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Двухфазная модель вращающихся ядер

На основе метода генераторной координаты сформулирована модель, в которой ядро представляется как трехосный ротатор, имеющий нормальную и сверхпроводящую фазы, связанные между собой. Диагонализуя матрицу трехосного ротатора при помощи когерентных состояний, получаем энергетический спектр ядра. Рассчитаны вероятности перехода в пределе больших I . Рассматриваются эффекты различных ориентаций массового квадруполоида относительно углового момента. Расчёты проведены для ядер ^{156,166}Er.

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Two-Phase Model of Rotating Nuclei

Using the generator-coordinate method the model is formulated in which the nucleus is treated as a triaxial rotator with coupled normal and superconductive phases. Averaging the model Hamiltonian over coherent states we study the effects of different orientations of the mass quadrupoloid with respect to the angular momentum for the case of ^{156,166}Er nuclei.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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The investigation of spectroscopic properties at moderate values of angular momentum I are carried out usually suggesting that the average field is axially symmetric. Although supported by the calculation of the equilibrium deformation, this suggestion may be invalid at finite I due to the strong deformation dependence of inertia properties. The possibility to describe the spectra of some transitional odd-mass nuclei in the model with a quasiparticle coupled to a triaxial core[1] gives an empirical indication to that.

The onset of triaxial deformation induced by the rotation may be investigated starting with the generator-coordinate method. Starting from the superposition of different "intrinsic" states[2] one writes

$$\Psi_{IM} = \sum_{ik} c^{I}_{ik} P^{I}_{Mk} |\Delta_i\rangle . \qquad (1)$$

In eq. (1) $|\Delta_i\rangle$ is a ECS-function with the gap parameter Δ_i,

$$P^{I}_{Mk} = \frac{2I+1}{8\pi^2} \int d\Omega D^{*I}_{Mk}(\Omega) \hat{R}(\Omega), \qquad (2)$$

where $\hat{R}(\Omega) = e^{-i\alpha\hat{J}_z} e^{-i\beta\hat{J}_y} e^{-i\gamma\hat{J}_z}$ is the operator of rotation through the Euler angles $\Omega = (\alpha, \beta, \gamma)$. Using the standard technique of the angular projection method one introduces the overlap matrices

$$\begin{matrix} \tilde{h}^{I}_{ik,i'k'} \\ \tilde{n}^{I}_{ik,i'k'} \end{matrix} \equiv \langle \Delta_i | \overset{H}{1} P^{I}_{kk'} | \Delta_{i'} \rangle = \frac{2I+1}{8\pi^2} \int d\Omega \, D^{*I}_{kk'}(\Omega) \langle \Delta_i | \overset{H}{1} R(\Omega) | \Delta_{i'} \rangle \tag{3}$$

and the Hamiltonian matrix

$$\mathcal{H} = \tilde{n}^{-1/2} \tilde{h} \tilde{n}^{-1/2} . \tag{4}$$

The eigenvalue problem for $\mathcal{H}$ determines the energies E_I and the eigenvectors f^I

$$f^{I}_{ik} = \sum_{i'k'} \tilde{n}^{1/2\,I}_{ik,i'k'} c^{I}_{i'k'} . \tag{5}$$

The knowledge of f^I or c^I may be used in order to calculate the matrix elements of physical operators.

Assuming the field essentially triaxial one writes

$$\tilde{n}^{I}_{ik,i'k'} = n^{I}_{ii'} \delta_{kk'} \quad (k, k' \geq 0). \tag{6}$$

Then, using the procedure of Kamlah [3] for the evaluation of $\langle \Delta_i | HR(\Omega) | \Delta_i \rangle$ one comes to

$$\tilde{h}^{I}_{ik,i'k'} = V_{ii'} \delta_{kk'} + \frac{1}{2} \sum_{\nu=1}^{3} A^{(\nu)}_{ii'} [(\mathcal{Q}^2_\nu)_{kk'} + (\mathcal{Q}^2_\nu)_{k,-k'}] \tag{7}$$

where the following symmetry properties of the ECS function were taken into account

$$e^{i\pi\hat{J}_\mu}\ |\Delta_i>=|\Delta_i>\quad(\mu=1,2,3),\ <\Delta_i|\hat{J}_\mu|\Delta_i>=0\,. \qquad (8)$$

Thus, the generator-coordinate method leads to the model of a triaxial rotator having several coupled phases marked by the index i. We note that the coefficients f^I_{ik} have the meaning of amplitudes in the expansion for the wave function of such a rotator in the functions having fixed angular momentum projections to one of the axis. The parameters in eq. (7) may be found from the many-body Hamiltonian in a straightforward way giving the microscopical description of the spectroscopic properties of nuclear states. This program is quite involved and may call for the improvement of the theory in many important details. For this reason we do not follow this line but use some additional approximations in order to study the general properties of the model.

First, we limit the sum in eq. (1) to two terms: $\Delta_1=\Delta\neq 0$, $\Delta_2=0$. The overlap between such states is considered to be small, i.e., $n_{12}=<\Delta|0>\equiv 2\eta \ll 1$. Then $c^I_{ik}\approx f^I_{ik}$. The Hamiltonian H can be written as a 2 x 2 matrix with the elements operating in the space of states with different

$$H=\begin{pmatrix} \frac{1}{2}\sum_{\nu=1}^{3} A^{(\nu)}_{11}\hat{\mathcal{J}}^2_\nu + V_1 & \xi \\ \xi & \frac{1}{2}\sum_{\nu=1}^{3} A^{(\nu)}_{22}\hat{\mathcal{J}}^2_\nu + V_2 \end{pmatrix} \qquad (9)$$

Second, we treat the inertia parameters as phenomenological ones. The moments of inertia in the correlated phase

$$\mathcal{J}_1^{(\nu)} = (A_{11}^{(\nu)})^{-1} = B\,\frac{(a_\mu^2 - a_\eta^2)^2}{(a_\mu^2 + a_\eta^2)}, \qquad (10)$$

where B is the fitting parameter and a_ν are the nuclear semiaxes. In the normal phase the rigid body moments of inertia are used. The coupling between the two phases is approximated by a constant ξ.

The third simplification we introduce consists in the use of coherent states [4] which in the limit $I \gg 1$ are expected to be a good approximation to the yrast state eigenvectors f^I. We write

$$c_{ik}^I = c_i D_{kI}^I(\phi,\theta,0) \qquad (11)$$

and pass from the eigenvalue problem to the variational problem in which the Hamiltonian $\mathcal{H}$ from eq. (9) is averaged over coherent states (11) and then the expectation value is minimized with respect to the deformation parameters β, γ and the angles ϕ,θ.

To calculate the B(E2) values the following approximate relation is used for the matrix elements of the charge-quadrupole operator taken between the intrinsic states:

$$\langle \Delta_i | \hat{\mathfrak{M}}_{2\kappa} \hat{R}(\Omega) | \Delta_{i'} \rangle = \frac{5}{4\pi} Ze \langle r^2 \rangle \beta \{ \delta_{\kappa,0} \cos\gamma + (\delta_{\kappa,2} + \delta_{\kappa,-2}) \frac{1}{\sqrt{2}} \sin\gamma \} n_{ii'}(\Omega), \qquad (12)$$

where β and γ are functions of semiaxes a_ν in eq. (10). If the angles ϕ, θ are the same for the states involved in a transition, then

$$B(E2, I+2\to I) = \frac{5}{16\pi}(eQ_0)^2 \Big| \sum_{i=1}^{2} c_{I+2,i}\, c_{I,i} \Big|^2 \Big[\cos\gamma \cdot D^2_{02}(\phi,\theta,0) + \\ + \frac{\sin\gamma}{\sqrt{2}}\big[D^2_{22}(\phi,\theta,0) + D^2_{2-2}(\phi,\theta,0)\big]\Big]^2 . \qquad (13)$$

In particular, if $\theta = \frac{\pi}{2}$, eq. (13) reads

$$B(E2, I+2\to I) = \Big(\frac{3}{8\sqrt{2\pi}} ZeR^2\beta\Big)^2 \Big| \sum_{i=1}^{2} c_{I+2,i}\, c_{I,i} \Big|^2 (\cos\gamma\sqrt{3} + \sin\gamma\cos 2\phi)^2, \qquad (14)$$

where Ze is the electric charge, $R = 1.2A^{1/3}$ fm. In the case of small γ and moderate values of I eq. (11) does not hold and is to be substituted by

$$c^I_{ik}(\text{axial}) = c^I_i \delta_{k,0} . \qquad (15)$$

The choice between (11) and (15) can be made remembering that in the case of the triaxial rotator the projection of the angular momentum on the axis 1 of the body-fixed frame of reference is approximately conserved if I satisfies the inequality

$$I \ll \frac{A_3 + A_2 - 2A_1}{2\sqrt{(A_3 - A_1)(A_2 - A_1)}} (A_1 \le A_2 \le A_3). \qquad (16)$$

The difference between (11) and (15) does not change essentially the energies of states and the B(E2) values, but leads to a hindrance factor in the E2-transition if the transition leads from the state with the c^I_{ik} coefficients given by eq. (11) to the state for which eq. (15) is valid

$$B'(E2, I+2 \to I) = \alpha B(E2, I+2 \to I). \qquad (17)$$

The hindrance factor α is

$$\alpha = |\langle \gamma \neq 0 | \gamma \approx 0 \rangle|^2 = (D^I_{0I}(0, \frac{\pi}{2}, 0))^2 \approx \frac{1}{\sqrt{\pi I}} . \qquad (18)$$

The model was applied to the $^{156,166}Er$ isotopes using the calculations of the potential energy $V_i(\beta, \gamma)$ with the shell correction method of Strutinsky. In the transitional nucleus ^{156}Er γ increases rapidly with I, reaches about 25° at I =10 and then drops down to very small values. This effect is accompanied by the decrease of the probability to find the nucleus in the correlated phase. A more detailed information concerning the yrast band of this nucleus is displayed in the table. The estimates for the B(E2) values in ^{156}Er have been calculated with eq. (14) for all the transitions but $12^+ \to 10^+$. In the latter case the γ-deformation changes in the transition and eqs. (17), (18) are used. Contrary to ^{156}Er the other isotope ^{166}Er appears to be quite stable in β and γ with I . The axiality is not broken here essentially up to $I \gtrsim 20$. In the case of ^{166}Er no hindrance is expected. This corresponds nicely to the measurements of life-times of states in Er isotopes reported in [7] . Experimentally it was

Table

Calculated deformation parameters β and γ, energy values E_I, transition ratios $B(E2, I+1 \to I)/B(E2, 2 \to 0)$ and the experimental energies[6] of the yrast band in ^{156}Er.

I	β	$\lvert\gamma\rvert^\circ$	E_I^{theor} (keV)	E_I^{exp}	$\frac{B(E2, I+2 \to I)}{B(E2, 2 \to 0)}_{theor.}$
2	0.213	17	545	344.2	1
4	0.219	20	973	797	1.080
6	0.226	23	1465	1340	1.168
8	0.234	26	2017	1959	1.265
10	0.237	25	2638	2633	1.294
12	0.213	6	3740	3315	0.15
14	0.213	5	4152	3838	0.899
16			4618	4383	
18			5614	5008	
20	0.213	5	5725	5718	1
22			6365	6490	
24			7064	7317	

found that the $12^+ \to 10^+$ transition in ^{156}Er is slowed down by about a factor of four while in ^{166}Er no deviation from the Alaga rules is found.

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