolubov introduced quasiaverages in the strict mathematical formulation [17/].

2. To pass to the following stage of transformations, from the set of quantum numbers f we exclude $\sigma = \pm 1$ so that the states $f = \rho\sigma$ which differ by sign σ will be conjugate with respect to the operation of time reflection. For instance, σ can be the sign of the momentum projection onto the symmetry axis.

In fact for any type of the interaction between nucleons one can find such a unitary linear transformation which transforms simultaneously the functions $\rho(f, f')$ and $\Phi(f, f')$ to the diagonal and canonical form, respectively, i.e.,

$$\begin{aligned} \phi(f, f') &= \phi(f)\,\delta_{f,-f'} = \phi(q)\,\delta_{q,q'}\,\delta_{\sigma,-\sigma'}\,, \\ \rho(f, f') &= \rho(f)\,\delta_{f,f'} = \rho(q)\,\delta_{q,q'}\,\delta_{\sigma,\sigma'}\,. \end{aligned} \tag{9}$$

Then the functions $\Phi(q)$ and $\rho(q)$ are bound by the condition

$$\rho(q) = \rho^2(q) + \phi^*(q)\,\phi(q). \tag{10}$$

In this representation the average value of the energy operator has the form

$$\begin{aligned} \langle | H | \rangle &= \sum_f \Big\{ T'(f) - \frac{1}{2}\sum_{f'} G(f, f'; f', f)\,\rho(f') \Big\}\rho(f) - \\ &- \sum_{q,q'} G(q+, q-; q'-, q'+)\,\phi^*(q)\,\phi(q'). \end{aligned} \tag{11}$$

The main equations are

$$2[E(q) - \lambda]\Phi(q) - [1 - 2\rho(q)]\sum_{q'} G(q+, q-; q'-, q'+)\Phi(q') = 0, \tag{12}$$

$$N = 2\sum_q \rho(q). \tag{13}$$

Here N is the number of protons and neutrons. Equation (12) containq the energies of the single-particle levels of an average field which are

$$E(q) = T(q) - \sum_{q'\sigma'} G(q+, q'\sigma', q'\sigma', q+)\,\rho(q'). \tag{14}$$

Within the microscopic method expressions of the type (14) are calculated on the basis of the experimental data on the nucleon-nucleon scattering.

Therefore, from the general form of the potential describing the interactions between nucleons there are extracted the average nuclear field and interactions resulting in superconducting pairing correlations. It is postulated in the nuclear theory that the average nuclear field corresponds to such a representation when the density matrix $\rho(f, f')$ is diagonal for the ground states of even-even nuclei lying in the β -stability zone. In this representation the residual forces are reduced completely to the interactions resulting in superconducting pairing correlations. Therefore, other residual interactions should not be taken into account.

The possibility of extraction of the average nuclear field is the reflection of the fundamental properties of the atomic nucleus but not a mathematical method. The extraction of the average nuclear field is possible, first, due to the Pauli principle, and second, to the relation of the Fermi surface momentum with the momentum of the repulsive core of the nucleon-nucleon potential. The variety of properties of atomic nuclei is caused by the average field or nuclear shells. Therefore, nuclei should not be considered as different fragments of the nuclear matter, but one should study the structure of each nucleus. This is the difference, for instance, from a crystal where it is senseless to study the structure of crystals of one and the same type but of different sizes.

It should be noted that the conditions of extraction of the average field may not be fulfilled for hypothetical superdense nuclear state and instead of a variety of nuclear properties this state may contain nondistinguishable nuclei as pieces of the nuclear matter of different sizes.

From formulae (9)-(14) we obtain the main equations of the theory of superconducing pairing correlations. It is assumed that the function $G(q+, q-; q'-, q'+)$ is independent of q

and q' and changes from nucleus to nucleus as A^{-1}. These assumptions are justified, and the theory uses two constants G_N and G_Z determined from the experimental data on pairing energies. Now we introduce the functions U_q and V_q

$$\rho(q) = V_q^2, \quad \Phi(q) = \Phi^*(q) = U_q V_q, \tag{15}$$

then the condition(10) has the form:

$$1 = V_q^2 + U_q^2, \tag{10'}$$

and equations (12) and (13) are

$$2[E(q) - \lambda] U_q V_q - (U_q^2 - V_q^2) G \sum_{q'} U_{q'} V_{q'} = 0,$$

$$N = 2 \sum_q V_q^2.$$

Let us introduce the correlation function

$$C = G \sum_{q'} U_{q'} V_{q'} \tag{16}$$

and after simple transformations we obtain

$$1 = \frac{G}{2} \sum_q \frac{1}{\sqrt{C^2 + \{E(q) - \lambda\}^2}}, \tag{17}$$

$$N = \sum_q \left\{1 - \frac{E(q) - \lambda}{\mathcal{E}(q)}\right\}, \tag{18}$$

$$V_q^2 = \frac{1}{2}\left\{1 - \frac{E(q) - \lambda}{\mathcal{E}(q)}\right\}, \quad \mathcal{E}(q) = \sqrt{C^2 + \{E(q) - \lambda\}^2}. \tag{19}$$

3. There are two types of excited states (except for rotational ones), for one of which, quasiparticle excitations, the conditions (9) are fulfilled, and for the other, vibrational states, are not.

It is shown in ref.[18] that the vibrational states are connected with off-diagonal parts of the density matrix. In that paper the off-diagonal increments $\delta\rho(q\sigma, q'\sigma')$, $\delta\Phi(q\sigma, q'\sigma')$ have been introduced and equations have been obtained for them. In ref.[19] the average field has explicitly been extracted and the equations reduced to the following form:

$$
\begin{aligned}
&[\mathcal{E}(q_1)+\mathcal{E}(q_2)]\, Z^{(\pm)}(q_1,q_2) - \omega Z^{(\mp)}(q_1,q_2) - \\
&- \sum_{q_1',q_2'} G^{\xi}(q_1,q_2;q_2',q_1')\, v^{(\mp)}_{q_1,q_2} v^{(\mp)}_{q_1',q_2'} Z^{(\pm)}(q_1',q_2') - \\
&- 2\sum_{q_1',q_2'} G^{\omega}(q_1,q_2;q_2',q_1')\, U^{(\pm)}_{q_1,q_2} U^{(\pm)}_{q_1',q_2'} Z^{(\pm)}(q_1',q_2') = 0,
\end{aligned}
\tag{20}
$$

$$
U^{(\pm)}_{q_1,q_2} = U_{q_1} v_{q_2} \pm U_{q_2} v_{q_1}, \quad v^{(\pm)}_{q_1,q_2} = U_{q_1} U_{q_2} \pm v_{q_1} v_{q_2} . \tag{21}
$$

When studying the excited states of complex nuclei one should remember that the interaction G manifests itself in two channels. The collective effects related to the quadrupole, octupole and other vibrations of this type and to the giant multipole resonances are generated by the interactions in the particle-hole channel which are denoted by $G^{\omega}(q_1,q_2;q_2',q_1')$. The interactions in the particle-particle channel are denoted by $G^{\xi}(q_1,q_2;q_2',q_1')$. The interactions of this type with the total moment equal to zero generate superconducting pairing correlations. In some cases the interactions with the total moment different from zero are taken into account in the particle-particle channel. The terms of eq.(20) comprising $U^{(\pm)}_{q_1,q_2}$ describe the interaction in the particle-hole channel. The terms comprising $v^{(\pm)}_{q_1,q_2}$ describe the interactions in the particle-particle channel.

To derive equation (20) the interaction between quasiparticles is taken in the most general way. It is shown in ref.[19] that all equations in nonphenomenological theories which are used to describe nuclear vibrations are particular cases of equation (20).

Note that equations analogous ') have been obtained in ref.[20]. In ref.[19] equations J) h has been generalized for the case of external field and ed for d iving equations of the theory of finite Fermi-systems[21]. In r .[22] equations of ref.[20] have been derived from equations of the theory of finite Fermi-systems.

4. A quasiparticle-phonon model is constructed within the semimicroscopic nuclear theory. The average field and residual or effective interactions are not calculated within the semimicroscopic theories but are given in a definite form on the basis of our knowledge about the nuclear structure. Therefore, relative quantities rather than absolute ones are calculated within the semimicroscopic theories. For instance, the excitation energies ut not the total nuclear energies in the ground and excited states are calculated. In particular the change of the nuclear energy with increasing deformation parameter is calculated.

There are many different variants of the semimicroscopic description. We use one of them. The main assumptions of the semimicroscopic description of the nuclear structure are as follows:[23]

1) The Hartree-Fock-Bogolubov method is used for obtaining the closed system of equations for the density and correlation functions. This is the main approximation in the many-body problem.

2) A representation is chosen where the density matrix is diagonal and the correlation function has a canonical form. In this representation all interactions between nucleons in a nucleus are reduced to the average field and interactions resulting in pairing.

3) The average field is extracted, which is described by the Saxon-Woods potential. It is postulated that the choice of an average field corresponds to the aforesaid representation for some doubly even nuclei lying in the β -stability zone. The average field defines directly many nuclear properties and makes it possible for residual forces to appear.

4) The excited states are defined as one-, two-, three-, and so on quasiparticle states.

5) The low-lying vibrational states are connected with off-diagonal elements of the density matrix. To describe them the multipole-multipole and spin-multipole-spin-multipole forces are introduced. The mathematical treatment is based on different variants of the second quantization method developed by N.N.Bogolubov[24].

6) The rotational, quasiparticle and phonon excited states are related to each other by the Coriolis and quasi-particle-phonon interaction.

2. The Model Hamiltonian

In the semimicroscopic nuclear theory the Hamiltonian describing different types of nuclear motions has the following form:

$$H = H_{av} + H_{pair} + T_{rot} + H_{cor} + H_Q + H_{\sigma Q} + H' . \tag{22}$$

Here H_{av} is the average field of the neutron and proton systems, H_{pair} are the interactions resulting in superconducting pairing correlations, T_{rot} is the kinetic energy of rotation, H_{cor} is the Coriolis interaction describing the coupling of intrinsic motion and rotation, H_Q is the multiple-multipole interaction, $H_{\sigma Q}$ is the spin-multipole-spin-multipole interaction, and H' are other interactions including, for instance, interaction of the Gamow-Teller type.

To describe the states the structure of which is related to off-diagonal parts of the density matrix, we introduce residual interactions. The central residual interaction is

$$V(|\vec{r}_1 - \vec{r}_2|) + V_\sigma(|\vec{r}_1 - \vec{r}_2|)(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)}) + \tag{23}$$
$$+\{V_\tau(|\vec{r}_1 - \vec{r}_2|) + V_{\tau\sigma}(|\vec{r}_1 - \vec{r}_2|)(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)})\}(\vec{\tau}^{(1)}\vec{\tau}^{(2)}) .$$

We expand it in a series of spherical functions, and as a result we obtain

$$V(|r_1 - r_2|) = \sum_{\lambda=0}^{\infty} R_\lambda(r_1, r_2)\frac{4\pi}{2\lambda+1}\sum_{\mu=-\lambda}^{\lambda}(-1)^\mu Y_{\lambda\mu}(\theta_1, \varphi_1)\, Y_{\lambda,-\mu}(\theta_2, \varphi_2), \tag{24}$$

$$V_\sigma(|r_1 - r_2|)(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)}) = \sum_{s=0}^{\infty} R_s^\sigma(r_1, r_2)\ \frac{4\pi}{2s+1}\sum_{\lambda=s,s\pm 1}(-1)^{s+1-\lambda} \times$$
$$\times \sum_{\mu=-\lambda}(-1)^\mu\{\vec{\sigma}^{(1)}\vec{Y}_s(\theta_1, \varphi_1)\}_{\lambda\mu}\cdot\{\vec{\sigma}^{(2)}\vec{Y}_s(\theta_2, \varphi_2)\}_{\lambda,-\mu} . \tag{25}$$

Of a similar form are the expansions of the functions V_τ and

$V_{\tau\sigma}$. Here

$$\{\vec{\sigma}\vec{Y}_s(\theta_1,\varphi_1)\}_{\lambda\mu} = \sum_{\rho=0,\pm1}\sum_{p=-s}^{s}(1\rho\, sp|\lambda\mu)\sigma_\rho Y_{sp}(\theta_1,\varphi_1), \qquad (25')$$

where r_i , θ_i , φ_i , $\sigma^{(i)}$ determine the position and the spin of a particle, and the functions $R_\lambda(r_1, r_2)$, $R_s^\sigma(r_1, r_2)$ describe the radial dependence. Therefore, the most general form of the central potential is given as series over multipoles and spin-multipoles.

From the existence of the static quadrupole deformation in the rare-earth and actinide regions one may conclude that the quadrupole-quadrupole interactions are very important. The part of this interaction which is not reduced to the average field should describe the interaction between quasiparticles. Thus, a part of the residual interaction can be approximated by the terms of the multipole-multipole and spin-multipole-spin-multipole interactions.

The radial part of the interaction is chosen in different ways. In order to obtain a simple secular equation instead of the diagonalization of the matrix of higher order, one should take the functions $R_\lambda(r_1, r_2)$ and $R_s^\sigma(r_1, r_2)$ in a factorized form

$$R_\lambda(r_1, r_2) = \varkappa_0^{(\lambda)} R(r_1) R(r_2),$$
$$R_s^\sigma(r_1, r_2) = \varkappa_{\sigma_0}^{(s)} R(r_1) R(r_2).$$

The functions $R_\lambda(r_1, r_2)$ and $R_s^\sigma(r_1, r_2)$ are often chosen as

$$R_\lambda(r_1, r_2) = \varkappa_0^{(\lambda)} r_1^\lambda r_2^\lambda,$$
$$R_s^\sigma(r_1, r_2) = \varkappa_{\sigma 0}^{(s)} r_1^s r_2^s. \qquad (26)$$

The corresponding expansions of the functions $V_\tau(|\vec{r}_1-\vec{r}_2|)$ and $V_{\tau\sigma}(|\vec{r}_1-\vec{r}_2|)$ involve the constants $\varkappa_1^{(\lambda)}$ and $\varkappa_{\sigma_1}^{(\lambda)}$. The neutron-neutron $\varkappa_{nn}$, proton-proton $\varkappa_{pp}$ and neutron-proton $\varkappa_{np}$ constants are connected with the isoscalar $\varkappa_0$ and isovector $\varkappa_1$ constants as follows:

$$\varkappa_{nn} = \varkappa_{pp} = \varkappa_0 + \varkappa_1 ,$$
$$\varkappa_{np} = \varkappa_0 - \varkappa_1 . \tag{27}$$

The interaction (26) is especially strong when both particles are near the nuclear surface. At $r > R_0$ the corresponding single-particle matrix elements decrease rapidly with increasing r and the strength of the interaction weakens sharply. Inside the nucleus the interaction strength decreases gradually. Often when studying the coherent effects $r_1 = r_2$, and the radial part is $R_\lambda = \varkappa^{(\lambda)} r^{2\lambda}$. Thus, the interaction (26) has the largest strength near the nuclear surface. Therefore the results of calculation (26) for the interaction of low-lying vibrational states are close to those for the surface delta-interaction[25]. The results of calculation of the multipole giant resonances are close to those with a new Skyrme interaction[26].

The calculations of nuclear characteristics performed with different residual forces give similar results. This indicates the fact that owing to the single-particle wave functions the detailed radial dependence does not manifest itself strongly in the calculation of matrix elements. It has been demonstrated in ref.[27] that the two-particle transition density behaves as a filter leaving only certain Fourier components of the effective forces. It has also been shown there that the residual interactions used in a certain limited configurational space are *almost the same if their Fourier-components* are *similar in* a comparatively narrow region of momenta transferred. It can be concluded that the use of the radial dependence of residual forces in the form of eq.(26) is justified. There are no convincing arguments in favour of another (quite definite) radial dependence.

2. Now we pass to the construction of the Hamiltonian of the quasiparticle-phonon nuclear model. This model is used for the calculation of few-quasiparticle components of the wave functions of spherical and deformed complex nuclei at low, intermediate and high excitation energies. For the sake of definiteness we give formulae for the deformed nuclei, however they can unambiguously be rewritten for spherical nuclei.

As is known, the rotational motion and its coupling with the quasiparticle and phonon excitations play an important role in atomic nuclei. The rotations, especially with large angular momenta, are described in detail in refs.[28-30]. In the quasiparticle-phonon model in many cases the coupling with rotation is neglected and rotation itself is described phenomenologically. This is due to the fact that the high-lying states with very large angular momenta are not considered. When studying the low-lying states the coupling with rotation can be taken into account in every concrete case. In the states with small momenta at intermediate and high excitation energy the coupling with rotation does not result in a considerable redistribution of strength of the few-quasiparticle components of the wave functions. Therefore, we do not introduce explicitly the kinetic energy of rotation and the Coriolis interaction into the model Hamiltonian; they can be added when necessary.

To construct the general form of the model Hamiltonian, we use formulae from ref.[16]. From the Hamiltonian (22) we choose the necessary terms and write as

$$H_M = H_p + H_Q + H_{\sigma Q} , \tag{28}$$

where

$$H_p = H_0 + H_{pp} \tag{29}$$

includes the interactions resulting in superconducting pairing correlations and those in the particle-particle channel with moment different from zero.

$$H_0 = H_0(n) + H_0(p) , \tag{30}$$

$$H_0(n) = \sum_s \epsilon(s) B(s,s) + H_c^{\beta}(n) + H_s'(n) , \tag{30'}$$

where (see page 207 in ref.[16])

$$H_c^{\beta}(n) = -\frac{G_N}{2} \sum_{s,s'} [U_s^2 A^+(s,s) - V_s^2 A(s,s)][U_{s'}^2 A(s',s') - V_{s'}^2 A^+(s',s')] , \tag{31}$$

$$H_s'(n) = -\frac{G_N}{\sqrt{2}} \sum_{s,s'} (U_s^2 - V_s^2) U_{s'} V_{s'} [A^+(s,s) B(s',s') + B(s's') A(s,s)] . \tag{31'}$$

We use the notation

$$A(q,q') = \frac{1}{\sqrt{2}} \sum_\sigma \sigma \alpha_{q\sigma} \alpha_{q-\sigma} \quad \text{or} \quad \frac{1}{\sqrt{2}} \sum_\sigma \alpha_{q\sigma} \alpha_{q'\sigma} \tag{32}$$

$$B(q,q') = \sum_\sigma \alpha^+_{q\sigma} \alpha_{q'\sigma} \quad \text{or} \quad \sum_\sigma \sigma \alpha^+_{q-\sigma} \alpha_{q'\sigma} . \tag{33}$$

Here $\alpha^+_{q\sigma}$ is the quasiparticle creation operator, the single-particle state is specified by the quantum numbers: for neutron $s\sigma$, for proton $r\sigma$ and for both systems $q\sigma$, $\sigma = \pm 1$.

The interactions in the particle-particle channel with moment different from zero are

$$H_{pp} = -\sum \frac{G_\lambda}{2} \sum_\mu P^+_{\lambda\mu} P_{\lambda\mu} , \tag{34}$$

$$\begin{aligned} P_{\lambda\mu} &= \sum_{\substack{qq' \\ \sigma\sigma'}} \langle q\sigma | \bar{\bar{f}}^{\lambda\mu} | q'\sigma' \rangle a^+_{q'\sigma'} a^+_{q\sigma} = \\ &= \sum_{qq'} \bar{\bar{f}}^{\lambda\mu}(qq') \Big\{ \frac{1}{\sqrt{2}} (A^+(qq') + A(q,q')) v^{(-)}_{qq'} + \\ &+ \frac{1}{\sqrt{2}} (A^+(qq') - A(qq')) v^{(+)}_{qq'} - B(qq')(U^{(+)}_{qq'} + U^{(-)}_{qq'}) \Big\} . \end{aligned} \tag{35}$$

The multiple-multipole interactions have the form

$$\begin{aligned} H_Q = \frac{1}{2} \sum_\lambda \sum_{\mu \geq 0} \Big\{ & (\varkappa^{(\lambda)}_0 + \varkappa^{(\lambda)}_1) [Q^+_{\lambda\mu}(n) Q_{\lambda\mu}(n) + Q^+_{\lambda\mu}(p) Q_{\lambda\mu}(p)] + \\ & + (\varkappa^{(\lambda)}_0 - \varkappa^{(\lambda)}_1) [Q^+_{\lambda\mu}(n) Q_{\lambda\mu}(p) + Q^+_{\lambda\mu}(p) Q_{\lambda\mu}(n)] \Big\} , \end{aligned} \tag{36}$$

where

$$\begin{aligned} Q_{\lambda\mu}(n) &= \sum_{\substack{ss' \\ \sigma\sigma'}} \langle s\sigma | f^{\lambda\mu} | s'\sigma' \rangle a^+_{s\sigma} a_{s'\sigma'} = \\ &= \sum_{ss'} f^{\lambda\mu}(ss') \Big\{ U^{(+)}_{ss'} (A(ss') - A^+(ss')) \frac{1}{\sqrt{2}} + \\ &+ v^{(-)}_{ss'} B(ss') \Big\} . \end{aligned} \tag{37}$$

Here

$$f^{\lambda\mu} = \frac{r^\lambda}{\sqrt{2(1+\delta_{\mu 0})}} (Y_{\lambda\mu} + (-1)^\mu Y_{\lambda -\mu})$$

in contrast with ref. [16], we do not distinguish between matrix elements $f^{\lambda\mu}(q,q')$ and $\bar{f}^{\lambda\mu}(q,q')$.

The spin-multipole-spin-multiple interaction has the form

$$H_{\sigma Q} = -\frac{1}{2} \sum_\lambda \sum_{\mu=-\lambda}^{\lambda} \sum_{s=\lambda,\lambda\pm 1} \Big\{ (\varkappa^{(s)}_{\sigma 0} + \varkappa^{(s)}_{\sigma 1}) [T^+_{\lambda\mu s}(n) \, T_{\lambda\mu s}(n) + \tag{38}$$

$$+T^{+}_{\lambda\mu s}(p)\,T_{\lambda\mu s}(p)\Big]+(\varkappa^{(s)}_{\sigma 0}-\varkappa^{(s)}_{\sigma 1})\Big[T^{+}_{\lambda\mu s}(n)\,T_{\lambda\mu s}(p)+T^{+}_{\lambda\mu s}(p)\,T_{\lambda\mu s}(n)\Big]\Big\},$$

where

$$T_{\lambda\mu s}(n)=\sum_{\substack{s_2 s_2' \\ \sigma\sigma'}}\langle s_2\sigma|\,i^{s}\big[\{\vec{\sigma}\,\vec{Y}_s\}_{\lambda\mu}+(-1)^{\mu}\{\vec{\sigma}\,\vec{Y}_s\}_{\lambda,-\mu}\big]\,|s_2'\sigma'\rangle\, a^{+}_{s_2\sigma}\,a_{s_2'\sigma'}= \tag{39}$$

$$=\sum_{s_2 s_2'} f^{\lambda\mu}_{s}(s_2 s_2')\Big\{\frac{1}{\sqrt{2}}\,u^{(-)}_{s_2 s_2'}\big(\mathcal{A}(s_2 s_2')+\mathcal{A}^{+}(s_2 s_2')\big)+v^{(-)}_{s_2 s_2'}\,\mathcal{B}(s_2 s_2')\Big\}$$

Here

$$\mathcal{A}(s,s')=\frac{1}{\sqrt{2}}\sum_{\sigma}\alpha_{s\sigma}\,\alpha_{s'-\sigma}\qquad\text{or}\qquad\frac{1}{\sqrt{2}}\sum_{\sigma}\sigma\,\alpha_{s\sigma}\,\alpha_{s'\sigma}\,, \tag{40}$$

$$\mathcal{B}(s,s')=\sum_{\sigma}\sigma\,\alpha^{+}_{s-\sigma}\,\alpha_{s'-\sigma}\qquad\text{or}\qquad\sum_{\sigma}\alpha^{+}_{s-\sigma}\,\alpha_{s'\sigma}\,. \tag{40'}$$

3. One-Phonon States

1. Let us consider the one-phonon states generated by the multipole-multipole forces in the particle-hole channel. Let us take a part of the model Hamiltonian

$$H_v=\sum_{q}\varepsilon(q)\,B(q,q)+H_M^{\beta}+H_Q^{v}, \tag{41}$$

where H_Q^{v} is a part of H_Q (36) without terms describing the quasiparticle-phonon interactions.

Now we introduce the phonon creation operator

$$Q_t^{+}=\frac{1}{2}\sum_{qq'}\Big\{\psi^{t}_{qq'}A^{+}(qq')-\varphi^{t}_{qq'}A(qq')\Big\}, \tag{42}$$

where $t=\lambda\mu i$, i being the state number with given $\lambda\mu$. After simple transformations[16], we obtain H_v in the form

$$H_v=\sum_{q}\varepsilon(q)\,B(qq)-$$

$$\cdot\frac{1}{4}\delta_{\lambda\mu,20}\sum_{ii'}\{G_N\sum_{ss'}[(U_s^2-v_s^2)(U_{s'}^2-v_{s'}^2)g_{ss'}^{20i}g_{ss'}^{20i'}+w_{ss'}^{20i}w_{ss'}^{20i'}]+$$

$$+G_Z\sum_{rr'}[(U_r^2-v_r^2)(U_{r'}^2-v_{r'}^2)g_{rr'}^{20i}g_{rr'}^{20i'}+w_{rr'}^{20i}w_{rr'}^{20i'}]\}Q_{20i}^{+}Q_{20i'}-$$

$$-\frac{1}{2}\sum_{tt'}\{(\varkappa_0^{(\lambda)}-\varkappa_1^{(\lambda)})[\sum_{\substack{ss'\\ s_2s_2'}}U_{ss'}^{(+)}U_{s_2s_2'}^{(+)}f^t(ss')g_{ss'}^{t}f^{t'}(s_2s_2')g_{s_2s_2'}^{t'}+$$

$$+\sum_{\substack{rr'\\ r_2r_2'}}U_{rr'}^{(+)}U_{r_2r_2'}^{(+)}f^t(rr')g_{rr'}^{t}f^{t'}(r_2r_2')g_{r_2r_2'}^{t'}]+ \tag{43}$$

$$+(\varkappa_0^{(\lambda)}-\varkappa_1^{(\lambda)})\sum_{\substack{ss'\\ rr'}}U_{ss'}^{(+)}U_{rr'}^{(+)}[f^t(ss')g_{ss'}^{t}f^t(rr')g_{rr'}^{t}+f^{t'}(ss')g_{ss'}^{t'}f^t(rr')g_{rr'}^{t}]\}Q_t^{+}Q_{t'},$$

where $g_{qq'}^t=\psi_{qq'}^t+\varphi_{qq'}^t$, $w_{qq'}^t=\psi_{qq'}^t-\varphi_{qq'}^t$, and the matrix element $f^t(qq')\equiv f^{\lambda\mu}(qq')$ does not depend on i.

The wave function of the one-phonon state is

$$Q_t^{+}\Psi, \tag{44}$$

where the wave function of the ground state Ψ_0 of doubly-even nucleus is the phonon vacuum

$$Q_t\Psi_0=0. \tag{44'}$$

The normalization condition (44) is

$$\sum_{qq'}g_{qq'}^{t}w_{qq'}^{t'}=2\delta_{tt'} \tag{45}$$

Following ref.[16] the energies ω_t of the one-phonon states with fixed values of $\lambda\mu$ or K^π are found using the variational principle

$$\delta\{\langle Q_t H_v Q_t^{+}\rangle-\frac{\omega_t}{2}[\sum_{qq'}g_{qq'}^{t}w_{qq'}^{t}-2]\}=0. \tag{46}$$

As a result we obtain the secular equation

$$\begin{vmatrix} (\varkappa_0^{(\kappa)}+\varkappa_1^{(\kappa)})X^t(n)-1 & (\varkappa_0^{(\kappa)}-\varkappa_1^{(\kappa)})X^t(n) \\ (\varkappa_0^{(\kappa)}-\varkappa_1^{(\kappa)})X^t(p) & (\varkappa_0^{(\kappa)}+\varkappa_1^{(\kappa)})X^t(p)-1 \end{vmatrix} = 0, \tag{47}$$

which coincides with (6.134) in ref.[16]. Here

$$X^t(n) = 2\sum_{ss'} \frac{f^t(ss')\,\bar{f}^t(ss')\,U^2_{ss'}\,\mathcal{E}(ss')}{\mathcal{E}^2(ss')-\omega_g^2}, \tag{48}$$

where

$$\bar{f}^t(ss') = f^t(ss') - \frac{\Gamma_n^t(s)}{\gamma_n^t}\delta_{s,s'}, \quad \mathcal{E}(ss') = \mathcal{E}(s)+\mathcal{E}(s'), \tag{48'}$$

$$\gamma_n^t = \sum_{ss'} \frac{4C_n^2-\omega_t^2+4\epsilon(s)\epsilon(s')}{\mathcal{E}(s)(4\mathcal{E}^2(s)-\omega_t^2)\mathcal{E}(s')(4\mathcal{E}^2(s')-\omega_t^2)}, \tag{48''}$$

$$\Gamma_n^t(s) = \sum_{s_2 s_2'} \frac{f^t(s_2 s_2')[4C_n^2-\omega_t^2+4\epsilon(s_2)\epsilon(s_2')-4\epsilon(s)\epsilon(s_2)+4\epsilon(s)\epsilon(s_2')]}{\mathcal{E}(s_2)(4\mathcal{E}^2(s_2)-\omega_t^2]\mathcal{E}(s_2')[4\mathcal{E}^2(s_2')-\omega_t^2]}, \tag{48'''}$$

$\epsilon(s) = \mathcal{E}(s)-\lambda_n$. It is seen from eq.(48') that for $\lambda\mu\neq 0$ $\bar{f}^t(ss')$ coincides with $f^t(ss')$.

Equation (47) can be written as follows:

$$\mathcal{F}(\omega) = \varkappa_0^{(\kappa)}\varkappa_1^{(\kappa)}(X^t(n)-X^t(p))^2-(1-\varkappa_0^{(\kappa)}X^t)(1-\varkappa_1^{(\kappa)}X^t) = 0, \tag{47'}$$

where $X^t = X^t(n)+X^t(p)$.

Note, that the influence of the constant $\varkappa_1^{(\lambda)}$ on the first one-phonon states has been investigated in ref.[31]. It has been shown there that the introduction of $\varkappa_1^{(\lambda)}$ results in renormalization of the constant $\varkappa_0^{(\lambda)}$ but the state structure is not essentially changed.

To find the functions $g_{rr'}^t$ and $w_{rr'}^t$, we use the normalization condition (45) and after cumbersome calculations obtain[32]

$$g_{\tau\tau'}^t = \sqrt{\frac{2}{Y_t}}\; y_p^t \; \frac{\tilde{f}^t(\tau\tau') U_{\tau\tau'}^{(+)} \varepsilon(\tau\tau')}{\varepsilon^2(\tau\tau') - \omega_t^2} , \tag{49}$$

$$w_{\tau\tau'}^t = \sqrt{\frac{2}{Y_t}}\; y_p^t \left\{ \frac{\tilde{f}^t(\tau\tau') U_{\tau\tau'}^{(+)} \omega_t}{\varepsilon^2(\tau\tau') - \omega_t^2} - \delta_{\tau,\tau'} \frac{C_p \Xi_p^t}{\varepsilon(\tau)\omega_t \gamma_p^t} \right\} , \tag{49'}$$

where

$$y_p^t = \frac{(\varkappa_0^{(\lambda)} - \varkappa_1^{(\lambda)}) X^t(n)}{1 - (\varkappa_0^{(\lambda)} + \varkappa_1^{(\lambda)}) X(p)} , \tag{50}$$

$$\Xi_p^t = \sum_{\tau\tau'} \frac{f^t(\tau\tau')}{\varepsilon(\tau)(4\varepsilon^2(\tau) - \omega_t^2)} \cdot \frac{4C_p^2 - \omega_t^2 + 4\varepsilon(\tau)\varepsilon(\tau')}{\varepsilon(\tau')(4\varepsilon^2(\tau') - \omega_t^2)} . \tag{50'}$$

The expressions for $g_{ss'}^t$ and $w_{ss'}^t$ are of an analogous form after the substitution of C_p, Ξ_p^t, γ_p^t and y_p^t by the corresponding quantities for neutrons C_n, Ξ_n^t, γ_n^t, $y_n^t = 1$.

Here

$$Y_t = Y_t(n) + (y_p^t)^2 Y_t(p) = \frac{1}{4} \frac{y_p^t}{\varkappa_0^{(\lambda)} - \varkappa_1^{(\lambda)}} \frac{\partial \mathcal{F}(\omega)}{\partial \omega} , \tag{51}$$

$$Y_t(n) = \sum_{ss'} \frac{(\tilde{f}^t(ss') U_{ss'}^{(+)})^2 \varepsilon(ss')\omega_t}{[\varepsilon^2(ss') - \omega_t^2]^2} = \frac{1}{4} \frac{\partial X^t(n)}{\partial \omega} , \tag{51'}$$

the expression for $Y_t(p)$ is analogous to (51'). If the isovector component of forces is absent ($\varkappa_1^{(\lambda)}=0$) then $y_p^t=1$ and all the expressions (49)-(51') have the form given in section 8 of ref.[16], and the secular equation is

$$1-\varkappa_0^{(\lambda)} X^t(\omega_t)=0. \tag{47'}$$

2. When considering the spin-multipole phonons, we use the Hamiltonian in the form

$$\begin{aligned} H_v^S &= \sum_q \mathcal{E}(q) B(qq) = \\ &-\frac{1}{2}\sum_{tt'} \Big\{ (\varkappa_{\sigma 0}^{(S)}+\varkappa_{\sigma 1}^{(S)}) \Big[\sum_{\substack{ss_1'\\ s_2 s_2'}} U_{s s_1'}^{(-)} U_{s_2 s_2'}^{(-)} f_S^t(s s_1) f_S^{t'}(s_2 s_2') g_{ss'}^t g_{s_2 s_2}^{t'} + \\ &+ \sum_{\substack{\tau\tau'\\ \tau_2\tau_2'}} U_{\tau\tau}^{(-)} U_{\tau_2\tau_2'}^{(-)} f_S^t(\tau\tau') f_S^{t'}(\tau_2\tau_2') g_{\tau\tau}^t g_{\tau_2\tau_2}^{t'} \Big] + \\ &+ (\varkappa_{\sigma 0}^{(S)}-\varkappa_{\sigma 1}^{(S)}) \sum_{\substack{s\tau\\ s_2 s_2}} U_{ss'}^{(-)} U_{\tau\tau'}^{(-)} \Big[f_S^t(s s_1') g_{ss_1}^t f_S^{t'}(\tau\tau') g_{\tau\tau'}^{t'} + \\ &+ f_S^t(s_1 s_1') g_{s_1 s_1'}^{t'} f_S^t(\tau\tau') g_{\tau\tau'}^t \Big] \Big\} Q_t^+ Q_{t'} . \end{aligned} \tag{52}$$

In this case the secular equation is

$$\varkappa_{\sigma 0}^{(S)} \varkappa_{\sigma 1}^{(S)} (S^t(n)-S^t(p))^2-(1-\varkappa_{\sigma 0}^{(S)} S^t)(1-\varkappa_{\sigma 1}^{(S)} S^t)=0, \tag{53}$$

where

$$S^t = S^t(n) + S^t(p) , \tag{54}$$

$$S^t(n) = 2\sum_{ss'} \frac{(f^t(ss')U_{ss'}^{(-)})^2 \mathcal{E}(ss')}{\mathcal{E}^2(ss')-\omega_t^2}. \tag{54'}$$

The expressions for $g_{qq'}^t$, $w_{qq'}^t$ and Y_t^S are derived from (49), (49') and (51) by substituting $U^{(+)}_{qq'}$ and $f^{t}(qq')$ by $U^{(-)}_{qq'}$ and $f^{t}_{s}(qq')$, and y^{ts}_{p} is derived from (50) by substituting $X^{t}(n)$ by $S^{t}(n)$ and $X^{t}(p)$ by $S^{t}(p)$.

If we take into account the multipole-multipole and spin-multipole-spin-multipole forces simultaneously in the particle-hole channel for $\lambda = S$, then for $\varkappa_{1}^{(\lambda)} = \varkappa_{\sigma 1}^{(S)} = 0$ we have the following expression:

$$(1-\varkappa_{0}^{(\lambda)}X^{t})(1-\varkappa_{\sigma_0}^{(S)}S^{t}) = \varkappa_{0}^{(\lambda)}\varkappa_{\sigma 0}^{(S)}(W^{t})^{2}, \tag{55}$$

where

$$W^{t} = 2\sum \frac{f_{s}^{t}(qq')f^{t}(qq')U^{(+)}_{qq'}U^{(-)}_{qq'}\omega_{t}}{\mathcal{E}^{2}(qq')-\omega_{t}^{2}} . \tag{56}$$

3. Let us consider the one-phonon states when the multipole-multipole forces are taken into account in the particle-particle and particle-hole channel simultaneously. We introduce the phonon operators, and if $G_{\lambda} \equiv G_{\lambda}(n) = G_{\lambda}(p) = G_{\lambda}(pn)$ we write the corresponding part of the Hamiltonian (34) in the form

$$H_{v}^{pp} = -\frac{G_{\lambda}}{2}\sum_{i}\Big\{\Big(\sum_{qq'}\tilde{\tilde{f}}^{\lambda\mu}(qq')g^{\lambda\mu i}_{qq'}v^{(-)}_{qq'}\Big)^{2} + \Big(\sum_{qq'}\tilde{\tilde{f}}^{\lambda\mu}(qq')w^{\lambda\mu i}_{qq'}v^{(+)}_{qq'}\Big)^{2}\Big\}\cdot \tag{57}$$
$$\cdot Q_{t}^{+}Q_{t} .$$

If the multiplle-multipole forces are taken into consideration in the particle-particle and particle-hole channel simultaneously, then the secular equation is

$$\begin{vmatrix} X^{\lambda\mu i} - \frac{1}{\varkappa^{(\lambda)}} & \mathcal{L}_1^{\lambda\mu i} & \mathcal{L}_2^{\lambda\mu i} \\ \mathcal{L}_1^{\lambda\mu i} & M^{(-)}_{\lambda\mu i} - \frac{1}{G_\lambda} & M_{\lambda\mu i} \\ \mathcal{L}_2^{\lambda\mu i} & M_{\lambda\mu i} & M^{(+)}_{\lambda\mu i} - \frac{1}{G_\lambda} \end{vmatrix} = 0, \tag{58}$$

where

$$M^{(-)}_{\lambda\mu i} = 2\sum_{q_1 q_2} \frac{(\bar{\bar{f}}^{\lambda\mu}(q_1,q_2)\, v^{(-)}_{q_1 q_2})^2 \mathcal{E}(q_1,q_2)}{\mathcal{E}^2(q_1,q_2) - \omega_i^2}, \tag{59}$$

$$M^{(+)}_{\lambda\mu i} = 2\sum_{q_1 q_2} \frac{(\bar{\bar{f}}^{\lambda\mu}(q_1,q_2)\, v^{(+)}_{q_1 q_2})^2 \mathcal{E}(q_1,q_2)}{\mathcal{E}^2(q_1,q_2) - \omega_i^2}, \tag{59'}$$

$$M_{\lambda\mu i} = 2\sum_{q_1 q_2} \frac{(\bar{\bar{f}}^{\lambda\mu}(q_1,q_2))^2 v^{(-)}_{q_1 q_2} v^{(+)}_{q_1 q_2}\, \omega_i}{\mathcal{E}^2(q_1,q_2) - \omega_i^2}, \tag{59''}$$

$$\mathcal{L}_1^{\lambda\mu i} = 2\sum_{q_1 q_2} \frac{f^{\lambda\mu}(q_1,q_2) U^{(+)}_{q_1 q_2}\, \bar{\bar{f}}^{\lambda\mu}(q_1,q_2) v^{(-)}_{q_1 q_2} \mathcal{E}(q_1,q_2)}{\mathcal{E}^2(q_1,q_2) - \omega_i^2}, \tag{60}$$

$$\mathcal{L}_2^{\lambda\mu i} = 2\sum_{q_1 q_2} \frac{f^{\lambda\mu}(q_1,q_2) U^{(+)}_{q_1 q_2}\, \bar{\bar{f}}^{\lambda\mu} v^{(+)}_{q_1 q_2}\, \omega_i}{\mathcal{E}^2(q_1,q_2) - \omega_i^2} \tag{60'}$$

Equation (58) can easily be derived from eq.(20) by substituting G^{ω} and G^{ξ} by the expressions from H_{g} (43) and H^{σ}_{pp} (57). The analogous expressions have been obtained in ref.[33] for spherical nuclei.

4. Phonon Description

1. The secular equation, determining the energies of one-

phonon states with fixed K^{π} ($K^{\pi} \neq 0^{+}$), for the multipole-multipole forces is the following:

$$2\mathcal{X}_{0}^{(\lambda)} \sum_{qq'} \frac{(f^{\lambda\mu}(qq')\, U_{qq'}^{(+)})^{2} \mathcal{E}(qq')}{\mathcal{E}^{2}(qq') - \omega_{i}^{2}} = 1 \qquad (61)$$

For each solution ω_i of eq.(61) the wave function has the form (44). The number of roots of this equation is equal to the number of two-quasiparticle states with the same values of K^{π} in the neutron and proton system. The energies of two- quasiparticle states are the poles of eq.(61). If the root ω_i is far from the corresponding pole, then the state is collective. As the root approaches the pole the state becomes the two-quasiparticle one. In many cases the roots ω_i are rather close to the poles $\mathcal{E}(qq')$, and the states are weakly collectivized.

In the secular eq.(61) the interaction between quasiparticles in the particle-hole channel is taken into account. If q is the particle state and q' is the hole state, then $(U_{qq'}^{(+)})^2 \geq 0.5$, in many cases $(U_{qq'}^{(+)})^2$ being approximately equal to unity. If both single-particle states q and q' are either particle or hole states, the quantitites $(U_{qq'}^{(+)})^2$ are very small and such states appear as purely two-quasiparticle states. These states do not in fact influence the collective properties of nuclei. To take these states into account, one should introduce interactions in the particle-particle channel.

Thus, the roots of the secular equation (61) and the corresponding one-phonon wave functions describe the whole system of states with given K^{π}. They involve the collective, weakly collective and two-quasiparticle states.

The secular equation of the type (61) is widely used for the calculation of energies of the first quadrupole and octupole collective states. The isoscalar constants $\varkappa^{(\lambda)}$ are fixed by the requirement for the calculated and experimental values of the first state energies to be close to each other. In deformed nuclei one and the same constant is used for the description of one-phonon states of all nuclei in each zone of A /34/. The study of the low-lying states allows one to fix the parameters of the Saxon-Woods potential, the pairing constants, the isoscalar constants of the quadrupole-quadrupole $\varkappa_0^{(2)}$ and octupole-octupole $\varkappa_0^{(3)}$ interactions. The isovector constants $\varkappa_1^{(\lambda)}$ are determined from the isovector resonance energies. One and the same ratio $\varkappa_1^{(\lambda)}/\varkappa_0^{(\lambda)}$ is used for a large group of deformed and spherical nuclei.

2. The definition of phonons is generalized within the quasiparticle-phonon model. The collective and weakly collective as well as two-quasiparticle states are described by means of phonons. This generalization is performed along two lines: the first one implies the calculation of all (not only the first) roots of secular equations of the type (61) and their wave functions are treated as the one-phonon functions, the second one means the description of the one-phonon states with any values of K^{π} in deformed nuclei and any I^{π} in spherical nuclei, the multipole-multipole and spin-multipole-spin-multipole forces with any λ and S including large multipolarities are introduced. All states with fixed K^{π} and I^{π} can easily be treated as the one-phonon ones if the constants $\varkappa_0^{(\lambda)}$ and $\varkappa_1^{(\lambda)}$ in eq.(47) or the constants $\varkappa_{\sigma 0}^{(\lambda)}$ and $\varkappa_{\sigma 1}^{(\lambda)}$ in eq.(53) are fixed. In deformed nuclei it is unnecessary to take into account interactions in the particle-particle chan-

nel except for the description of states with $K^{\pi}=0^{+}$. In spherical nuclei for the description of 0^{+} and some 2^{+} states one should take into account the interactions in the particle-particle channel and therefore solve a secular equation of the type (58) (see ref.[33]).

The use of equations of the type (47) and (53) and of the wave functions (44) for the description of states in deformed nuclei with $K^{\pi}=1^{+},3^{+},4^{+},5^{+},\ldots$ and $4^{-},5^{-},6^{-},\ldots$ and in spherical nuclei with the corresponding values of I^{π} requires the introduction of new constants $\varkappa_0^{(\lambda)}$, $\varkappa_1^{(\lambda)}$, $\varkappa_{\sigma_0}^{(s)}$ and $\varkappa_{\sigma_1}^{(s)}$. These constants are fixed rather arbitrary[35]. This arbitrariness is due to the fact that nonrotational states of high multipolarity are poorly studied experimentally. Are there strongly collectivized states of high multipolarity? This question is still open. It should be noted that if there exist such states, they can hardly be observed experimentally. Obviously, there is an upper limit for the constants $\varkappa_0^{(\lambda)}$ and $\varkappa_{\sigma_0}^{(s)}$. In even-even nuclei there are many low-lying states with K^{π} or I^{π} equal to $1^{+},3^{+},4^{-},\ldots$, and the choice of $\varkappa_0^{(\lambda)}$ and $\varkappa_{\sigma_0}^{(s)}$ is limited since these states should not lower very much.

When studying the states with high multipolarity one should take into account that besides maximum at energy where the poles corresponding to the matrix elements with $\lambda=\Delta N$ dominate, there should exist maxima at lower energies with the poles corresponding to the matrix elements with $\Delta N<\lambda$. For instance, the low-lying octupole resonances with the energy 5-10 MeV have been detected[36] in some spherical and deformed nuclei, the matrix elements with $\Delta N=1$ dominating in their wave functions. It is interesting that the calculations performed in refs.[37,38] without any fitting have confirmed the existence of the low-lying octupole resonances in spherical and deformed nuclei.

For deformed nuclei one should remember that the multipole-multipole interactions with large λ describe phonons not only with $\lambda = K$ but also with $K < \lambda$. Therefore the phonons with fixed K are determined by the multipole-multipole forces with $\lambda = K$, $K+2$, $K+4$, ... The constants $\varkappa_0^{(\lambda)}$ for large λ should be chosen as to prevent a strong change of states with $K < \lambda$ which are determined by the interaction with smaller multipolarity λ. For instance, when studying the $K^{\pi} = 2^+$ states one may also take into account the multipole-multipole forces with $\lambda = 4, 6, \ldots$. The constants $\varkappa_0^{(4)}$, $\varkappa_0^{(6)}$, ... should be taken such that the energy and structure of the first $K^{\pi} = 2^+$ states determined by the forces with $\lambda = 2$ should not be changed strongly. There is no experimental information concerning the influence on the one-phonon states of forces with different λ. It may be assumed that due to the arbitrariness in introduction of interactions in nuclei the states with fixed K^{π} are studied on the basis of the multipole-multipole (spin-multipole-spin-multipole) forces with the only value λ. When calculating quantities of the type $B(E\lambda)$ one should take into account the transitions to the rotational states with $I = \lambda$ and $K < \lambda$ since they give the coherent contribution.

If the multipole and spin-multipole forces are taken into account simultaneously, then the secular equation has a more complex form (55). The study has shown that the spin-multipole forces slightly influence the first quadrupole and octupole states in deformed and especially spherical nuclei. The spin-multipole forces have not been experimentally detected in these states. Since the choice of interaction is arbitrary, it is unnecessary to take into account the multipole and spin-multipole forces simultaneously when calculating the one-phonon states.

It may be confirmed that to find the energies of the one-phonon states it is sufficient to solve equations of the type (47) and (53) and sometimes (58) instead of solving the secular equation (20).

3. The construction of the model Hamiltonian is very arbitrary. The arbitrariness is due to the form of the average field potential as well as to that of the residual forces. Therefore, we may introduce some limitations to the description of one-phonon states. Further, some of these limitations can be removed due to the availability of the corresponding experimental data or theoretical considerations.

Now let us formulate the following rules for the description of one-phonon states with fixed K^{π} in deformed nuclei and I^{π} in spherical nuclei.

1) To find the energies the following equations are solved:

a) The secular equations (47) with the multipole-multipole forces with the minimal values of λ.

b) The secular equations (53) with the spin-multipole-spin-multipole forces with the minimal values of s if the multipole forces do not exist or they are of higher multipolarity.

2) The forces of different multipolarities or multipole and spin-multipole forces are not taken into account simultaneously.

3) In deformed nuclei when calculating the $B(E\lambda)$ values, spectroscopic factors and other functions, the transitions to the rotational states are calculated, i.e.,to all the states with $I=\lambda$ and different values of K .

4) The interaction in the particle-hole channel is taken into account. The interaction in particle-particle channel is taken into account in the calculation of: a) the 0^+ states in all nuclei, b) the 2^+ states in individual spherical nuclei (equation of the type (58)).

5) The isoscalar constants $\varkappa_0^{(\lambda)}$ for $\lambda < 4$ are determined from the first state energy, and for $\lambda \leq 4$ they are taken so small as to prevent the lowering and strong collectiveness of the first states . The ratio of $\varkappa_1^{(\lambda)}/\varkappa_0^{(\lambda)}$ is determined: a) from the position of the corresponding isovector resonance, b) from the phenomenological estimates.

In some cases the secular equations can be complicated in order to exclude the spurious states.

The good agreement of the calculated density of nuclear states[39] with the experimental data at the neutron binding energy B_n justifies the completeness of the phonon space.

5. The Quasiparticle-Phonon Interaction

1. In the quasiparticle-phonon nuclear model all two-quasiparticle and vibrational states are given in terms of the phonon operators. If there is no interaction between phonons, the whole set of nonrotational states of even-even nucleus is

written as a series of one-phonon states, a series of two-phonon states and a series of n -phonon states. The set of nonrotational states of odd-A nucleus is given as a series of one-quasiparticle states, a series of quasiparticle plus phonon states, a series of quasiparticle plus two-phonon states, and so on. The nonrotational states of odd-odd nuclei comprise a series of states with proton and neutron quasiparticles which then are added by one, two and more phonons. Such a picture of excited states has been used in ref.[39] for the calculation of the density of excited states at different excitation energies up to the neutron binding energy B_n ,and the good agreement with experiment has been obtained.

The set of noninteracting quasiparticles and phonons gives a wrong picture of nuclear excited states.

The correct wave functions of nuclear excited states are written as superpositions of components with different number of phonons. The wave function components which differ by one phonon are related by the interaction of quasiparticles with phonons. If phonons are fixed, the corresponding parts of the multipole-multipole and spin-multipole-spin-multipole forces which describe the quasiparticle-phonon interactions are uniquely determined. If the secular equations for phonons are solved, all model parameters turn out to be fixed. The larger the quasiparticle-phonon interaction connecting, for instance, the one-quasiparticle and quasiparticle plus phonon states the stronger a phonon is collectivized.

The quasiparticle-phonon interaction has the following advantages as compared to other types of effective interactions:

1) A consistent description of quasiparticle and phonon states and their coupling.

2) A unique choice of the form and constants of the interaction.

3) Applicability for the description at low, intermediate and high excitation energies.

2. The Hamiltonian of the multipole-multipole interaction H_Q (36) comprises, besides a part entering into (43) which is used for the calculation of one-phonon states, the terms containing the operators of the form

$$\alpha^+_{q\sigma} \alpha_{q'\sigma'} (Q^+_t + Q_t),$$

which are responsible for the description of the quasiparticle-phonon interaction. We denote the corresponding part of the Hamiltonian (36) by H'_{vq} and write as follows:

$$\begin{aligned}
H'_{vq} = & -\frac{1}{2\sqrt{2}} \sum_t \Big\{ (\varkappa_0^{(\lambda)} + \varkappa_1^{(\lambda)}) \Big[\sum_{ss'} f^t(ss') U^{(+)}_{ss'} g^t_{ss'} \sum_{s_2 s'_2} v^{(-)}_{s_2 s'_2} f^t(s_2 s'_2) \\
& ([Q^+_t + Q_t] B(s_2 s'_2) + B(s_2 s'_2) [Q^+_t + Q_t]) + \\
& + \sum_{rr'} f^t(rr') U^{(+)}_{rr'} g^t_{rr'} \sum_{r_2 r'_2} f^t(r_2 r'_2) v^{(-)}_{r_2 r'_2} ([Q^+_t + Q_t] B(r_2 r'_2) + \\
& + B(r_2 r'_2) [Q^+_t + Q_t]) \Big] + \\
& + (\varkappa_0^{(\lambda)} - \varkappa_1^{(\lambda)}) \Big[\sum_{ss'} f^t(ss') U^{(+)}_{ss'} g^t_{ss'} \sum_{rr'} f^t(rr') v^{(-)}_{rr'} ([Q^+_t + Q_t] B(rr') + \\
& + B(rr') [Q^+_t + Q_t]) + \sum_{rr'} f^t(rr') U^{(+)}_{rr'} g^t_{rr'} \cdot
\end{aligned} \tag{62}$$

$$\cdot \sum_{ss'} f^t(ss') v_{ss'}^{(-)} ([Q_t^+ + Q_t] B(ss') + B(ss)[Q_t^+ + Q_t])] \}$$

Let us transform H'_{vq} taking into account the fact that the one-phonon energies are determined from the solutions of secular equations (47) and their wave functions are expressed through $g_{qq'}^t$ and $w_{qq'}^t$, in the form of (49')-(51'). As a result, we obtain

$$H_{vq} = -\frac{1}{2} \sum_t \{ \sum_{ss'} \Gamma_{ss'}^t(n) [B(ss')(Q_t^+ + Q_t) + (Q_t^+ + Q_t) B(ss')] +$$

$$+ \sum_{\tau\tau'} \Gamma_{\tau\tau'}^t(p) [B(\tau\tau')(Q_t^+ + Q_t) + (Q_t^+ - Q_t) B(\tau\tau')] \} , \tag{63}$$

where

$$\Gamma_{ss'}^t(n) = \frac{v_{ss'}^{(-)}}{2\sqrt{Y_t}} f^t(ss') ,$$

$$\Gamma_{\tau\tau'}^t(p) = \frac{v_{\tau\tau'}^{(-)}}{2\sqrt{Y_t}} y_p^t f^t(\tau\tau') , \tag{64}$$

Y_t and y_p^t being defined by eqs.(51) and (50).

For the spin-multipole interaction, to the Hamiltonian H_v^s (52) one should add a part corresponding to the quasiparticle-phonon interaction in the form

$$H^{s}_{\nu q} = \frac{1}{2}\sum_{g}\Big\{\sum_{ss'}\Gamma^{tn}_{ss'}(n)\big[\mathcal{B}(ss')(Q^{+}_{t}+Q_{t})+(Q^{+}_{t}+Q_{t})\mathcal{B}(ss')\big]+ \tag{65}$$

$$+\sum_{\tau\tau'}\Gamma^{tn}_{\tau\tau'}(p)\big[\mathcal{B}(\tau\tau')(Q^{+}_{t}+Q_{t})+(Q^{+}_{t}+Q_{t})\mathcal{B}(\tau\tau')\big]\Big\},$$

where

$$\Gamma^{ts}_{ss'}(n) = \frac{1}{2}\frac{v^{g(-)}_{ss'}}{\sqrt{Y^{s}_{t}}}\, f_{s}(ss'),$$

$$\Gamma^{ts}_{\tau\tau'}(p) = \frac{1}{2}\frac{v^{g(+)}_{\tau\tau'}}{\sqrt{Y^{s}_{t}}}\, y^{t}_{p} f^{t}_{s}(\tau\tau'). \tag{66}$$

For the multipole-multipole forces with non-zero moment of the particle-particle channel for the isoscalar interaction the corresponding part of the Hamiltonian is

$$H^{pp'}_{\nu q} = \frac{G_{\lambda}}{2\sqrt{2}}\sum_{i}\Big(\Big\{\sum_{qq'}\bar{\bar{f}}^{\lambda\mu}(qq')\big[g^{t}_{qq'}v^{(-)}_{qq'}+w^{t}_{qq'}v^{(+)}_{qq'}\big]Q^{+}_{t}+$$

$$+\sum_{qq'}\bar{\bar{f}}^{\lambda\mu}(qq')\big[g^{t}_{qq'}v^{(-)}_{qq'}-w^{t}_{qq'}v^{(+)}_{qq'}\big]Q_{t}\Big\}\sum_{q_2q_2'}f^{\lambda\mu}(q_2q_2')(U^{(+)}_{q_2q_2'}+U^{(-)}_{q_2q_2'})B(qq')+\text{e.c.}\Big). \tag{67}$$

The model Hamiltonian taking into account the secular equations for phonons is the following:

$$H_M = \sum_q \varepsilon(q) B(qq') - \frac{1}{2}\sum_t \frac{1}{Y_t}\Big\{ \sum_{ss'} \frac{(f^t(ss')U^{(+)}_{ss'})^2 \varepsilon(ss')}{\varepsilon^2(ss') - \omega_t^2} +$$

$$+ y_p^t \sum_{rr'} \frac{(f^t(rr')U^{(+)}_{rr'})^2 \varepsilon(rr')}{\varepsilon^2(rr') - \omega_t^2} \Big\} Q_t^+ Q_t -$$

$$- \frac{1}{2}\sum_t \sum_{qq'} \Gamma^t_{qq'} \{ B(qq')(Q_t^+ + Q_t) + (Q_t^+ + Q_t) B(qq') \} - \tag{68}$$

$$- \frac{1}{2}\sum \frac{1}{Y_t^s} \Big\{ \sum_{ss'} \frac{(f_s^t(ss')U^{(+)}_{ss'})^2 \varepsilon(ss')}{\varepsilon^2(ss') - \omega_t^2} + y_p^{ts} \sum \frac{(f_s^t(rr')U^{(+)}_{rr'})^2 \varepsilon(rr')}{\varepsilon^2(rr') - \omega_t^2} \Big\} Q_t^+ Q_t +$$

$$+ \frac{1}{2}\sum_t \sum_{qq'} \Gamma^{ts}_{qq'} \{ \mathcal{B}(qq')(Q_t^+ + Q_t) + (Q_t^+ + Q_t) \mathcal{B}(qq') \}$$

Here

$\Gamma^t_{qq'}$ is equal to $\Gamma^t_{ss'}(n)$ and $\Gamma^t_{rr'}(p)$,

$\Gamma^{ts}_{qq'}$ is equal to $\Gamma^{ts}_{ss'}(n)$ and $\Gamma^{ts}_{rr'}(p)$.

With additional consideration of interactions in the particle-particle channel with moment different from zero, the Hamiltonian (68) should be added by the terms (57) and (67) transformed in accordance with the secular equation (58).

6. The System of Basic Equations and Solutions

To obtain the basic equations of the model with the Hamiltonian (68), we use the variational principle. Let us consider first odd-mass deformed nuclei. The wave function of a nucleus with an odd number of neutrons can be represented in the form of expansion

$$\Psi_n(K^\pi)=\frac{1}{\sqrt{2}}\sum_\sigma\Big\{\sum_s C_s^n \alpha^+_{s\sigma}+\sum_g D_g^n(\alpha^+ Q^+)_g+ \tag{69}$$
$$+\frac{1}{\sqrt{2}}\sum_G F_G^n(\alpha^+ Q^+ Q^+)_G+\dots\Big\}\Psi_0,$$

where Ψ_0 is the wave function of the ground state of an even-even nucleus which has one nucleon less, determined by formula (44), and n is the number of an excited state with given K^π, $g=qt$, $G=qt_1t_2$, $t=\lambda\mu i$.

Following the accepted procedure we find the average value H_M over the state (69), and on the basis of the variational principle obtain a chain of coupled equations. In ref.[40] it has been shown that this chain of equations is equivalent to that for the corresponding Green functions. The cut-off of a chain for the wave functions (69) corresponds to the cut-off of equations for the Green functions.

The system of equations with the wave function (69) involving all the terms up to the quasiparticle plus three phonon terms is given in refs.[9,10] and studied in refs.[41-43]. The corresponding equations for spherical nuclei have been derived in ref.[44].

Let us describe the multipole and spin-multipole interactions. To this end we introduce the following notation[45]:

$$\Gamma_{qq'}=\begin{cases}\Gamma^t_{qq'} & \text{for the multipole interactions}\\ -\Gamma^{ts}_{qq'} & \text{for the spin-multipole interactions}\end{cases} \tag{70}$$

$$\Gamma_{g G} = \begin{cases} \frac{1}{\sqrt{2}} \left\{ \Gamma^{t_2}_{g g_2} \delta_{t, t_1} + \Gamma^{t_1}_{g g_2} \delta_{t, t_2} \right\} & \text{for the multipole interactions} \\ -\frac{1}{\sqrt{2}} \left\{ \Gamma^{t_2 S}_{g g_2} \delta_{t t_1} + \Gamma^{t_1 S}_{g g_2} \delta_{t, t_2} \right\}, & \text{for the spin-multipole interactions} \end{cases} \quad (71)$$

The wave function of an odd-N nucleus is given in the form

$$\Psi_n (K^\pi) = \frac{1}{\sqrt{2}} \sum_\sigma \Big\{ \sum_s C^n_s \alpha^+_{s\sigma} + \sum_g D^n_g (\alpha^+ Q^+)_g + \frac{1}{\sqrt{2}} \sum_G F^n_G (\alpha^+ Q^+ Q^+)_G \Big\} \Psi_0 , \quad (72)$$

with the normalization condition

$$\sum_s (C^n_s)^2 + \sum_g (D^n_g)^2 + \sum_G (F^n_G)^2 = 1. \quad (73)$$

The expectation value H_M (68) over the state (72) has the form

$$(\Psi^*_n (K^\pi) H_M \Psi_n(K^\pi)) = \sum_s \mathcal{E}(s) (C^n_s)^2 + \sum_g P(g) (D^n_g)^2 + \sum P(G) (F^n_G)^2 + 2 \sum_{s,g} \Gamma_{sg} C^n_s D^n_g - 2 \sum_{g,G} \Gamma_{gG} D^n_g F^n_G , \quad (74)$$

where the fundamental poles are denoted by

$$P(g) = \mathcal{E}(q) + \omega_t ,$$
$$P(G) = \mathcal{E}(q) + \omega_{t_1} + \omega_{t_2} .$$

Using the variational principle

$$\delta \Big\{ (\Psi^*_n (K^\pi) H_M \Psi_n (K^\pi)) - \eta_n [(\Psi^*_n (K^\pi) \Psi_n (K^\pi)) - 1] \Big\} = 0. \quad (75)$$

we obtain the following system of equations:

$$(P(g) - \eta_n) D^n_g - \sum_s \Gamma_{sg} C^n_s - \sum_G \Gamma_{gG} F^n_G = 0 , \quad (76)$$

$$(\mathcal{E}(s) - \eta_n) C^n_s - \sum_g \Gamma_{sg} D^n_g = 0 , \quad (77)$$

$$(P(G)-\eta_n)F_G^n-\sum_g \Gamma_{gG}D_g^n=0. \qquad (78)$$

Now we introduce the notation

$$K(g,g')=\sum_s\frac{\Gamma_{sg}\Gamma_{sg'}}{\mathcal{E}(s)-\eta_n}+\sum_G\frac{\Gamma_{gG}\Gamma_{g'G}}{P(G)-\eta_n} \qquad (79)$$

and rewrite equation (76) as

$$(P(g)-\eta_n)D_g^n-\sum_{g'}K(g,g')D_{g'}^n=0 \qquad (76')$$

The secular equation has the form

$$\theta(\eta_n)=\det\|\delta_{gg'}(P(g)-\eta_n)-K(g,g')\|=0. \qquad (80)$$

The rank of the determinant is equal to the number of the quasiparticle plus phonon components in the wave function (72). If one takes rather a large space of single-particle states, the rank of this determinant is of an order of 10^4-10^5. It has been shown in ref.[41] that the determinant of the system of equations (76) can be represented as follows:

$$\prod_g(P(g)-\eta)^{-1}\cdot\theta(\eta)=1-\sum_s\frac{A_s}{\mathcal{E}(s)-\eta}-\sum_g\frac{A_g}{P(g)-\eta}-\sum_G\frac{A_G}{P(G)-\eta}, \qquad (80')$$

where the coefficients A_s, A_g and A_G, which are the sums of determinants of different ranks, are independent of η. Therefore, the secular equation

$$\theta(\eta)=0$$

comprises the poles of the first order only.

The above system of equations is used to study the fragmentation of single-particle states (see refs.[46,47]). To study the fragmentation of an individual single-particle state S_0 , we transform it by introducing the following functions:

$$\tilde{C}_{\xi}^{n} = \frac{C_{\xi}^{n}}{C_{S_0}^{n}}, \quad \tilde{D}_{g}^{n} = \frac{D_{g}^{n}}{C_{S_0}^{n}}, \quad \tilde{F}_{G}^{n} = \frac{F_{G}^{n}}{C_{S_0}^{n}},$$

where $\xi \neq S_0$. The system of equations (76),(77) and (78) can be rewritten as (see ref.[46])

$$\tilde{\mathcal{F}}_{S_0}(\eta_n) = \mathcal{E}(S_0) - \eta_n - \sum_g \Gamma_{S_0 g} \tilde{D}_g^n = 0, \tag{81}$$

$$(\mathcal{E}(\xi) - \eta_n)\tilde{C}_{\xi}^n - \sum_g \Gamma_{\xi g} \tilde{D}_g^n = 0, \tag{82}$$

$$(P(g) - \eta_n)\tilde{D}_g^n - \sum_{\xi} \Gamma_{\xi g} \tilde{C}_{\xi}^n - \sum_G \Gamma_{gG} \tilde{F}_G^n = \Gamma_{S_0 g}, \tag{83}$$

$$(P(G) - \eta_n)\tilde{F}_G^n - \sum_g \Gamma_{gG} \tilde{D}_g^n = 0. \tag{84}$$

The normalization condition of the wave function (72) is rewritten as follows:

$$(C_{S_0}^n)^{-2} = 1 + \sum_{\xi} (\tilde{C}_{\xi}^n)^2 + \sum (\tilde{D}_g^n)^2 + \sum_{G} (\tilde{F}_G^n)^2 .$$

Equation

$$(P(g) - \eta_n)\tilde{D}_g^n = \sum K_{S_0}(g,g')\tilde{D}_{g'}^n = \Gamma_{S_0 g}, \tag{83'}$$

where

$$K_{S_0}(g,g') = \sum_{\xi} \frac{\Gamma_{\xi g}\Gamma_{\xi g'}}{\mathcal{E}(\xi) - \eta_n} + \sum_G \frac{\Gamma_{gG}\Gamma_{g'G}}{P(G) - \eta_n}, \tag{79'}$$

corresponds to eq.(76).

Let us denote the determinant of the system (83') by $\Theta(S_0, \eta)$. The relation

$$(C^n_{s_c})^{-2} = -\left.\frac{\partial \mathcal{F}_{s_c}(\eta)}{\partial \eta}\right|_{\eta=\eta_n} \tag{85}$$

is strictly fulfilled.

The solution of equation (83') has the form

$$\tilde{D}^n_g = \frac{\theta_g(s_c;\eta)}{\theta(s_c;\eta)}, \tag{86}$$

where $\theta_g(s_c;\eta)$ is obtained from $\theta(s_c;\eta)$ by substituting the column g by free terms (83'). Let us substitute (86) into (81) and after transformations we have

$$\mathcal{F}_{s_c}(\eta) = \frac{1}{\theta(s_c;\eta)}\begin{vmatrix} \varepsilon(s_c)-\eta & \Gamma_{s_c g_1} & \dots & \Gamma_{s_c g_N} \\ \Gamma_{s_c g_1} & \rho(g_1)-\eta-K_{s_c}(g_1 g_1) & \dots & -K_s(g_1 g_N) \\ \dots & \dots & \dots & \dots \\ \Gamma_{s_c g_N} & -K_{s_c}(g_N g_1) & \dots & \rho(g_N)-\eta-K_{s_c}(g_N g_N) \end{vmatrix}, \tag{87}$$

where N is the number of states g. Now we calculate the determinant and rearrange the terms. So, we have

$$\mathcal{F}_{s_c}(\eta) = (\varepsilon(s_c)-\eta)\,\frac{\theta(\eta)}{\theta(s_c;\eta)}. \tag{87'}$$

Let us find the function $\tilde{C}^n_{\bar{s}}$ and substitute (86) into (82)

$$\tilde{C}^n_{\bar{s}} = \frac{1}{\varepsilon(\bar{s})-\eta}\sum_g \Gamma_{\bar{s}g}\tilde{D}^n_g = \frac{1}{(\varepsilon(\bar{s})-\eta_n)\theta(s_c;\eta_n)}\sum_g \Gamma_{\bar{s}g}\theta_g(s_c;\eta_n)$$

We calculate the determinant, rearrange the terms, and then obtain

$$\tilde{C}_{\bar{s}}^{n} = -\frac{1}{(\mathcal{E}(\bar{s})\eta_n)\,\theta(s_0;\eta_n)} \begin{vmatrix} 0 & \Gamma_{\bar{s},g_1} & \cdots & \Gamma_{\bar{s},g_N} \\ \Gamma_{s_0 g_1} & p(g_1)-\eta_n-K_{s_0}(g_1 g_1) & \cdots & -K_{s_0}(g_1 g_N) \\ \cdots & \cdots & \cdots & \cdots \\ \Gamma_{s_0 g_N} & -K_{s_0}(g_N, g_1) & \cdots & p(g_N)-\eta_n-K_{s_0}(g_N g_N) \end{vmatrix} \equiv \frac{\Delta(s_0,\bar{s};\eta_n)}{\theta(s_0,\eta_n)}. \tag{88}$$

Let us give formulae for a simplified case when the wave function has the form

$$\Psi_n(K^{\pi}) = \frac{1}{\sqrt{2}}\sum_{\sigma}\Big\{\sum_{s} C_s^n \alpha_{s\sigma}^{+} + \sum_{g} D_g^n (\alpha^{+} Q^{+})_g\Big\}\Psi_0 \tag{89}$$

and its normalization is

$$1 = \sum_{s}(C_s^n)^2 + \sum_{g}(D_g^n)^2 .$$

In this case the equations can be written as

$$(\mathcal{E}(s)-\eta_n)C_s^n - \sum_{g}\Gamma_{sg} D_g^n = 0, \tag{90}$$

$$(p(g)-\eta_n)D_g^n - \sum_{s}\Gamma_{sg} C_s^n = 0 . \tag{91}$$

They can be rewritten as

$$(\mathcal{E}(s)-\eta_n)C_s^n - \sum_{s'} K(ss')C_{s'}^n = 0, \tag{92}$$

where

$$K(ss') = \sum_{g}\frac{\Gamma_{sg}\Gamma_{s'g}}{p(g)-\eta_n} \tag{92'}$$

The secular equation has the form

$$\theta(\eta_n) = det \| \delta_{ss'}(\varepsilon(s) - \eta_n) - K(ss') \| = 0 \; . \tag{93}$$

The rank of this determinant equals the number of one-quasiparticle components in the first sum in (89). For deformed nuclei 10-15 terms and for spherical 1-3 are sufficient.

Now we extract the state s_o and write the equations as

$$\mathcal{F}_{s_o}(\eta_n) = \varepsilon(s_o) - \eta_n - \sum_g \Gamma_{s_o g} \tilde{D}^n_g = 0, \tag{94}$$

$$(\varepsilon(\bar{s}) - \eta_n)\tilde{C}^n_{\bar{s}} - \sum_g \Gamma_{\bar{s}g} \tilde{D}^n_g = 0, \tag{95}$$

$$(\rho(g) - \eta_n)\tilde{D}^n_g - \sum_{\bar{s}} \Gamma_{\bar{s}g} \tilde{C}^n_{\bar{s}} = \Gamma_{s_o g} \, , \tag{96}$$

$$(C^n_{s_o})^{-2} = 1 + \sum_{\bar{s}} (\tilde{C}^n_{\bar{s}})^2 + \sum_g (\tilde{D}^n_g)^2. \tag{97}$$

Equations (95) and (96)can be rewritten as

$$\sum_{\bar{s}'} \left\{ (\varepsilon(\bar{s}) - \eta_n)\delta_{\bar{s}\bar{s}'} - K(\bar{s}, \bar{s}') \right\} \tilde{C}^n_{\bar{s}'} = K(s_o, \bar{s}). \tag{95'}$$

The solution of this equation can be written in the form

$$\tilde{C}^n_{\bar{s}} = \frac{\theta_{s_o}(\bar{s})}{\theta_{s_o}} \; , \tag{98}$$

where θ_{s_o} is the determinant of the system (95') and $\theta_{s_o}(\bar{s})$ is obtained from it by substituting the column $\bar{s}$ by the r.h. side of eq.(95'),

$$\mathcal{F}_{s_o}(\eta) = \frac{\theta}{\theta_{s_o}} \; . \tag{99}$$

If only one single-particle state s_0 with given K^{π} is taken into account, then the wave function is represented as

$$\Psi_n(K^{\pi}) = C^n_{s_0} \frac{1}{\sqrt{2}} \sum_{\sigma} \left\{ \alpha^+_{s_0\sigma} + \sum_g \tilde{D}^n_g (\alpha^+ Q^+)_g \right\} \Psi_0 , \qquad (100)$$

and the secular equation and expression for $(C^n_{s_0})^2$ have the form

$$\mathcal{F}_{\rho_0}(\eta_n) = \mathcal{E}(s_0) - \eta_n - \sum_g \frac{\Gamma^2_{s_0 g}}{\rho(g) - \eta_n} = 0, \qquad (100')$$

$$(C^n_{s_0})^{-2} = 1 + \sum_g \frac{\Gamma^2_{s_0 g}}{\rho(g) - \eta_n} . \qquad (100'')$$

In order to find the energies of the states described by the wave function (72), one should diagonalize the matrix of a very high rank (80). Mathematically, this problem is very complex. The approximate methods of solution of equations of the type (80) have been studied in refs./41-43/. The approximations have been found, which describe satisfactorily the largest components of the wave functions (72). For intermediate and high excitation energies, the few-quasiparticle components, we are interested in, compose a small part in the normalization (73) and are poorly described. Therefore, the methods of approximate solutions of equations of the type (80) developed in refs./41-43/ cannot be used to study the fragmentation of single-particle states.

To study the fragmentations of single-particle states and to calculate the neutron strength functions and spectroscopic factors of the one-nucleon transfer reactions, we propose the following approximate approach consisting of four steps.

The first step:

The wave function is taken in the form of (89) and the solution for the secular equation (93) is found. This problem is very simple, and it has been solved in refs.[46-48]. From all the states g in (80) the selection rules choose the set of states g' the number of which is two orders less than the total number of the states g .

The second step:

We select the set of states g' the number of which is equal to the total number of solutions of equation (93) minus the number of one-quasiparticle components in the first term (89). For each solution η_n corresponding to the pole $p(g')$ we find the quantities $(C^n_{s_0})$

The third step:

From the set of states g' we select such a set of states g'' for which the corresponding quantities $(C^n_{s_0})^2$ are larger than the definite value of C_0^2 . If one takes $C_0^2 = 0.002$ then the number of states g'' is 15-20.

The fourth step:

In the determinant (80) we take only the set of states g' and then diagonalize them. Thus we find the state energies and quantities $(C^n_{s_0})^2$. This problem can be solved at computer for many nuclei, since one could diagonalize the matrices of the rank from 10 to 100.

2. Let us consider an even-even deformed nucleus. The model Hamiltonian (68) can be rewritten as follows:

$$H_M = \sum_t \omega_t Q_t^+ Q_t -$$

$$-\frac{1}{2}\sum_t \sum_{qq'} \Gamma_{qq'}^t \left\{ B(qq')(Q_t^+ + Q_t) + (Q_t^+ + Q_t) B(qq') \right\} +$$

$$+\frac{1}{2}\sum_t \sum_{qq'} \Gamma_{qq'}^{ts} \left\{ \mathcal{B}(qq')(Q_t^+ + Q_t) + (Q_t^+ + Q_t) \mathcal{B}(qq') \right\} \tag{101}$$

The wave function can be represented as the expansion

$$\Psi_n(K^\pi) = \left\{ \sum_t R_t^n(\lambda\mu) Q_t^+ + \frac{1}{\sqrt{2}} \sum_{t_1 t_2} P_{t_1 t_2}^n(\lambda\mu) Q_{t_1}^+ Q_{t_2}^+ + \right. \tag{102}$$

$$\left. + \mathcal{T}_{t_1 t_2 t_3}^n(\lambda\mu) Q_{t_1}^+ Q_{t_2}^+ Q_{t_3}^+ + \cdots \right\} \Psi_0 .$$

Now we find the average value H_M over the state (102) and using the variational principle obtain a chain of coupled equations. The case when the wave function takes into account the terms comprising one, two, three and four phonons and neglects those comprising more than four phonons has been investigated in ref.[49] In this paper the system of basic equations has been obtained and the method of their solution has been proposed. Formulae for spherical nuclei have been derived in ref.[50].

Let us study in detail the problem with a simple wave function, which is taken in the form

$$\Psi_n(K^\pi) = \left\{ \sum_i R_i^n(\lambda\mu) Q_t^+ + \frac{1}{\sqrt{2}} \sum_{t_1 t_2} P_{t_1 t_2}^n(\lambda\mu) Q_{t_1}^+ Q_{t_2}^+ \right\} \Psi_0 . \tag{103}$$

Its normalization condition is

$$\sum_i (R_i^n(\lambda\mu))^2 + \sum_{t_1 t_2} (P_{t_1 t_2}^n(\lambda\mu))^2 = 1 \qquad (103')$$

Now we find the average value H_M (101) over the state (103)

$$(\Psi_n^*(K^\pi) H_M \Psi_n(K^\pi)) = \sum_i \omega_i (R_i^n(\lambda\mu))^2 + \\ + \sum_{t_1 t_2} \omega_{t_1 t_2} (P_{t_1 t_2}^n(\lambda\mu))^2 - 2 \sum_{i t_1 t_2} U_{t_1 t_2}(\lambda\mu) R_i^n(\lambda\mu) P_{t_1 t_2}^n(\lambda\mu) , \qquad (104)$$

where $\omega_{t_1 t_2} = \omega_{t_1} + \omega_{t_2}$,

$$U_{t_1 t_2}(\lambda\mu) = \frac{1}{\sqrt{2}} \langle Q_{\lambda\mu i} H_M Q_{t_1}^+ Q_{t_2}^+ \rangle \equiv U_{t_1 t_2}(t) , \qquad (105)$$

for the multipole-multipole interaction the explicit form of $U_{t_1 t_2}(\lambda\mu i)$ is given by formula (9.75) in ref.[16].

Using the variational principle

$$\delta \{ (\Psi_n^*(K^\pi) H_M \Psi(K^\pi)) - \eta_n [(\Psi_n^*(K^\pi) \Psi_n(K^\pi)) - 1] \} = 0, \qquad (106)$$

we derive the system of basic equations

$$(\omega_i - \eta_n) R_i^n - \sum_{t_1 t_2} U_{t_1 t_2}(t) P_{t_1 t_2}^n = 0 , \qquad (107)$$

$$(\omega_{t_1 t_2} - \eta_n) P_{t_1 t_2}^n - \sum_i U_{t_1 t_2}(t) R_i^n = 0 , \qquad (107')$$

or

$$(\omega_i - \eta_n) R_i^n - \sum_{i'} K_{ii'} R_{i'}^n = 0 , \qquad (108)$$

where

$$K_{ii'} = \sum_{t_1 t_2} \frac{U_{t_1 t_2}(\lambda\mu i)\, U_{t_1 t_2}(\lambda\mu i')}{\omega_{t_1 t_2} - \eta_n} . \qquad (108')$$

Therefore the secular equation has the form

$$\theta(\eta_n) = \det \| (\omega_i - \eta_n)\delta_{ii'} - K_{ii'} \| = 0, \tag{109}$$

the rank of the determinant being equal to the number of one-phonon states in the first sum in (103).

Using the normalization condition for the wave function, we derive the following expressions for its coefficients:

$$R_i^n = -\frac{M_{ii}}{N} \tag{110}$$

$$P^n_{t_1 t_2} = \frac{1}{N} \sum_i \frac{U_{t_1 t_2}(\lambda\mu i) M_{ii}}{\omega_{t_1 t_2} - \eta_n}, \tag{110'}$$

where M_{ii} – is the minor of determinant (109),

$$N = \Big(\sum_i (M_{ii})^2 + \sum_{t_1 t_2} \Big\{ \sum_i \frac{U_{t_1 t_2}(\lambda\mu i) M_{ii}}{\omega_{t_1 t_2} - \eta_n} \Big\}^2 \Big)^{-1}. \tag{110''}$$

Let us rewrite equation (108) as

$$\mathcal{F}_{i_0}(\eta_n) = \omega_{i_0} - \eta_n - K_{i_0 i_0} - \sum_{i'} K_{i_0 i'} \tilde{R}^n_{i'} = 0, \tag{108''}$$

$$(\omega_i - \eta_n)\tilde{R}^n_i - \sum_{i'} K_{ii'} \tilde{R}^n_{i'} = K_{i_0 i'}. \tag{108'''}$$

It can easily be shown that

$$\mathcal{F}_{i_0}(\eta) = \frac{\theta(\eta)}{M_{i_0 i_0}}, \tag{109'}$$

$$\tilde{R}^n_i = \frac{R^n_i}{R^n_{i_0}} = \frac{(-1)^{i_0+i} M_{i i_0}}{M_{i_0 i_0}}, \tag{111}$$

$$(R^n_{i_0})^{-2} = -\frac{\partial}{\partial \eta}\left\{\mathcal{F}_{i_0}(\eta)\right\}\Big|_{\eta=\eta_n}, \qquad (111')$$

where the determinant M_{ii_0} is obtained from the minor $M_{i_0 i_0}$ by substituting the i -th column by a column of free terms (108').

It is not very difficult to solve the system (109) and to find the functions R^n_i and $P^n_{t_1 t_2}$. For a limited number of states i and $t_1 t_2$ in deformed nuclei this problem has been solved in ref.[51]. For spherical nuclei this problem has been solved in ref.[37] which also gives the reduced probabilities $B(E\lambda)$ of excitation of the giant multipole resonances. The E1 radiative strength functions for semimagic nuclei have been calculated in ref.[52].

If only one one-phonon state, is taken into account in eq. (103), the wave function has the form

$$\Psi_n(K^\pi) = R^n(\lambda\mu)\left\{Q^+_{t_0} + \sum_{t_1 t_2} P^n_{t_1 t_2}(\lambda\mu) Q^+_{t_1} Q^+_{t_2}\right\}\Psi_0 \quad . \qquad (112)$$

The secular equation and expression for $(R^n(\lambda\mu))^2$ have the form

$$\omega_{t_0} - \eta_n = \frac{1}{2}\sum \frac{(U_{t_1 t_2}(\lambda\mu n))^2}{\omega_{t_1 t_2} - \eta_n}, \qquad (112')$$

$$(R^n(\lambda\mu))^{-2} = 1 - \frac{1}{2}\sum_{t_1 t_2} \frac{(U_{t_1 t_2}(\lambda\mu n))^2}{(\omega_{t_1 t_2} - \eta_n)^2} . \qquad (112'')$$

3. The wave functions (72) and (103) have the terms containing the product of two phonon creation operators. Since the phonon operators are constructed from the product of quasiparticle operators satisfying the fermion commutation rela-

tions, one can observe a certain violation of the Pauli principle in the productions in two phonon operators. The problem of exclusion the terms violating the Pauli principle has been studied in many papers, for instance in refs.[53,54,55]. Let us show that within the quasiparticle-phonon model the problem can be formulated so as to avoid the violation of the Pauli principle.

The mathematical method is demonstrated for the case when the wave functions have not more than two phonons. We introduce the operators of "true" bosons $b^+(q,q')$, $b(q,q')$ satisfying the commutation relations

$$[b(q,q'), b^+(q_2q_2')] = \delta_{qq_2}\delta_{q'q'} + \delta_{qq_2'}\delta_{q'q_2}, \qquad [b(q,q'), b(q_2q_2')] = 0, \tag{113}$$

and the condition

$$b(q,q') = b(q',q). \tag{113'}$$

Using the exact commutation relations, we express the operators $A^+(q,q')$ and $B(q,q')$ by the boson operators as follows:

$$B(q,q') = \sum_{q_2} b^+(q,q_2)b(q',q_2), \tag{114}$$

$$A^+(q,q') = b^+(q,q') + x\, b^+(q,q')\sum_{q_2q_2'} b^+(q_2q_2')b(q_2q_2') + y\sum_{q_2q_2'} b^+(q,q_2)b^+(q'q_2')b(q_2q_2') \tag{114'}$$

where $x = -\frac{3-\sqrt{6}}{6}$, $y = -\frac{1}{\sqrt{6}}$.

We introduce the phonon operators

$$\tilde{Q}_t = \frac{1}{2}\sum_{q,q'}\left\{\psi^t_{qq'}\, b(q,q') - \varphi^t_{qq'}\, b^+(q,q')\right\}. \tag{115}$$

and express the operators of multipole moment (37) by them.

For the sake of simplicity we take the Hamiltonian as

$$H'_M = \sum_q \mathcal{E}(q) B(q,q) - \frac{1}{2}\sum_{\lambda,\mu\neq 0} \varkappa_0^{(\lambda)} \{ Q^+_{\lambda\mu}(n) Q_{\lambda\mu}(n) + Q^+_{\lambda\mu}(p) Q_{\lambda\mu}(p) + 2Q^+_{\lambda\mu}(n) Q_{\lambda\mu}(p) \}, \tag{116}$$

where $Q_{\lambda\mu}(n)$ is defined by formula (37). Taking into account the secular equation (47) we obtain the Hamiltonian, expressed by new phonons $\tilde{Q}_t$, in the following form

$$\begin{aligned} H'_M = \sum_t \omega_t \tilde{Q}^+_t \tilde{Q}_t - \frac{1}{2}\sum_{t_2} \frac{1}{\sqrt{Y_{t_2}}} \sum_{q_2 q'_2 q_3} f^{t_2}(q_2 q'_2) v^{(-)}_{q_2 q'_2} \cdot \\ \sum_{t,t'} [\psi^t_{q_2 q_3} \psi^{t'}_{q'_2 q_3} \tilde{Q}^+_{t'} \tilde{Q}^+_t \tilde{Q}_{t_2} + \varphi^t_{q_2 q_3} \psi^{t'}_{q'_2 q_3} \tilde{Q}^+_{t_2} \tilde{Q}_t \tilde{Q}_{t'} + \\ + (\psi^t_{q_2 q_3} \psi^{t'}_{q'_2 q_3} + \varphi^t_{q'_2 q_3} \varphi^{t'}_{q_2 q_3})(\tilde{Q}^+_{t_2} \tilde{Q}^+_t \tilde{Q}_{t'} + \tilde{Q}^+_{t'} \tilde{Q}_t \tilde{Q}_{t_2})] - \\ - \frac{1}{4}\sum_{\substack{tt' \\ t_2 t'_2}} \{ V^{tt'}_{t_2 t'_2} \tilde{Q}^+_{t'_2} \tilde{Q}^+_t \tilde{Q}_{t'} \tilde{Q}_{t_2} + \tilde{V}^{tt'}_{t_2 t'_2} \tilde{Q}^+_t \tilde{Q}^+_{t'} \tilde{Q}_{t_2} \tilde{Q}_{t'_2} \}. \end{aligned} \tag{116'}$$

where the functions $V^{tt'}_{t_2 t'_2}$ and $\tilde{V}^{tt'}_{t_2 t'_2}$ comprise the sum of productions of matrix elements and functions $\psi^t_{qq'}$ and $\varphi^t_{qq'}$.

We take the wave function as (112) and the average value $H_{M'}$ over this state is

$$\begin{aligned} (\Psi^*_n(K^\pi) H'_M \Psi_n(K^\pi)) = (R^n)^2 \{ \omega_{t_0} + 2\sum_{t,t'} \omega_{tt'} (P^n_{tt'})^2 - \\ - 2\sum_{t,t'} U_{tt'}(t_0) P^n_{tt'} - \sum_{\substack{tt' \\ t_2 t'_2}} [V^{tt'}_{t_2 t'_2} P^n_{tt'_2} P^n_{t' t_2} + \tilde{V}^{tt'}_{t_2 t'_2} P^n_{tt'} P^n_{t_2 t'_2}] \}. \end{aligned} \tag{117}$$

Using the variational principle, we obtain the following system of equations:

$$\omega_{t_0} - \eta_n - \sum_{t,t'} U_{tt'}(t_0) P^n_{tt'} = 0, \tag{118}$$

$$P^n_{tt'} = \frac{1}{2}\frac{U_{tt'}(t_0)}{\omega_{tt'} - \eta_n} + \frac{1}{2}\frac{1}{\omega_{tt'} - \eta_n}\sum_{t_2 t_2'} \left(V^{tt_2}_{t't_2'} + \tilde{V}^{tt'}_{t_2 t_2'} \right) P^n_{t_2 t_2'} \tag{118'}$$

When we take bosons instead of quasibosons we are led to the second term in eq.(118'). If it is neglected and substituted into (118), we obtain the secular equation (112').

Now we extract the coherent terms in eq.(118') and then rewrite this equation as follows:

$$\begin{aligned} \left\{ \omega_{tt'} - \eta_n - \frac{1}{2}\left(V^{tt}_{t't'} + \tilde{V}^{tt'}_{tt'} \right) \right\} P^n_{tt'} = \\ = \frac{1}{2} U_{tt'}(t_0) - \frac{1}{2} \sum_{t_2 t_2' \neq tt'} \left(V^{tt_2}_{t't_2'} + \tilde{V}^{tt'}_{t_2 t_2'} \right) P^n_{t_2 t_2'} \end{aligned} \tag{119}$$

In the first approximation when incoherent terms are neglected in eq.(119) the use of bosons instead of quasibosons means some shift of energies of the two-phonon states. Using the perturbation theory one can take into account noncoherent terms which represent the rescattering of phonons on the phonons.

Therefore, within the quasiparticle-phonon nuclear model one can work with true bosons without violating the Pauli principle. Within each approximation, i.e., in a certain cutoff of expansions in the wave functions (69) and (102), the corresponding expression for the operators $A(q,q')$ and $A^+(q,q')$ is chosen, i.e., the subsequent terms in the expansion (114') are taken and the system of basic equations is found.

7. The Method of Strength Functions

1. The wave functions of the type (72) and (103) at intermediate and high excitation energies of complex nuclei do not describe correctly the state structure due to the absence of many-phonon components in them. So, for instance, to describe the excited states with energy of about 4 MeV in ^{239}U the wave function (69) should comprise the quasiparticle plus four phonon components. When formulating the quasiparticle-phonon model we did not aim at finding the correct wave functions of highly excited states. The quasiparticle-phonon model is formulated so as to obtain the most correct description of few-quasiparticle components of the wave functions averaged over some energy interval.

For intermediate and high excitation energies the results of calculation of the characteristics for each state can hardly be represented clearly. For instance, in ^{239}U at excitation energies of (3-5) MeV, 10-20 poles (and the corresponding solutions) of the quasiparticle plus phonon type are in the interval of 100 keV. Therefore, when calculating the fragmentation of one-quasiparticle states /48,56/, the sums of the type $\sum_n (C^n_{s})^2$ have been calculated for the states lying in the interval from 200 to 400 keV, and the results have been represented as a histogram. The energy of each state has been found, the components (many thousand of them) of the wave functions have been calculated and the value of one of them has been used for the calculation of quantities of the type $\sum_n (C^n_{s_0})^2$. Only a small part of information obtained has been used. Therefore, it became necessary to construct such a mathematical apparatus which could be used for the calculation of aquired quantities in a certain interval of excitation energies. Such an apparatus is the method of strength functions, i.e., the method of direct calculation of averaged characteristics without a detailed calculation of each state. This method has been used in refs.[46,57]

Let us consider the fragmentation of the single-particle state described by the wave function (100) with the secular equation (100') and the expression $(C_{s_0}^n)^2$ in the form as (100").

Now we construct the function

$$\Phi_{s_0}(\eta) = \sum_n (C_{s_0}^n)^2 \rho\,(\eta_n - \eta), \tag{120}$$

where

$$\rho\,(\eta_n - \eta) = \frac{1}{2\pi}\,\frac{\Delta}{(\eta - \eta_n)^2 + (\Delta/2)^2} \tag{121}$$

The way of presentation of the results of calculation depends on the value of the energy interval of averaging Δ. In ref.[58] the strength functions have been calculated using the function $\rho(\eta_n - \eta)$. These functions are widely used in the quasiparticle-phonon nuclear model for the study of the fragmentation of one-quasiparticle states, for the calculation of neutron strength functions and giant multipole resonances[23,32,37,38,46,47,52,59,60]. The results similar to those which are used in the Green function method[61] can be obtained by introducing the functions $\rho(\eta_n - \eta)$. The probabilities of excitation of the giant multipole resonances have been calculated using the expressions which can be obtained by introducing the function $\rho(\eta_n - \eta)$ [26,61].

Bearing in mind formula (85) the function $\Phi_{s_o}(\eta)$ can be written as

$$\Phi_{s_o}(\eta) = -\sum_n \left(\frac{\partial \mathcal{F}_{s_o}(\eta)}{\partial \eta}\right)^{-1}_{\eta=\eta_n} \rho(\eta_n - \eta) \tag{120'}$$

Using the theory of residues, we express the function (120') in terms of the contour integral around the poles which are the solutions of eq.(100'). As a result we obtain

$$\Phi_{s_o}(\eta) = \frac{-1}{2\pi i} \frac{\Delta}{2\pi} \oint \frac{dz}{\mathcal{F}_{s_o}(z)} \frac{1}{(\eta - z)^2 + (\Delta/2)^2}, \tag{122}$$

the contour ℓ_p being represented in Fig.1. Since the contour integral along the circle of infinite radius in the complex plane z is equal to zero, we change the integral over the contour ℓ_p to two contour integrals ℓ_1 and ℓ_2 around the poles $z_1 = \eta + i\Delta/2$, $z_2 = \eta - i\Delta/2$,

$$\Phi_{s_o}(\eta) = \frac{1}{2\pi i} \frac{\Delta}{2\pi} \oint_{\ell_1, \ell_2} \frac{dz}{\mathcal{F}_{s_o}(z)} \frac{1}{(\eta - z)^2 + (\Delta/2)^2} \tag{122'}$$

After simple calculations

$$\Phi_{s_o}(\eta) = -\frac{\Delta}{2\pi} \frac{1}{2(\eta - z)} \frac{1}{\mathcal{F}_{s_o}(z)} \Bigg|_{z = \eta \pm i\Delta/2} = \tag{122''}$$

$$= \frac{1}{\pi} \mathcal{I}m \left(\frac{1}{\mathcal{F}_{s_o}(\eta + i\frac{\Delta}{2})} \right).$$

and using $\mathcal{F}_{s_0}(\eta + i\frac{\Delta}{2})$ as (100'), we obtain

$$\Phi_{s_0}(\eta) = \frac{\Delta}{2\pi} \frac{\Gamma(\eta)}{(\mathcal{E}(s_0)-\gamma(\eta)-\eta)^2+(\Delta/2)^2\Gamma^2(\eta)}, \qquad (123)$$

where

$$\Gamma(\eta) = 1 + \sum_g \frac{\Gamma^2_{s_0 g}}{(p(g)-\eta)^2+(\Delta/2)^2}, \qquad (123')$$

$$\gamma(\eta) = \sum_g \frac{\Gamma^2_{s_0 g}(p(g)-\eta)}{(p(g)-\eta)^2+(\Delta/2)^2} \qquad (123'')$$

The function $\Phi_{s_0}(\eta)$ is given in the Breit-Wigner form. However the dependence of $\Gamma(\eta)$ and $\gamma(\eta)$ on energy η is very important, since it causes a strong deviation from the $\Phi_{s_0}(\eta)$ Breit-Wigner form[47]. Usually[62] the functions of the form (123) with constant Γ and γ are used, and this cannot be justified.

2. The method of strength functions is an important part of the quasiparticle-phonon nuclear model. The method of strength functions has been used in refs.[46,47] to study the main regularities of the fragmentation of single-particle states in odd-A deformed nuclei. Taking the wave function as (72) and the function $\mathcal{F}_{s_0}(\eta)$ as (81) and performing the same calculations as in eqs.(120),(122) and (122'), we obtain

$$\Phi_{s_0}(\eta) = \frac{1}{\pi} \mathcal{J}m \left\{ \frac{1}{\mathcal{F}_{s_0}(\eta + i\frac{\Delta}{2})} \right\}, \qquad (124)$$

i.e.,the same form as (122"). To obtain $\Phi_{s_0}(\eta)$ one should calculate the matrix of high order g' or g'' rather than diagonalize, which is much easier mathematically.

The fragmentation of one-quasiparticle states allows one to calculate the strength functions of neutron resonances and of one-nucleon transfer reactions, the formulae for which include the expressions of the form

$$(\sum_s a_{\ell I}^{sK} U_s C_s^n)^2, \qquad (125)$$

$$(\sum_s a_{\ell I}^{sK} V_s C_s^n)^2 . \qquad (125')$$

The single-particle wave functions φ_s^K are given as the expansions over the spherical basis

$$\varphi_s^K = \sum_{N\ell I} a_{N\ell I}^{sK} \varphi_{N\ell I}, \quad a_{\ell I}^{sK} = \sum_N a_{N\ell I}^{sK} . \qquad (126)$$

In ref.[46] the strength functions

$$S_{\ell I}^{pK}(\eta) = \sum_n \rho(\eta_n - \eta) \left| \sum_s a_{\ell I}^{sK} U_s C_s^n \right|^2, \qquad (127)$$

$$S_{\ell I}^{hK}(\eta) = \sum_n \rho(\eta_n - \eta) \left| \sum_s a_{\ell I}^{sK} V_s C_s^n \right|^2 \qquad (127')$$

are introduced, and the following transformations

$$S_{\ell I}^{pK}(\eta) = \sum_n \rho(\eta_n - \eta) \sum_s (a_{\ell I}^{sK} U_s)^2 (C_s^n)^2 +$$

$$+ \sum_n \rho(\eta_n - \eta) \sum_{\substack{s s' \\ s \neq s'}} a_{\ell I}^{sK} a_{\ell I}^{s'K} U_s U_{s'} (C_s^n)^2 \frac{C_{s'}^n}{C_s^n} =$$

$$= - \sum \rho(\eta_n - \eta) \sum_s (a_{\ell I}^{sk} U_s)^2 \left(\frac{\partial \mathcal{F}_s(\eta_n)}{\partial \eta} \right)^{-1} -$$

$$- \sum_n \rho(\eta_n - \eta) \sum_{\substack{s,s' \\ s \neq s'}} a_{\ell I}^{sk} a_{\ell I}^{s'k} U_s U_{s'} \left(\frac{\partial \mathcal{F}_s(\eta_n)}{\partial \eta} \right)^{-1} \frac{\Delta(s,s';\eta)}{\theta(s,\eta)} .$$

are performed. Here we have used formulae (85) and (86). Making the same operations as when passing from eq.(120) to eq.(122"), we obtain

$$S_{\ell I}^{pk}(\eta) = \frac{1}{\pi} \sum_s (a_{\ell I}^{sk} U_s)^2 \, \mathrm{Im} \left\{ \frac{1}{\mathcal{F}_s(\eta + i\Delta/2)} \right\} + \qquad (128)$$

$$+ \frac{2}{\pi} \sum_{s>s'} a_{\ell I}^{sk} a_{\ell I}^{s'k} U_s U_{s'} \, \mathrm{Im} \left\{ \frac{\Delta(s,s';\eta + i\Delta/2)}{\theta(s,\eta + i\Delta/2)} \right\} .$$

The expression for $S_{\ell I}^{hk}(\eta)$ differs from eq.(128), since the functions U_s and $U_{s'}$ are substituted by V_s and $V_{s'}$. Using the condition of completeness the authors of ref.[46] have obtained the following expression of the sum rule type:

$$\sum_n \left(\sum_s a_{\ell I}^{sk} U_s C_s^n \right)^2 = \sum_s U_s^2 (a_{\ell I}^{sk})^2 . \qquad (129)$$

The right-hand side of eq.(129) is the upper limit of the strength function (128). Calculating eq.(128) in a certain energy interval and comparing it with the r.h.side of eq.(129), one may determine which part of the strength function is exhausted in this energy interval.

When calculating the strength functions instead of diagonalizing the matrix of high order g' or g'' for each state there are calculated the imaginary parts of the determinants of an order of g' or g'' at different values of energy η with step Δ. The calculation of the strength functions instead of calculating the values for each state reduces the computational time by a factor of 10^2–10^3.

3. Within the quasiparticle-phonon nuclear model the strength functions for the reduced $E\lambda$ -transition probabilities are widely used to study the giant multipole resonances and neutron resonances.

We dirive the expression for the strength function of $E\lambda$ -excitation of a doubly even deformed nucleus, the excited states of which are described by the wave function (103). The reduced $E\lambda$ -transition probability has the form

$$B(E\lambda; 0^+0 \to I_f^{\pi_f} K_{f_n}) \equiv B(E\lambda; \eta_n) = (00\lambda\mu / I_f K_f)^2 \mathcal{M}_n^2 , \qquad (130)$$

$$\mathcal{M}_n = \frac{1}{2} \sum R_i^n(\lambda\mu) \left(\frac{2-\delta_{\mu,0}}{Y_i}\right)^{1/2} \left[e_{eff}^{(\lambda)}(p) X^i(p) y_p^i + e_{eff}^{(\lambda)}(n) X^i(n) \right] = \qquad (131)$$

$$= \sum_i R_i^n(\lambda\mu, n) L_I(\lambda\mu) ,$$

where $e_{eff}^{(\lambda)}(p)$ and $e_{eff}^{(\lambda)}(n)$ are the effective electric charges, and for $E1$ -transitions they are equal to

$$e_{eff}^{(\lambda)}(p) = \frac{N}{A} e , \quad e_{eff}^{(\lambda)}(n) = -\frac{Z}{A} e ,$$

The functions Y_i, X^i and y_p^i are determined by formulae (51), (48) and (50). Then

$$\mathcal{M}_n^2 = \sum_{i,i'} R_i^n(\lambda\mu) R_{i'}^n(\lambda\mu) L_i(\lambda\mu) L_{i'}(\lambda\mu) = \qquad (131')$$

$$= \sum_{i,i'} (R_i^n(\lambda\mu))^2 \widetilde{R}_{i'}^n(\lambda\mu) L_i(\lambda\mu) L_{i'}(\lambda\mu).$$

Let us introduce the strength function

$$b(E\lambda,\eta) = \sum_n B(E\lambda;\eta_n)\rho(\eta_n-\eta) \tag{132}$$

where $\rho(\eta_n-\eta)$ is determined by formula (121). It is easy to show that the relation

$$\int_{\eta-\Delta/2}^{\eta+\Delta/2} b(E\lambda,\eta')\,d\eta' = \sum_{n\Delta} B(E\lambda;\eta_n) \tag{132'}$$

is fulfilled with a sufficiently good accuracy. The summation in it is performed over all the states with given K^π in the energy interval Δ. Using formulae (111) and (111') and substituting (130) and (131) into (132), we obtain

$$b(E\lambda,\eta) = -(00\lambda\mu)I_fK_f)^2 \sum_n \sum_{ii'} \frac{L_i(\lambda\mu)L_{i'}(\lambda\mu)}{\frac{\partial}{\partial\eta_n}\mathcal{F}_i(\eta_n)} \frac{(-1)^{i+i'}M_{ii'}}{M_{ii}} \frac{1}{(\eta_n-\eta)^2+(\Delta/2)^2} =$$

$$= -\frac{\Delta}{2\pi}\frac{(00\lambda\mu)I_f(K_f)^2}{} \oint_{L} \sum_{ii'} \frac{L_i(\lambda\mu)L_{i'}(\lambda\mu)}{\mathcal{F}_i(Z)} \frac{(-1)^{i+i'}M_{ii'}(Z)}{M_{ii}(Z)} \frac{dz}{(\eta-Z)^2 (\Delta/2)^2}. \tag{132''}$$

Here the integration is over the contour, given in the Figure. Using the same procedure as when passing from eq.(122) to eq.(122") and formula (109), we obtain

$$b(E\lambda,\eta) = \frac{1}{\pi}(00\lambda\mu|I_fK_f)^2 \sum_{ii'} (-1)^{i+i'} L_i(\lambda\mu)L_{i'}(\lambda\mu)\, \mathrm{Im}\left\{\frac{M_{ii'}(\eta+i\frac{\Delta}{2})}{B(\eta+i\frac{\Delta}{2})}\right\}. \tag{133}$$

When calculating the strength functions $b(E\lambda,\eta)$ one should not diagonalize the matrices θ and M_{ii} but should calculate their imaginary part at different values of η. This reduces the computational time by a factor of 10^2–10^3. The rank of the determinants θ can be chosen within the following limits: 10–20 for spherical nuclei and 20–100 for deformed nuclei. It follows from the aforesaid that the calculation of many giant multipole resonances for a large number of nuclei needs reasonable time at the computer.

The calculation of the energy weighted sum rule (EWSR) is very important for determining the regions of location of the giant multipole resonances. The energy weighted sum rule is as follows:

$$S_\lambda(\eta) = \sum_{\mu,n} \eta_n(\lambda\mu) B(E\lambda,\eta_n) \rho(\eta_n - \eta) =$$
$$= \frac{1}{\pi} \sum_{\mu} (00\lambda\mu | I_f K_f)^2 \sum_{ii'} (-1)^{i+i'} L_i(\lambda\mu) L_{i'}(\lambda\mu)\, \mathrm{Im} \left\{ \frac{(\eta + i\Delta/2) M_{ii'}(\eta + i\Delta/2)}{\theta(\eta + i\Delta/2)} \right\} \qquad (134)$$

For spherical nuclei there is no summation over μ or K.

The model independent energy weighted sum rule is useful for determining the completeness of the single-particle basis used. The model independent dipole sume rule is

$$\sum_n B(E1,\eta_n)\eta_n = 0{,}18 \frac{Z \cdot N}{A} e^2 \cdot \text{barn MeV}. \qquad (135)$$

For $\lambda > 1$

$$\sum_n B(E\lambda, \eta_n)\eta_n = 4.8\lambda(3+\lambda)^2 \frac{Z}{A^{2/3}} B(E\lambda)_{s.p} \text{ MeV}. \qquad (136)$$

8. Conclusion

1. Many properties of complex nuclei at low, intermediate and high excitation energies can be calculated within the quasiparticle-phonon model. A part of these calculations has already been performed. It is obvious that in future more complicated variants of the model will be used, by including new terms in the wave functions (69) and (102) and by taking into account new forces.

2. It should be noted that the main contribution to the wave functions of highly excited states comes from many-quasiparticle components. It is undoutful that in future new properties of highly excited states due to many-quasiparticle components will be demonstrated. So far, there is no data on the magnitude and distribution of many-quasiparticle components of the highly excited state wave functions. Even for neutron resonances it is shown[14,15,63] that direct experimental evidence for many-quasiparticle components of their wave functions is absent. The contribution of the few-quasiparticle components to the wave function normalization is only 10^{-4}–10^{-6}.

3. With increasing excitation energy the state structure becomes complicated. A variety of properties of the high-lying states may be expected as compared to the low-lying ones. One can hardly imagine the structure of nuclear states at very high excitation energies. Will it be the state of nondistinguishable nuclear matter or something else?

4. It is doubtfull whether a simple and clear description of complex nuclei will be found. The atomic nucleus is a very complex system, but this complexity can be understood, the known properties can be described and the new ones predicted within the nuclear theory based on the computational technique which is rapidly developing.

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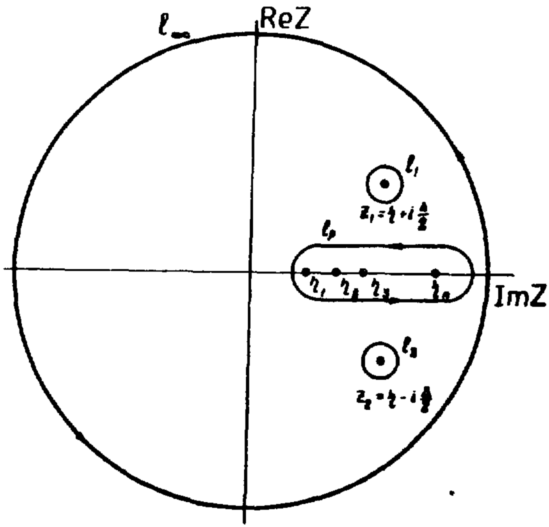

Figure. Integration contours in complex plane Z.

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