ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

ДУБНА

E4 - 10961

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1977

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Submitted to "Physics Letters"

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E4 - 10961

Вывод "ядерной теории поля" для схематической модели

Используя функциональные методы, мы получаем гамильтониан так называемой ядерной теории поля для схематической модели. Кроме того получаются диаграммные правила ядерной теории поля.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1977

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E4 - 10961

Functional Derivation of the "Nuclear Field Theory" for a Schematic Model

Using functional methods we derive the Hamiltonian of the so-called Nuclear Field Theory for a schematic model. As a byproduct the graphical rules of the diagrammatic nuclear field treatment are obtained.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1977

In the last few years a Nuclear Field Theory (NFT) has been developed to handle the complex interplay between collective and single particle excitations of the nuclear many-body system in a sophisticated way [1-4]. Rules for a diagrammatic perturbation expansion in terms of both single-particle and collective excitation modes have first been formulated for some schematic models [1,2]. In this perturbation treatment proper care is taken of the overcompleteness of the basis and of the identity of the nucleons appearing in the collective modes as well as in the single-particle excitations. Moreover, for a general two-body interaction the equivalence between the nuclear field treatment and the usual Feynman diagrammatic perturbation expansion (involving only fermions) has been shown in refs. [3,4] for processes connecting intermediate states. Recently [5] an attempt has been made to derive the graphical rules of the NFT via functional methods in a simple model. However, the collective field employed there is not yet the phonon field of the nuclear field expansion. Furthermore, the corresponding effective Lagrangian does not contain explicitly the fermion fields. In the present note using functional methods* we formulate a modified perturbation theory in terms of collective fields for a schematic model from which, finally, the NFT is obtained. The NFT Hamiltonian is strictly derived. The diagrammatic rules of the NFT come out quite naturally.

The model under consideration consists of $N=2\Omega$ fermions distributed over two single-particle levels, each with degeneracy 2Ω, and which interact via a schematic monopole particle-hole force

$$H = H_{sp} + H_{tb} \,, \tag{1a}$$

where

$$H_{sp} = \bar{\epsilon} \sum_{\sigma, m} \sigma a^{+}_{m\sigma} a_{m\sigma} \tag{1b}$$

and

$$H_{tb} = -V(A + A^{+})^2 \tag{1c}$$

with

$$A^{+} = \sum_{m=1}^{2\Omega} a^{+}_{m1} a_{m,-1} \,. \tag{1d}$$

The index σ takes the values $\sigma=1$ and $\sigma=-1$ for the upper and the lower level, respectively, while the index m labels the degenerate states within each level. The level spacing is given by $2\bar{\epsilon}$, and V denotes the interaction strength. In the unperturbed ground state the lower level is filled and

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*Analogous methods have been applied for deriving effective Lagrangians of more complicated theories [6].

the upper one is empty. The monopole interaction (1c) contains Hartree-Fock contributions which can be absorbed in renormalized single-particle energies $\epsilon = \bar{\epsilon} + V$. The generating functional for the fermion Green function is given by the following path-integral

$$Z[\eta,\eta^+] = \mathfrak{N} \int DaDa^+ e^{i\int dt\{\mathcal{L}_f(t) + \eta^+ a + a^+ \eta\}}, \qquad (2)$$

where

$$\mathcal{L}_f(t) = \sum_{m\sigma} a^+_{m\sigma}(t)(i\partial_t - \sigma\epsilon)a_{m\sigma}(t) + V:(A(t) + A^+(t))^2: \qquad (3)$$

is the Lagrangian corresponding to the Hamiltonian of eq. (1), and $\mathfrak{N}$ is an irrelevant normalization factor. The integration over the fermion variables a, a^+ may easily be performed after linearizing the interaction term in eq. (3) with the help of a collective (boson) variable $\phi(t)$

$$e^{i\int dt\, V(A+A^+)^2} = \mathfrak{N}_1 \int D\phi\, e^{i\int dt\{-\frac{\phi^2(t)}{4V} + \phi(t)(A+A^+)\}}. \qquad (4)$$

Introducing the single-particle Green function in the external field $\phi(t)$ via

$$g^{-1}_{\sigma\sigma'}(t,t') = g^{0-1}_{\sigma}(t,t')\delta_{\sigma\sigma'} + (1-\delta_{\sigma\sigma'})\phi(t)\delta(t-t'), \qquad (5)$$

$$g^{0-1}_{\sigma}(t,t') = (i\partial_t - \sigma\epsilon)\delta(t-t')$$

the generating functional becomes

$$Z[\eta,\eta^{+}] = \mathfrak{N}_2 \int D\phi\, e^{i\{S[\phi] - \int dt dt' \eta^{+}(t) g(t,t') \eta(t')\}}, \qquad (6)$$

where the collective action is given by*

$$S[\phi] = \int dt \{ -\frac{\phi^2(t)}{4V} - i2\Omega\, \mathrm{tr}(\log g^{-1})(t,t)\}. \qquad (7)$$

From the principle of least action, $\delta S[\phi]/\delta\phi = 0$, we get the equation of motion of the collective field

$$\phi_0(t) = -i4\Omega V[g_{1,-1}(t,t') + g_{-1,1}(t,t')]_{t'=t+0}\,. \qquad (8)$$

Excluding phase transitions** we consider in the following only the trivial solution $\phi_0 = 0$. We may now expand the collective action around the stationary point $\phi_0 = 0$. This brings the second term in eq. (7) into the form

$$-i2\Omega \int dt\, \mathrm{tr}(\log g^{-1})(t,t) = -i2\Omega \int dt\, \mathrm{tr}(\log g_0^{-1})(t,t) + \sum_n L_n[\phi],$$

where the term***

$$L_n[\phi] = -i2\Omega\, \mathrm{tr}\{ \frac{(-)^{n+1}}{n} [g_0 \begin{pmatrix} 0 & \phi \\ \phi & 0 \end{pmatrix}]^n \} \qquad (9)$$

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* The trace has to be taken over the σ index.

** The necessary condition to ensure the stability of the normal ground state is $\epsilon > 4V(\Omega - \frac{1}{4})$.

*** Matrix multiplication implies here integration over intermediate times.

represents a closed fermion loop emitting or absorbing n (even) collective lines ϕ. The second order term yields the bubble diagrams. As usual the bubble processes are included into the free action

$$S_{free}[\phi] = \int dt\{ -\frac{\phi^2(t)}{4V} \} + L_2[\phi] .$$

Thus we have

$$S[\phi] = S_{free}[\phi] + S_{int}[\phi]$$

with

$$S_{int}[\phi] = \sum_{n=4}^{\infty} L_n[\phi] .$$

The free action part $S_{free}[\phi]$ can be cast into the form

$$S_{free}[\phi] = \frac{1}{2}\int dt\,dt'\phi(t)T^{-1}(t,t')\phi(t'),$$

where

$$T(t,t') = \int \frac{d\omega}{2\pi} e^{-i\omega(t-t')}T(\omega);\; T^{-1}(\omega) = -\left[\frac{1}{2V} + \frac{8\Omega\epsilon}{\omega^2 - 4\epsilon^2} \right]$$

is the free propagator of the collective field which is easily recognized as the T - matrix in the ladder approximation. From $T^{-1}(\omega) = 0$ we find the frequency of the collective excitation mode being equal to

$$\omega_0 = 2\epsilon\sqrt{1 - \frac{4V\Omega}{\epsilon}} . \tag{10}$$

For subsequent considerations it is convenient to rewrite the propagator as

$$T(\omega) = -2V + T_c(\omega), \quad T_c(\omega) = \Lambda D(\omega)\Lambda ,$$

$$D(\omega) = \frac{1}{\omega^2 - \omega_0^2}, \quad \Lambda = 4V\sqrt{2\Omega\epsilon} . \tag{11}$$

Our final aim is to obtain an effective Lagrangian in terms of fermion and collective (phonon) fields including their coupling. For this purpose, we express $\exp(iS_{int})$ again by an integral over fermion variables. Introducing a source term $j\phi$ in the exponent of eq. (6) and integrating over the ϕ variable the generating functional takes the form

$$Z[\eta,\eta^{+}] = \mathfrak{N}_3 e^{-iL_2[\frac{1}{i}\frac{\delta}{\delta j}]} \int DaDa^{+} \times$$

$$\times \exp\{i\int dt[jVj + 2Vj(A+A^{+}) + \mathfrak{L}_f(t) + \eta^{+}a + a^{+}\eta - \tag{12}$$

$$- \frac{1}{2}\int dt'(A+A^{+}+j)T_c(t,t')(A+A^{+}+j)]\}|_{j=0} .$$

In analogy to eq. (4) we can now introduce a new collective field $\varphi(t)$, the free propagator of which is given by $D(\omega)$, defined in eq. (11). Then, the generating functional reads

$$Z[\eta,\eta^{+}] = \mathfrak{N}_4 e^{-iL[2V(\tilde{A}+\tilde{A}^{+}) + \frac{1}{i}\frac{\delta}{\delta j}]} \times$$

$$\times \int DaDa^{+}\int D\varphi e^{i\int dt\{\mathfrak{L}_{nf}(t) + j\Lambda\varphi + \eta^{+}a + a^{+}\eta\}}|_{j=0}, \tag{13}$$

where

$$\mathfrak{L}_{nf} = \mathfrak{L}_f + \mathfrak{L}_{ph} + \mathfrak{L}_{int} . \tag{14}$$

is just the effective Lagrangian in the NFT which comprises besides the full fermion

Lagrangian (defined by eq. (3)) a free phonon and an interaction term which are given by

$$\mathcal{L}_{ph} = \frac{1}{2}\varphi(t) D^{-1}(t,t')\varphi(t')$$

and

$$\mathcal{L}_{int} = \Lambda\varphi(t)(A(t) + A^{+}(t))$$

respectively. $\tilde{A}$ is obtained from A by the replacement $a \to \frac{\delta}{i\delta\eta^{+}}$, $a^{+} \to \frac{\delta}{i\delta\eta}$. Quantization of the collective field yields

$$\varphi(t) = \frac{1}{\sqrt{2\omega_0}}(C(t) + C^{+}(t)) ,$$

where $C(t)$, $C^{+}(t)$ are the phonon operators in the interaction picture. Then the Hamiltonian corresponding to eq. (14) reads

$$H_{nf} = H_{sp} + H_{ph} + H_{tb} + H_{int}$$

with

$$H_{ph} = \omega_0 C^{+} C ,$$

$$H_{int} = -\Lambda'(C + C^{+})(A + A^{+}), \quad \Lambda' = \frac{\Lambda}{\sqrt{2\omega_0}} .$$

Thus H_{nf} is just the Hamiltonian of the NFT, which has previously been derived only empirically (see, e.g., ref. /1,3,4/). The functional method used in the present paper yields not only the NFT Hamiltonian but provides us also with the corresponding graphical rules for a diagrammatic perturbation theory based on this Hamiltonian: The term in front of the functional integral in eq. (13) (see also eq. (9)) projects out the bubble diagrams from the perturbation expansion, which have already been included in the definition of the collective field. The obtained results are not restricted to the schematic model considered here but can easily be generalized to more complicated situations.

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Received by Publishing Department
on September 16,1977.