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ДУБНА

E4 - 10995

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STRUCTURE OF LIGHT NUCLEI IN THE K-HARMONICS DESCRIPTION WITH DIFFERENT NUCLEON-NUCLEON POTENTIALS

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Структура легких ядер в методе К-гармоник для различных вариантов нуклон-нуклонного взаимодействия

В методе К-гармоник изучалась структура легких ядер. В связи с этим развивался формализм метода гиперсферических функций. Исследован вопрос о построении гиперсферических гармоник с заданной перестановочной симметрией, развита эффективная генеалогическая техника, рекуррентная по числу нуклонов в ядре, получены матричные элементы различных физических операторов. Затем эти формулы применялись для описания различных свойств ядер. Оценивалась зависимость результатов расчета от варианта реалистического нуклон-нуклонного потенциала. Изучались энергии связи, средние квадратичные радиусы, возбужденные состояния нормальной и аномальной четности, параметр сжимаемости и кластерная характеристика ядра. Исследовались ядра 1p -оболочки в приближении $K=K_{min}$ и $K=K_{min}+1$. Полученные результаты позволяют сделать вывод, что относительное положение в спектрах уровней менее чувствительно к выбору параметров потенциала, чем абсолютное значение энергии связи. В дипольном гигантском резонансе существенна примесь состояний три частицы - три дырки и пять частиц - пять дырок. Параметр сжимаемости легких ядер зависит от варианта нуклон-нуклонных сил, отличается для изовекторных и изоскалярных состояний, но не меняется при увеличении энергии возбуждения ядра.

Сообщение Объединенного института ядерных исследований. Дубна 1977

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E4 - 10995

Structure of Light Nuclei in the K -Harmonics Description with Different Nucleon-Nucleon Potentials

The structure of light nuclei is studied within the K-harmonics method. First we briefly consider the structure of the wave functional parentage coefficients and the matrix elements of various operators. Then several nuclear properties are described by the obtained formulas, using different realistic two-nucleon potentials. Binding energies, rms sizes, excited states of normal and anomalous parity, compressibilities of the nuclei, the cluster structure of the wave functions, etc., are studied. The 1p-shell nuclei in the $K=K_{min}$ and $K=K_{min}+1$ approximation are investigated.

The investigations have been performed at the Laboratory of the Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1977

1. The K-harmonics method has been widely used recently in the theory of light atomic nuclei. The first and successful applications cover the three- and four-nucleon problems[1]. Later on the method was generalized for arbitrary A [2] and applied for calculations of the binding energies and low-lying excitations in the light nuclei. Nevertheless, there is no systematic study available in the literature which would concern the diverse nuclear properties treated simultaneously within the technique of K-harmonics. Such a study was hampered by the difficulties met in constructing of the hyperspherical harmonics (HSH) with a definite permutation symmetry. This problem is reduced to the study of transformations which connect HSH expressed in the different sets of Jacobi coordinates. Recently [4] the problem was solved by applying an effective recurrence (in the nucleon number A) technique of the fractional-parentage type. As a matter of fact, this method is a natural extension of the translational-invariant shell-model techniques[5].

In the present paper we study the structure of the K-harmonic wave functions. Some useful formulae for the fractional-parentage coefficients are given together with the matrix elements of different physical operators. Subsequently these expressions have been used to estimate, e.g., the spectroscopic factors and compressibilities for all $1p$-shell nuclei using several choices of the nucleon-nucleon residual interaction. The $K = K_{min}$ and $K_{min} + 1$ approximation of the K-harmonics method has been employed in the calculation.

2. The wave function Ψ of a nucleus consisting of A nucleons must be translational-invariant and, therefore, the Jacobi coordinates $\vec{X}_1, \vec{X}_2 \ldots, \vec{X}_{A-1}$ are usually selected as its arguments. Each of these coordinates $\vec{X}_i$ is the distance between the centers of mass of two nucleon groups with p_i and q_i nucleons, respectively ($p_i, q_i \geq 1$). One can introduce the normalized Jacobi coordinates $\vec{x}_i = \vec{X}_i/\mu_i$ where $\mu_i = \frac{p_i q_i}{p_i + q_i} m$ is the reduced mass, and m is the nucleon mass. The standard set of the Jacobi coordinates is

$$\begin{aligned}
\vec{x}_1 &= \frac{1}{\sqrt{2}} (\vec{r}_1 - \vec{r}_2) \\
\vec{x}_2 &= \sqrt{\frac{2}{3}} \left[\frac{1}{2}(\vec{r}_1 + \vec{r}_2) - \vec{r}_3 \right] \\
&\cdots\cdots\cdots \\
\vec{x}_{A-1} &= \sqrt{\frac{A-1}{A}} \left(\frac{1}{A-1} \sum_{i=1}^{A-1} \vec{r}_i - \vec{r}_A \right).
\end{aligned} \qquad (1)$$

Here $\vec{r}_i$ are the coordinates of the i-th nucleon. Each coordinate of the standard set determines the distance between the i + 1-th nucleon and the center of mass of the group of nucleons numbered 1,2....i . The choice of the Jacobi coordinates may be graphically illustrated in the form of a "Jacobi

tree" /6/. The Jacobi tree shown in Fig. 1 corresponds to the standard set of the Jacobi coordinates (1). The non-standard Jacobi tree shown in Fig.2 describes the following set of the Jacobi coordinates

$$\begin{aligned}
\vec{x}_1 &= \frac{1}{\sqrt{2}} (\vec{r}_1 - \vec{r}_2) \\
\vec{x}_2 &= \frac{1}{\sqrt{2}} (\vec{r}_3 - \vec{r}_4) \\
\vec{x}_3 &= \frac{1}{2} (\vec{r}_1 + \vec{r}_2 - \vec{r}_3 - \vec{r}_4) .
\end{aligned} \qquad (2)$$

The K-harmonics method consists in that spherical coordinates (the collective hyperradius ρ and 3A-4 hyperspherical angles) are introduced in the 3(A-1)-dimensional space of the Jacobi coordinates and the wave function Ψ of the nucleus is expanded in the hyperspherical harmonics which are the standard functions of these angles /7/. Now we consider the structure of these functions. Let one have the Cartesian coordinates in the n-dimensional space $\vec{x}_1, \vec{x}_2, \vec{x}_3 \ldots \vec{x}_n$. The hyperspherical angles $\theta_1, \theta_2 \ldots \theta_{n-1}$ can be chosen so that the Cartesian and hyperspherical coordinates be related as

$$\begin{aligned}
x_1 &= \rho \sin\theta_{n-1} \ldots \sin\theta_2 \sin\theta_1 \qquad 0 \leq \rho \leq \infty \\
& \qquad\qquad\qquad\qquad\qquad\qquad 0 \leq \theta_1 < 2\pi \\
x_2 &= \rho \sin\theta_{n-1} \ldots \sin\theta_2 \cos\theta_1 \\
&\cdots\cdots\cdots\cdots \\
x_{n-1} &= \rho \sin\theta_{n-1} \cos\theta_{n-2} \\
& \qquad\qquad\qquad\qquad\qquad\qquad 0 \leq \theta_K < \pi \quad K \neq 1 \\
x_n &= \rho \cos\theta_{n-1} \qquad\qquad \rho^2 = \sum_{i=1}^{n} x_i^2
\end{aligned} \qquad (3)$$

This set of the hyperspherical coordinates can be graphically represented in the form of a hyperspherical tree (see Fig.3).

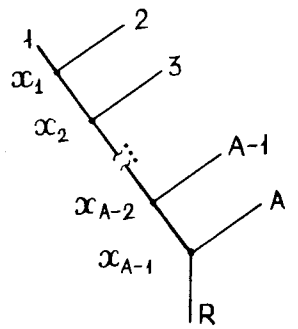

Fig.1.

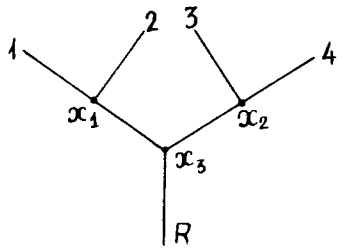

Fig.2.

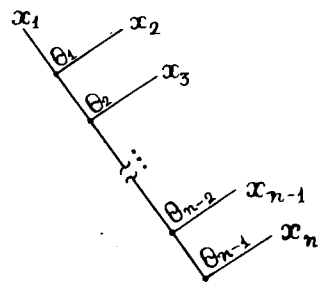

Fig.3.

If a set of hyperspherical coordinates (3) is used, the volume element dN in the n-dimensional space is of the form

$$dN = dx_1\, dx_2 \ldots dx_n = \rho^{n-1} d\rho\, d\Omega\,, \tag{4}$$

where the element of the solid angle $d\Omega$ is

$$d\Omega = \sin^{n-2}\theta_{n-1} \sin^{n-3}\theta_{n-2} \ldots \sin\theta_2\,. \tag{5}$$

The Laplacian is given by the expression

$$\Delta_n = \sum_n \frac{\partial^2}{\partial x_n^2} = \frac{1}{\rho^{n-1}} \frac{\partial}{\partial\rho}\left(\rho^{n-1}\frac{\partial}{\partial\rho}\right) + \frac{1}{\rho^2}\Delta_{\Omega_n}\,. \tag{6}$$

The hyperspherical harmonics are the eigenfunctions of the angular part of the Laplacian: Δ_{Ω_n}

$$\Delta_{\Omega_n} Y_{LM}(\theta_q) = -\ell_n(\ell_n + n - 2)\, Y_{LM}(\theta_q). \tag{7}$$

The value K is an analogue of the angular momentum at n=3 and is called the global moment. The subscript ν denotes all the quantum numbers necessary to distinguish various degenerate states of equation (7). For ν, it is expedient to use the Young diagram $[f]$, the Yamanochi symbol (λ), and LM the orbital moment of state and its projection.

The fractional parentage coefficients are used to calculate the matrix elements of various operators. Using the single-particle fractional parentage coefficients one can express the function of A particles through the products of the functions of A-1 particles by the function for motion of the last particle with respect to the center of mass of the A-1 particle

group. The formula of the fractional parentage coefficients $\langle A|A-1\rangle$ using $\langle A-1/A-2\rangle$ will be rewritten as /4/

$$\langle A\,K[f]\,L\,\ell \,|\, A-1\,K_1[f_1]\,L_1\,\ell_1,\ell_1\rangle =$$

$$= \frac{1}{n_{f_1}}\sqrt{\frac{n_f\,n_{f'}}{A}}\,\frac{1}{Q}\Big\{\delta_{[f_1][f']}\,\delta_{K,K'}\,\delta_{L,L'}\,\delta_{\ell\ell_2}+(A-1)\sum_{K_1'[f_1'][f']L_1'\ell_1'\ell_2'L_0K_0}(-1)^{\ell+\ell'+j_1+j_2+j_1'+j_2'}\frac{n_f}{n_{f'}}$$

$$\cdot\langle [f_1'][f_1]\,|\,P_{A-1,A}\,|\,[f_1'][f']\rangle^{[f]}\cdot\langle \ell_1\ell_2'\,|\,\ell_2\ell_1'\rangle_{\varphi}^{L_0K_0}\,U[J_1'j_1'J_{j_2};J'J_0]\cdot$$

$$\cdot\langle A-1\,K'[f']\,L'\,\varepsilon'\,|\,A-2\,K_1'[f_1']\,L_1'\,\varepsilon_1',\ell_2'\rangle\cdot$$

$$\cdot\langle A-1\,K'[f']\,L'\,\varepsilon'\,|\,A-2\,K_1'[f_1']\,L_1'\,\varepsilon_1',\ell_1'\rangle\cdot$$

$$\cdot U(L_1'\ell_2'L\ell_2;L'L_0)\,U(L_1'\ell_1'L\ell_2;L'L_0)\,U[J_1'j_2'J_{j_2};J'J_0] \qquad (8)$$

$$Q^2 = 1+(A-1)\sum_{L_1'\ell_1'K_1'[f_1'][f']\ell_2'\ell_1'L_0K_0}\frac{n_{f_1'}}{n_{f'}}\langle [f_1'][f']\,|\,P_{A-1,A}\,|\,[f_1'][f']\rangle^{[f]}\times$$

$$\cdot\langle A-1\,K'[f']\,L'\,\varepsilon'\,|\,A-2\,K_1'[f_1']\,L_1'\,\varepsilon_1',\ell_2'\rangle\cdot\,(-1)^{\ell_1'+\ell_2'}$$

$$\cdot\langle A-1\,K'[f']\,L'\,\varepsilon'\,|\,A-2\,K_1'[f_1']\,L_1'\,\varepsilon_1',\ell_1'\rangle\cdot(-1)^{j_1'-j_2'}\times$$

$$\cdot\langle \ell_2\ell_2'\,|\,\ell_2\ell_1'\rangle_{\varphi}^{L_0K_0}\,U(L_1'\ell_2'L\ell_2;L'L_0)\,U(L_1'\ell_1'L\ell_2;L'L_0)\cdot$$

$$\cdot U[J_1'j_2'J_{j_2};J'J_0]\,U[J_1'j_1'J_{j_2};J'J_0]$$

Here

$$\varphi = \operatorname{arctg}\sqrt{\frac{1}{A(A-2)}}\,,\quad j_i=\tfrac{1}{2}(\ell_i-\tfrac{1}{2})\,,\quad j_i'=\tfrac{1}{2}(\ell_i'-\tfrac{1}{2})$$

$$J_0=\tfrac{1}{2}(K_0+1),\quad J=\tfrac{1}{2}(K+\tfrac{3A-7}{2}),\quad J_1'=\tfrac{1}{2}(K_1'+\tfrac{3A-13}{2}),$$

$$J_1=\tfrac{1}{2}(K_1+\tfrac{3A-10}{2})\,,\quad J'=\tfrac{1}{2}(K'+\tfrac{3A-10}{2})$$

n_f is the dimensionality of the irreducible representation of the group S_A; $\langle [f_1'][f_1]\,|\,P_{A-1,A}\,|\,[f_1'][f']\rangle$ is the matrix element of the standard irreducible representation of the same group. The label $U[\dots]$ stands for the generalized Racah coefficients with quarter-integral moments, the label $U(\dots)$ denotes the usual Racah coefficients and $\langle \ell_1\ell_2'\,|\,\ell_2\ell_1'\rangle_{\varphi}^{L_0K_0}$ are the Raynal-Revai coefficients which are related to the Talmi coefficients using the 3j-symbols with quarter-integral moments.

$$\langle \ell_1\ell_2\,|\,\ell_1'\ell_2'\rangle_{\varphi}^{K\Lambda} = \sum_{\substack{n_1+n_2=K\\ n_1'+n_2'=K}}\langle n_1\ell_1\,n_2\ell_2\,|\,NK\rangle\cdot\langle n_1'\ell_1'\,n_2'\ell_2'\,|\,NK\rangle\cdot$$

$$\cdot\langle n_1\ell_1,n_2\ell_2:\Lambda\,|\,P^{2r}/q(p+q+2)\,|\,n_1'\ell_1',n_2'\ell_2':\Lambda\rangle. \qquad (9)$$

The two-particle fractional parentage coefficients are expressed as

$$\langle A\,K[f]\,L\,\ell\,|\,A-2\,K_2[f_2]\,L_2\,\ell_2,\Lambda\{L'K'\};L_0\rangle =$$

$$= \langle [f](2)\,|\,[f_2](2_2),[f_0](2_0)\rangle^{-1}\sum_{K_1\ell_1L_1\ell_1'\ell_2\lambda\varkappa}\langle A\,K[f]\,L\,\ell\,|\,A-1\,K_1[f_1]\,L_1\,\ell_1,\ell_1\rangle\cdot$$

$$\cdot\langle A-1\,K_1[f_1]\,L_1\,\ell_1\,|\,A-2\,K_2[f_2]\,L_2\,\ell_2,\ell_2\rangle(-1)^{\lambda-\ell_1-\ell_2+J_\varkappa-j_1-j_2-1}\times$$

$$\cdot U(L_2\ell_2L\ell_1;L_1\lambda)\,U[J_2j_2J_{j_1};J_1J_\varkappa]\,U(L_2\Lambda L L_0;L'\lambda)\cdot$$

$$\cdot U[J_2\,J_1\,J\,J_0;J'J_\varkappa]\,\langle \ell_1\ell_2\,|\,\Lambda L_0\rangle_{\varphi}^{\varkappa\lambda}, \qquad (10)$$

where $\varphi = arc\,tg\sqrt{\frac{A-2}{A}}$ the Young diagram $[f_1]$ and the corresponding Yamanochi symbols (2_2) and (2) are chosen in an arbitrary but fixed way.

Here L_0 is the moment of momentum of the nucleon pair A-1 and A (the degree of freedom

$$x'_{A-1} = \frac{1}{\sqrt{2}}(r_A - r_{A-1})\ )$$

Λ is the moment of this pair relative to the remaining A-2 particle (the degree of freedom $x'_{A-2} = \sqrt{\frac{2(A-2)}{A}}\left(\frac{r_A + r_{A-1}}{2} - \frac{1}{A-2}\sum_{i=1}^{A-2} r_i\right)$).

Thus, with in the K-harmonics method we may use the same formulas for the fractional parentage coefficients which were obtained earlier by the TISM[5]. In this case the Talmi-Moshinsky coefficients must be replaced by the Raynal-Revai coefficients, $6j$ -symbols must be added for overbinding of the global moments K, and additional phase multipliers are inserted.

Within the K-harmonics method [1] the wave function of nucleus A is sought in the form of expansion in the K-harmonics polynomials $|AK[f]\varepsilon LST>$:

$$\Psi(1,2,\ldots A) = \rho^{-\frac{3A-4}{2}} \sum_{K\gamma} \chi_{K\gamma}(\rho)\,|AK\gamma>, \qquad (11)$$

where $\gamma = [f]\varepsilon LST$. The Hamiltonian of the nucleus is the form

$$H = -\frac{\hbar^2}{2m}\frac{1}{\rho^{3A-4}}\frac{\partial}{\partial\rho}\left(\rho^{3A-4}\frac{\partial}{\partial\rho}\right) - \frac{\hbar^2}{2m\rho^2}\Delta_\theta + V\,. \qquad (12)$$

The Schrödinger equation for the radial functions can be written as

$$\left\{\frac{d^2}{d\rho^2} - \frac{\gamma_K(\gamma_K+1)}{\rho^2} - \frac{2m}{\hbar^2}\left(E + W^{K\gamma}_{K\gamma}(\rho)\right)\right\}\chi_{K\gamma}(\rho) = \frac{2m}{\hbar^2}\sum_{K'\gamma'\neq K\gamma} W^{K'\gamma'}_{K\gamma}(\rho)\,\chi_{K'\gamma'}(\rho)\,, \qquad (13)$$

where $\gamma_K = K + \frac{3A-6}{2}$, $W^{K'\gamma'}_{K\gamma}(\rho)$ are the matrix elements of the potential energy of the nucleon-nucleon interaction

$$V = \sum_{i<j}^{A} V(r_{ij}) \qquad V(r_{ij}) = f(r_{ij})\,W_{\sigma\tau} \qquad (14)$$

which can be expressed in terms of the two-particle fractional parentage coefficients in the form

$$W^{K\bar{\gamma}}_{K\gamma}(\rho) = \langle AK[f]\varepsilon LSTM_L M_S M_T | V | AK[\bar{f}]\bar{\varepsilon}\bar{L}\bar{S}\bar{T}M_L M_S M_T\rangle =$$
$$= \frac{A(A-1)}{2}\sum_{K'[f']\varepsilon'\Lambda'S'T'\lambda S_0 T_0} \langle AK[f]\varepsilon LST\,|\,A-2\,K'[f']\varepsilon'L'S'T';\,\lambda S_0 T_0\rangle \times$$
$$\times \langle A\bar{K}[\bar{f}]\bar{\varepsilon}\bar{L}\bar{S}\bar{T}\,|\,A-2\,K'[f']\varepsilon'L'S'T';\,\lambda S_0 T_0\rangle \times$$
$$\times \langle S_0 T_0\,|\,W_{\sigma\tau}\,|\,S_0 T_0\rangle \cdot R^{K\bar{K}}_{K'\lambda}(\rho) \qquad (15)$$

Here

$$R^{K\bar{K}}_{K'L_0}(\rho) = \int d\theta_1 \, (\sin\theta_1)^{3A-7} (\cos\theta_1)^2 f(\rho\cos\theta_1) \cdot$$
$$\cdot N_{KK'L_0} \, N_{\bar{K}K'L_0} \, (\sin\theta_1)^{2K'} (\cos\theta_1)^{2L_0} \cdot$$
$$\cdot P^{K'+\frac{3A-6}{2}-1, L_0+\frac{1}{2}}_{K-K'-L_0}(\cos 2\theta_1) \quad P^{K'+\frac{3A-6}{2}-1, L_0+\frac{1}{2}}_{\bar{K}-K'-L_0}(\cos 2\theta_1) .$$

(16)

The matrix element of the effective potential is calculated using the formula proven by Baz. According to this formula the matrix element in the K-harmonics method dependent on ρ is expressed in terms of the matrix element of the translational-invariant shell model. This matrix element is the following function of the oscillator parameter λ /2/.

$$W^{K'\gamma'}_{K\gamma}(\rho) = \frac{\Gamma\left(K+\frac{3A-6}{2}\right)}{2\pi i \, \rho^{2K+3A-3}} \int_C \frac{ds \; e^{s\rho^2}}{s^{K+\frac{3A-1}{2}}} W^{K'\gamma'}_{K\gamma}(s^{-\frac{1}{2}}) .$$

(17)

This method is asymptotical and may give accurate results only at sufficiently high mass numbers A (we confine ourselves to the lower term in powers of $(3A-4+2K)^{-1}$).

These formulas were used to find the binding energies and the wave functions of all p-shell nuclei for various versions of the central potential of the nucleon-nucleon interaction.

Then, we have calculated the RMS sizes, the matrix elements of the dipole transition operator, the compressibility of the nuclei, the spectroscopic form-factor of α-cluster. For example we give below the expressions for matrix elements of some physical operators.

a. The reduced width amplitude of the decay of nucleus A into nucleus A–4 and α-particle. This amplitude can be expressed as

$$\mathcal{J}(r) = \sum_{NN_1N_2n\mathcal{K}\mathcal{L}} \langle N_1 K_1 \, N-N_1 \, \mathcal{K} \,|\, NK\rangle \langle n\Lambda \, N_2 K_2 \,|\, N-N_1 \, \mathcal{K}\rangle \cdot$$
$$\cdot \langle AK[f]LST \,|\, A_1 K_1 [f_1] L_1 S_1 T_1 ; \Lambda, 4K_2[f_2]L_2 S_2 T_2 \{\mathcal{K}\mathcal{L}\}\rangle \cdot$$
$$\cdot C^N_A C^{N_1}_{A_1} C^{N_2}_4 R_{n\Lambda}\left(\frac{r}{r_0}\right) \qquad (18)$$

where r is the distance between the centers-of-mass of particles $A_1 = A-4$ and $A_2 = 4$; $\langle AK\ell \,|\, |A_1 K_1 \ell_1 ; \Lambda, 4K_2\ell_2\{\mathcal{K}\mathcal{L}\}\rangle$ are the fractional parentage coefficients for four particles in the K-harmonics method; $\langle N_i K_i \, N-N_i \, \mathcal{K} \,|\, NK\rangle$ are the generalized 3j-symbols for the global momentum; C^N_A, $C^{N_1}_{A_1}$, $C^{N_2}_4$ are the coefficients of the expansion of K-harmonics functions in the wave functions of the TISM.

b. The dipole transition operator from the ground state $J^\pi = 0^+$, $T = 0$ of O^{16} nucleus to the excited state $J^\pi = 1$, $T = 1$ /8/

$$\langle \Psi_{0^+} \,|\, \hat{d} \,|\, \Psi_{1^-}\rangle = \frac{3\cdot 69!!}{2^{36}\cdot 35!} e \left(\int_0^\infty \chi_0(\rho)\chi_{1s}(\rho)\rho\, d\rho + \right.$$
$$\left. + \sqrt{5} \int_0^\infty \chi_0(\rho)\chi_{1d}(\rho)\rho\, d\rho \right) . \qquad (19)$$

c. The compressibility of the nucleus /9/ may be written in the form

$$K = \frac{m}{\hbar^2} r^2 (\Delta E)^2, \qquad (20)$$

where r is the radius of the nucleus ground state; ΔE is the difference of excitation energies of the neighbouring monopole states.

3. Fig.4 shows the calculated binding energies of the p-shell nuclei within the K-harmonics description and the experimental values /10/.

It can be seen from the figure that the theoretical binding energies for the chosen potential (version 1 of Volkov's potential /11/) are strongly underestimated (excluding the O^{16} nucleus) as compared to the experimental values. This result is not unexpected since the calculations were made in the approximation $K=K_{min}$. So, the inclusion of the subsequent harmonics will result in an increase of the binding energy.

Besides, the absolute value of the binding energy depends strongly on the choice of the nucleon-nucleon potential. Indeed, as is seen from Table 1, the results of the calculations of the binding energies of certain nuclei strongly differ for the various two-nucleon potentials /11,12/.

Therefore, the disagreement between the theory and experiment may be associated not only with inefficiency of K_{min} approximation but also with inadequate choice of the nucleon-nucleon potential. The shape of the calculated A dependence of the binding energy reproduces that of the experimental curve so that the anomalous increase in the binding energy for the nuclei having the α-structure can be described using the K-harmonics.

Table 1. The results of the calculations of the binding energies of certain nuclei for the various two-nucleon potentials.

A	E_b^{th}, MeV	Versions of the potential	E_b^{exp}, MeV
He^4	23.5	1/10/	28.3
	25.8	6/10/	
	26.0	3/10/	
Li^6	17.0	1/10/	31.99
	20.9	6/10/	
	13.0	1/11/	
Li^7	26.1	1/10/	39.25
	13.7	1/11/	
O^{16}	142.4	1/10/	127.62
	158.4	2/10/	
	151.4	3/10/	

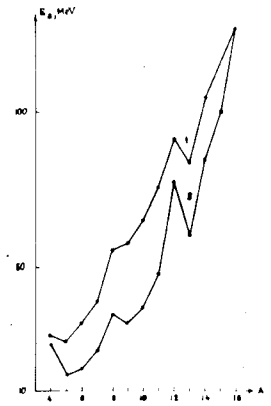

Fig.4. Comparison of the calculated binding energies in the K-harmonics description of the p-shell nuclei with the experimental values.
1. - Experimental values.
2. - Results of calculations in the $K=K_{min}$ approximation.

Table 2. The theoretical and experimental rms radii of the light nuclei.

A	$\langle r^2 \rangle^{th}_{1/2}$, fermi	$\langle r^2 \rangle^{exp.}_{1/2}$, fermi
4	1.81	1.708
5	2.31	-
6	2.74	2.535
7	2.65	-
8	2.71	-
9	2.84	2.26
10	2.72	-
11	2.87	2.25
12	2.94	2.496
13	2.92	-
14	2.85	2.48
15	2.80	-
16	2.75	2.741

Table 2 presents the results of calculations of the RMS sizes of the light nuclei for the version of Volkov's potential /11/. It can be seen that the theoretical values are overestimated. This is probably due to the inadequacy of the used version of potential. A deeper potential may give higher values of the binding energy and a smaller width of the well so that the values of the RMS sizes will also decrease.

Fig. 5 shows the spectra of the lower lying excited states of the ^{6}Li and ^{7}Li nuclei within the hyperspherical method

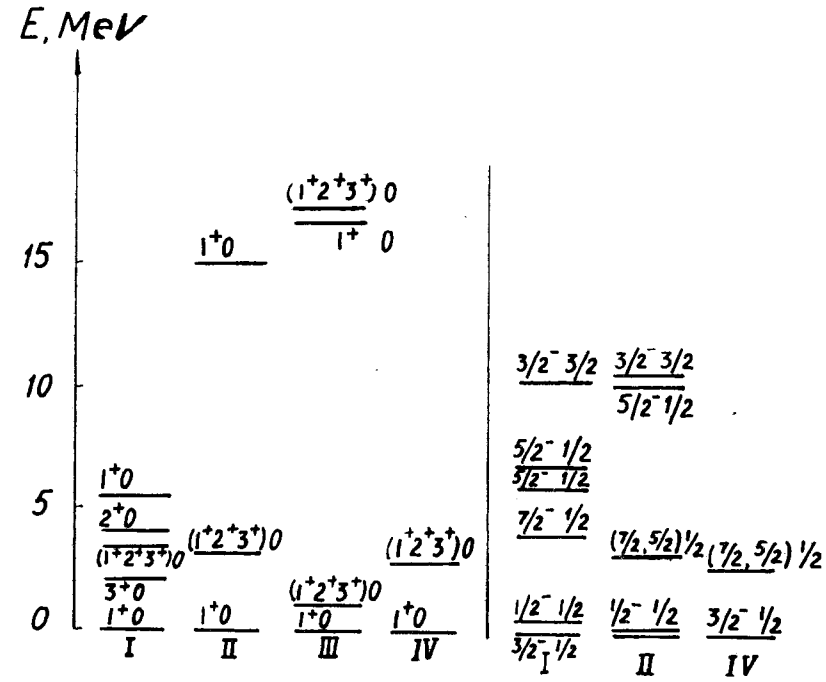

Fig.5. Spectra of the low-lying excited states of the nuclei in the hyperspherical method: a. Li^6, b. Li^7.
I. - Experimental values.
II.- Version 1 of the potential /11/.
III.-Version 6 of the potential /11/.
IV. -Eikemeier-Hackenbroich's central potential /12/.

for two kinds of the central potential (due to Volkov and Eikemeier-Hackenbroich) along with the corresponding experimental spectra. The calculations involved central forces, disregarding the spin-orbital splitting. Therefore, the relative positions of the calculated levels LST-010 and LST-210 in ^{6}Li; should be compared with the value of the center of

gravity of the lowest levels $J^\pi = 1^+, 2^+, 3^+$, $T = 0$ in experiment. For the chosen versions of the central potential the relative positions of the levels vary from 1 MeV to 3.5 MeV, while the experimental value is 3.74 MeV. In all cases, however, the sequence of the levels is correct. The first excited state with $J^\pi = 1^+$, $T = 0$ with respect to the collective variable ρ can be also found. This excited state has one node when plotted versus the hyperradius ρ.

For several versions of the potential this level lies in the 15-17 MeV range. The 15.6 MeV level with $J^\pi = 1^+$ is experimentally observed in this nucleus. The results obtained make it possible to conclude that the relative position of levels in the spectra is less sensitive to the choice of the potential parameters than the absolute value of the binding energy. Similar conclusions can be drawn from the results for the spectra of lower lying excited states of the Li^7 nucleus /3/.

Fig. 6 shows the results of calculation of the reduced width amplitude for the virtual decay of ^{16}O into ^{12}C and d-particle for the nuclear states $J^\pi = 0^+$ LST= 000 and the relative orbital momentum $\Lambda = 0$.

These results practically coincide with those for the potential of the (3A-3) - dimensional harmonic oscillator.

The excited states of anomalous parity can be obtained in the $K = K_{min} + 1$ approximation. We have calculated the states with $J^\pi = 1^-$, $T = 1$ for O^{16}. Fig. 7 shows the strengths of the dipole transitions $J^\pi = 1^-$, $T = 1$ of O^{16} in percent for the first version of Volkov's potential. As a result, 20% of the dipole transition strengths is exhausted by the first monopole excited state and 5% by the

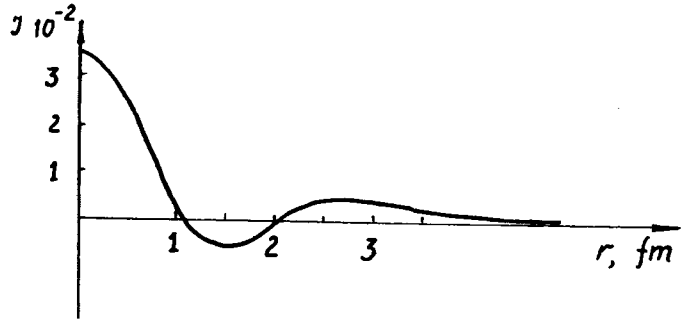

Fig.6. The amplitude of the reduced width for the virtual decay of ^{16}O into ^{12}C and d-particle in the hyperspherical method.

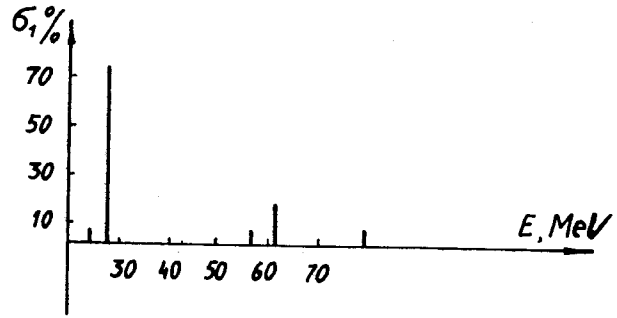

Fig.7. The strengths of the dipole transitions of O^{16} for the first version of Volkov's potential.

second. In terms of the particle-hole excitations, this corresponds to an essential admixture of the $|3p\,3h>$ and $|5p\,5h>$ states in the dipole resonance. These results are in qualitative agreement with experiment /13/. According to the experimental data about half of the integral cross-section determined by the dipole sum rule belongs to the

energy range of the giant resonance. The remaining part of the cross-section falls within the range of higher energies. The calculated nuclear compressibility is shown in fig. 8. As a function of A it grows about 3.5 times within the p-shell. The compressibility is anomalously large for the alpha-cluster nuclei. Naturally the compressibility parameter depends strongly on the choice of nucleon-nucleon potential ,e.g. the Yukawa and Gaussian potentials produce compressibilities in ^{16}O ($J^{\pi}T = 0^{+}0$) which differ by a factor of 1.3. The calculated compressibilities differ strongly(by a factor of 1.5) for the isovector ($J^{\pi}T = 1^{-}1$) and isoscalar ($J^{\pi}T = 0^{+}0$) states.

We have estimated the variations of the compressibility parameter with growing nuclear excitation. We see that $K = \frac{m}{\hbar^2} r^2 (\Delta E)^2$ does not change with increasing nuclear excitation. In fact, it is easy to show that the r_{ms} radius grows quickly enough (~ 10-13%) to compensate for the tendency of the nuclear levels to approach each other

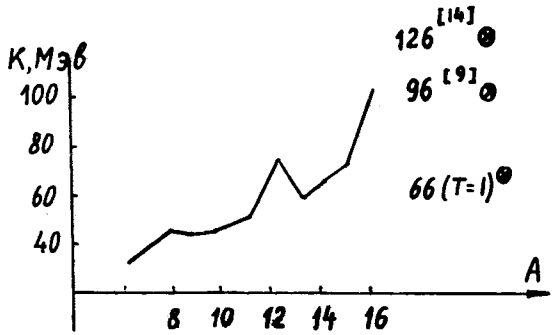

Fig.8. The calculated compressibility of the light nuclei in the K-harmonics method.

(decreasing ΔE) at larger excitations. In reference[15] giant dipole resonances were described in the shell model and a conclusion was made about the so-called loss of rigidity of nucleus in excited states. In that calculation the compressibility for the isoscalar states was taken from experiment and then it was used for the description of isovector states (J =1, T= 1). As we have shown ,the compressibility does not vary with the increasing energy, but it differs strongly for isoscalar and isovector states. We are apt to think that the estimations we have made in the K-harmonics formalism elucidate the cause of the necessity of increasing the compressibility encountered in the shell-model calculations[15].

It is a pleasure to express my gratitude to Prof. V.G.Soloviev and to Dr. Mikhailov for useful discussions and for their interest in this work.

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Received by Publishing Department
on October 5, 1977.