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Теоретические энергии деформации очень нейтронодефицитных ядер с $84 \le Z \le 89$

Проведены вычисления энергии деформации,основанные на методе оболочечной поправки Струтинского и модели аксиально-симметричного ротатора, для выяснения форм ираст-состояний с малым спином в очень нейтронодефицитных изотопах элементов с Z от 84 до 88, Рассчитывалась также энергия деформации основного состояния легких нечетных изотопов в засвинцовой области. Предсказывается сплюснутая форма основного состояния ($\epsilon \approx -0{,}22$) в изотопах с A вблизи 196 и Z = 86. Для ^{196}Rn отмечается необычное пересечение полос, не принадлежащих ираст-линии и построенных на состояниях, которые являются изомерами формы с малой ($\epsilon \approx 0{,}13$) и большой ($\epsilon \approx 0{,}31$) вытянутой деформацией. Предсказывается внезапное изменение среднеквадратичного радиуса в нечетных изотопах At и Fr.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Theoretical Deformation Energy for Very Neutron-Deficient Nuclei with $84 \le Z \le 89$

The theoretical deformation energy for very neutron-deficient isotopes of both even-even and odd-mass nuclei lying beyond lead has been calculated and discussed. The oblate ground state shape ($\epsilon \approx -0.22$) for isotopes with A around 196 and Z = 86 is predicted. For ^{196}Rn the unusual crossing of non-yrast bands based on shape isomeric states at small ($\epsilon \approx 0.13$) and large ($\epsilon \approx 0.31$) prolate deformations has been noticed. Predictions are made for sudden changes in the mean-square radius in odd-mass At and Fr isotopes.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

Neutron-deficient isotopes of elements adjacent to lead provide a good possibility of testing the standard deformation energy calculations. In this nuclear region the shape coexistence phenomenon becomes probable in some even-even nuclei, the nuclear shape being dependent on the spin of the state. Furthermore, the addition of an odd particle may also change the equilibrium deformation with respect to that of the even-mass neighbours. Estimations of this effect were first performed by Soloviev /1/. Thus, the most traditional interpretation of rather rich experimental data on irregularities in level spacings, B(E2)-values and charge radii of very neutron-deficient isotopes is given in terms of the transition from a slightly deformed shape to a well deformed one (see, e.g., the review by Hamilton /2/). In general, similar effects are expected to occur in the neighbouring nucleu. There is some hope to produce very neutron-deficient odd-mass Fr isotopes at the on-line isotope separator ISOLDE at CERN /3/ to look for possible shrinking effects within a long isotopic chain. Till now, no systematic deformation energy calculation has been made for very β-unstable nuclei with proton numbers between 82 and 90. In the present paper we complete the earlier investigation /4/ of deformation energy surfaces for the mass region mentioned with special emphasis on light even-mass Rn isotopes and neighbouring odd-mass nuclei.

After a brief discussion on the method used (sect. 2) we give a short review of the earlier theoretical predictions (sect. 3), describe the shape transitions in even-even Rn isotopes (sect. 4) and in odd-mass Po, At, Rn, Fr isotopes (sect. 6) and discuss a possible new type of band crossing in ^{196}Rn (sect. 5).

2. METHOD

In order to calculate the ground state (g.s.) of even-even and odd-mass nuclei as a function of axially symmetric deformation parameters we have applied the Strutinsky shell correction approach using the deformed Woods-Saxon potential. The pairing correlations of monopole type are included. Low-spin yrast states in even-even nuclei are considered taking into account the rotation of the nucleus in a perturbative manner. The further details of the method employed and of the choice of parameters are given in ref. /4/. The calculation reported here has been performed using the formerly proposed scheme /4/. It should be pointed out that for the nuclei considered the oscillating shell corrections for neutrons and protons are generally out of phase and their superposition leads to several local minima in the deformation energy.

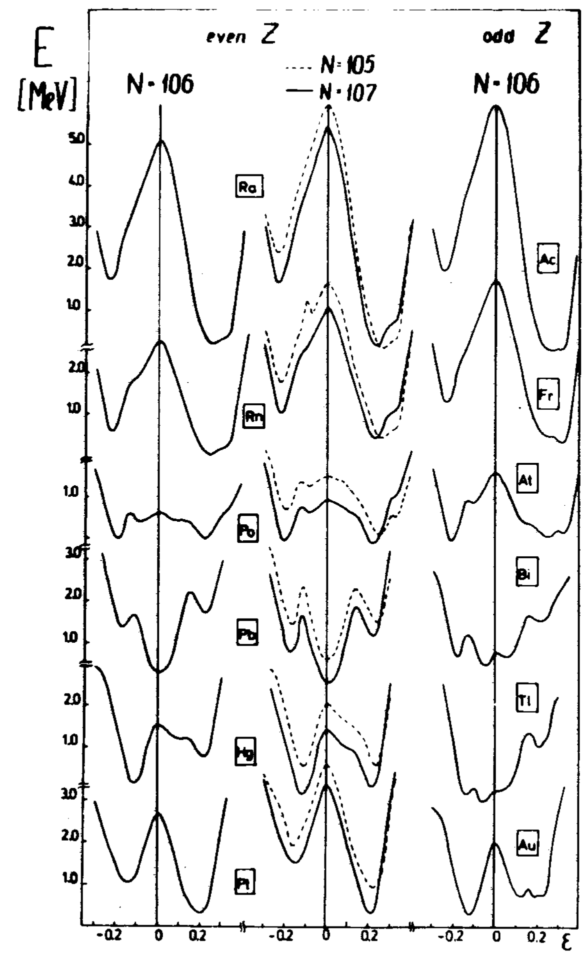

Fig. 1. The g.s. deformation energy for even-even and odd-mass nuclei in the Pb region with Z=78, 80, 82, 84, 86, 88; N=106/N=107(105) and Z=79, 81, 83, 85, 87, 89; N=106. For the equilibrium hexadecapole deformations and the pairing strengths used, see the text and ref. /4/.

3. SHAPES OF EVEN-EVEN Pt, Hg, Pb AND Po ISOTOPES

In this section the most important trends in g.s. energies of even-even neutron-deficient nuclei around lead, known from the previous investigation, are briefly summarized. Let us begin with the Pt isotopes with the proton number 4 units less than that of the closed spherical shell. As can be seen from fig. 1, very light isotopes with $A \approx 184$ have well deformed prolate shapes in the g.s. With increasing mass number the oblate and prolate minima take symmetric positions and come very close in energy at $A \approx 190$. These isotopes may become unstable against the non-axial deformation γ, which is not included in our calculation. Experiments with even-mass isotopes do not provide any evidence for the existence of bands belonging to separated minima. The appearance of decoupled bands in heavier odd-mass Pt nuclei supports the assumption concerning the oblate-like shape of the adjacent even-even ones.

The light Hg isotopes with $182 \leq A \leq 188$ are predicted to have slightly deformed oblate shapes in the g.s. /5/. New experimental results seem to indicate that there exists a barrier between the two minima /6/.

For the Pb isotopes no shape transition is predicted as the mass number decreases. Due to the magic proton number, even in very light isotopes with mass numbers around 184 the spherical minimum remains to be the deepest one.

Very light Po isotopes with $A < 190$ are predicted to have a well deformed prolate shape. The g.s. of ^{192}Po has an oblate shape with $\epsilon = -0.2$. For $A \geq 194$ this shape is nearly spherical.

The shapes of rotating nuclei may be different from the g.s. ones. Shape transitions from a slightly deformed oblate shape to a well deformed prolate one are known at angular momenta $I \approx 4$ in the light isotopes ^{184,186,188}Hg /5/ and from a spherical shape to an oblate or prolate deformed one in the case of light Pb isotopes with mass numbers around 188. No shape transition can be unambiguously predicted in the light Po isotopes.

IV. THE SHAPES OF LIGHT EVEN-EVEN Rn ISOTOPES

There is no close correlation between the deformation energy of nuclei with the same number of neutrons and 82 ± 2 protons, but the general features of the potential energy curve for Rn resemble that for Hg. The deformation energy $E_0(\epsilon)$ at $I = 0$ for the Rn isotopes is shown in fig.2. The curve has a pronounced minimum at $\epsilon \approx -0.22$ for $A \leq 198$. In the case of ^{198,196,194}Rn this oblate minimum is the absolute one. Such a comparatively large oblate g.s. deformation has already been mentioned for ^{192}Po, although the spherical maximum found in that case was only about 0.5 MeV above the minimum. At this deformation the only proton levels lying about 2 MeV above or below the Fermi level for $Z = 84$, 86, and 88 are four descending high angular momentum orbitals ($h_{9/2}$, $i_{13/2}$) and one ascending ($s_{1/2}$) level. In addition, contrary to the general case, the shell corrections both for protons and neutrons are in the phase that leads to a deep g.s. minimum at $\epsilon \approx -0.22$.

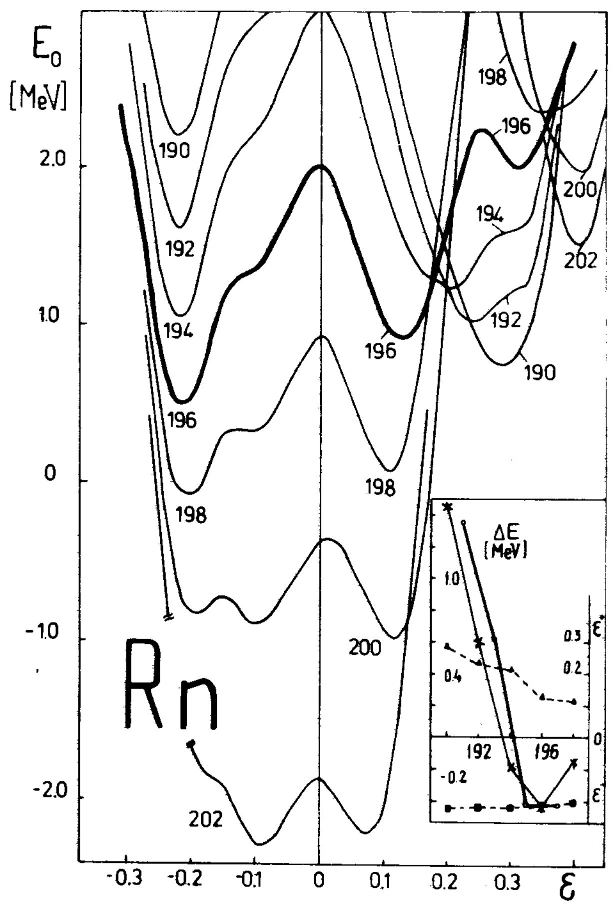

Fig. 2. The g.s. deformation energies $E_0(\epsilon)$ for even-even Rn isotopes in the interval $190 \le A \le 202$. The hexadecapole deformation $\epsilon_4 = 0$ is close to the equilibrium one. For the averaged gaps $\tilde{\Delta}_{p(n)} = 13(11)/\sqrt{A}$ MeV is used. In the lower right-hand corner of the figure the oblate-prolate energy difference ΔE for the even-even (*) and odd-neutron (o) isotopes and the equilibrium deformation of both minima (■, ▲) of the even-even isotopes (the right-hand side scale) are shown.

For $A = 198$, 196 a prolate minimum is seen at $\epsilon \approx 0.12$ about 0.3 MeV above the oblate one. Unlike the light Hg isotopes, it lies at a smaller deformation than the oblate one. Consequently, for these nuclei there may be no shape transitions in the lowest yrast states. Between mass numbers 194 and 192 the two minima change their relative positions and the g.s. deformation increases considerably. For ^{190}Rn the equilibrium deformation and the oblate-prolate energy difference ($\Delta E_I = E_I^{min}(obl) - E_I^{min}(prol)$) have the values of $\epsilon = 0.28$ and $\Delta E = 1.5$ MeV, respectively. We specially note that for $190 \le A \le 198$ the oblate g.s. minimum does not almost change its position. As in the experimentally investigated Pt isotopes, in Rn isotopes around $A = 196$ there are two possibilities. If a barrier exists between the two minima at $\gamma \neq 0$, two separated bands should be observed, the moment of inertia of the higher one being about twice smaller than that of the yrast band. If the prolate minimum is a saddle point, a single band with a disturbed level sequence should be expected.

In the g.s. band the smooth growth of level spacings is broken at $I = 10$, as is seen in the left-hand side of fig. 3. In the calculation this is due to the presence of high-spin orbitals close to the Fermi energy, that make the perturbative approach invalid, as already discussed in ref. /4/. An adequate description of this part of the band may presumably be obtained by taking rotational alignment into account.

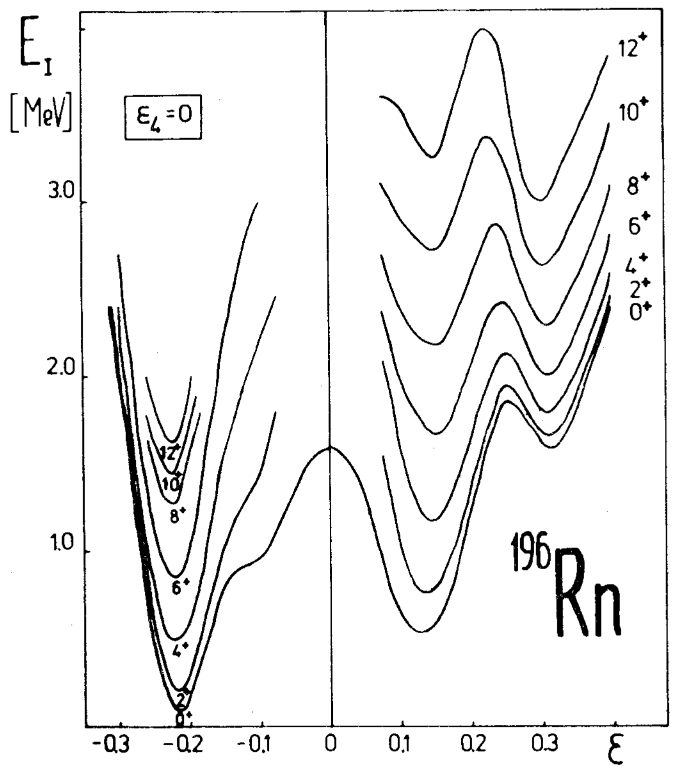

Fig. 3. Deformation energy $E_I(\varepsilon)$ (BCS) for angular momenta $I = 0(2)12$. The hexadecapole deformation and the pairing strengths are the same as indicated in the caption to fig. 2.

The variation of the smoothing parameter results in only a minor change of the shape of the deformation energy curve. For example, the oblate-prolate energy difference ΔE for ^{196}Rn changes by no more than 0.15 MeV at a 30% variation of the smoothing parameter.

Let us briefly consider the case of 6 particles above the spherical proton shell (Ra isotopes). As can be seen from fig. 1, ^{194}Ra and adjacent odd-mass nuclei are predicted to have well deformed prolate shapes in the g.s. Preliminary investigations have shown that the oblate-prolate transition is shifted to higher mass numbers ($A \approx 198$).

5. POSSIBLE NEW TYPE OF BAND CROSSING IN ^{196}Rn

On the g.s. energy curve of ^{196}Rn there is a third local minimum at $\varepsilon \approx 0.31$ (see fig.3), which is separated by a barrier of about 0.2 MeV from the earlier discussed prolate minimum. Of course, such a shallow barrier could not lead to a metastable state. However, the barrier increases with increasing spin. As is seen from fig. 3, for $I = 12$ its height is about 1 MeV. Thus, for higher angular momenta the possibility of a separated stronly deformed band cannot be ruled out.

After summarizing our results we suggest the following bands for ^{196}Rn. Besides the yrast band corresponding to the well deformed oblate shape with irregular level spacings around $I = 10$, there may exist another band starting at about 0.4 MeV. For this band the back-bending behaviour is predicted due to the crossing of the band of slightly deformed states with the second isomeric one corresponding to the large prolate deformation. The back-bending phenomenon occurs at about $I = 8$. The moment of inertia of strongly deformed states is 3-4 times as large as that of the slightly deformed ones.

In the case of heavier isotopes the two minima corresponding to small deformations fuse gradually. The third minimum is shifted to larger deformations, e.g. $\epsilon \approx 0.4$ for A=202. For lighter isotopes the third minimum is shifted to smaller deformations to fuse gradually with the second one. Thus, the suggested new type of band crossing is localized around ^{196}Rn with a possible uncertainty of about two mass units.

6. THE G.S. SHAPES OF ODD-MASS NUCLEI

In order to study the effect of the odd particle we calculate the g.s. deformation energy $E_0(\epsilon, \epsilon_4)$ for the odd-proton Au, Tl, Bi, At, Fr nuclei and the odd-neutron Pt, Pb, Po, Rn nuclei in the mass range $181 \leq A \leq 199$. The well-known results for light odd-mass Hg isotopes /5/ are not discussed here. The general trends of the equilibrium shapes for odd-proton nuclei with N =106 and for odd-neutron nuclei with N =105 and 107 can be seen in fig. 1. As may be expected, the light odd-mass Pb, Tl and Bi isotopes are almost spherical in the g.s. We shall discuss in more detail the light odd-mass Po, At, Rn, and Fr isotopes. Figure 4 shows the g.s. deformation energy curves for the odd-mass Fr isotopes in the interval $191 \leq A \leq 213$. The curves have at least two minima. For ^{191}Fr the oblate minimum is about 2 MeV higher than the prolate one. Thus, ^{191}Fr should have a well deformed prolate shape ($\epsilon = 0.325$, $\epsilon_4 = -0.02$). A transition from prolate deformed shapes for $A < 195$ ($\epsilon \geq 0.3$) to oblate ones ($\epsilon = -0.225$, $\epsilon_4 = 0$) for $A \gtrsim 195$ is easily seen. For ^{197}Fr, ΔE amouns to

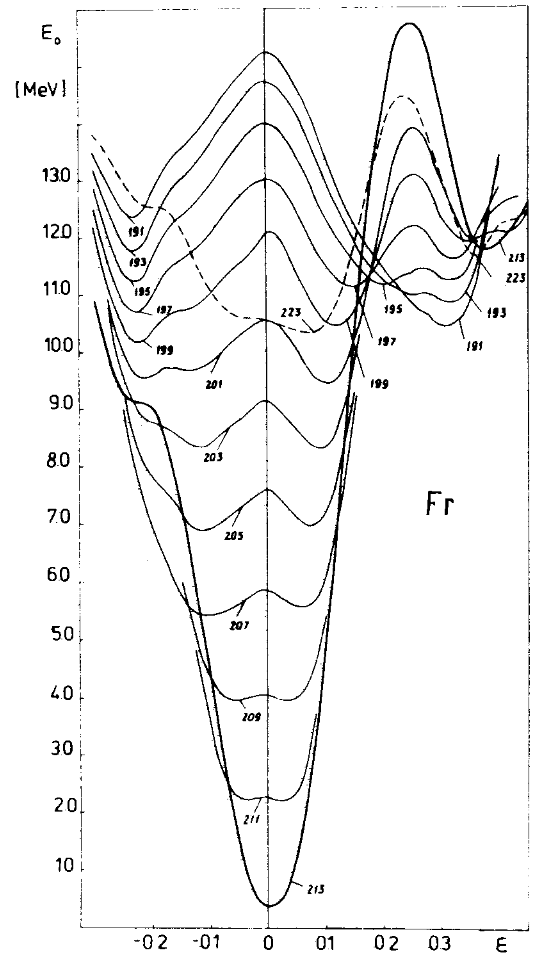

Fig. 4. The deformation energies $E_0(\epsilon)$ (ECS) of the odd-mass Fr isotopes. The hexadecapole deformation is fixed for all curves to 0. The pairing strengths are the same as indicated in the caption to fig. 2.

-0.44 MeV. With increasing neutron number both equilibrium points are moving in the direction of zero deformation ϵ. The value of ΔE becomes small and the curves for different isotopes approach the deep spherical pattern of ^{213}Fr (126 neutrons). A second minimum is seen at $\epsilon \approx 0.35$ lying 11 MeV above the g.s. one. For still heavier isotopes (e.g. ^{223}Fr) a shape close to the spherical one remains the lowest one, but the second minimum is lowered by several MeV. The usually measured values of $\delta\beta^2$ for Po, At, Rn, and Fr are given in table, where $\delta\beta^2(A) = \beta^2(A) - \beta^2(A')$ and A' is a reference nucleus. Our calculation predicts a sudden jump of the nuclear radius between ^{191}At and ^{189}At and between ^{197}Fr and ^{195}Fr. As seen from fig. 2, for the light odd-neutron Po and Rn isotopes the minima at oblate and prolate deformations take almost symmetric positions. Thus, for these nuclei neither the shrinking effect nor odd-even shape staggering is expected. The latter conclusion was drawn from the behaviour of the corresponding curves $\Delta E(A)$ in fig. 2. However, similarly to the light Hg isotopes, the odd-even shape staggering may exist in the odd-proton Fr isotopes with respect to the even-even isotopes. As seen from the table and fig. 2, ^{195}Fr is predicted to be prolate whereas ^{194}Rn is found to be oblate.

7. CONCLUSIONS

The deformation energy based on the Strutinsky shell correction approach and the axially symmetric rotor model is used to calculate the shapes of low-spin yrast states in very neutron-deficient isotopes of elements with $Z = 84-88$. The shrinking effect is predicted for light At and Fr nuclei. For the even-mass Rn isotopes with $A \approx 196$ a crossing

Table

Equilibrium deformations ϵ^+ and ϵ^- the oblate-prolate energy difference ΔE and the difference $\delta\beta^2$ between the equilibrium deformation squared of the given nucleus with that of the reference one for which $\delta\beta^2$ is not given(–) for odd-mass Po, At, Rn and Fr isotopes. The hexadecapole deformation is denoted in parentheses; otherwise $\epsilon_4 = 0$ is implied. Only those odd-mass isotopes for which $\Delta E \leq 1.5$ MeV are included in the table

Isotope	ϵ^+	ϵ^-	ΔE (MeV)	$\delta\beta^2$
^{187}Po	0.275	-0.195	1.48	0.063
^{189}Po	0.245	-0.195	0.60	0.040
^{191}Po	0.215	-0.20	0.06	0.021
^{193}Po	0.085	-0.20	-0.42	-
^{187}At	0.32	-0.22	1.40	0.101
^{189}At	0.285	-0.21	0.75	0.068
^{191}At	0.24	-0.21	-0.15	-
^{193}At	0.12	-0.205	-0.53	
^{195}At	0.105	-0.205	-0.36	
^{191}Rn	0.26	-0.22	1.34	0.044
^{193}Rn	0.23	-0.225	0.59	0.022
^{195}Rn	0.165	-0.22	-0.42	-
^{197}Rn	0.125	-0.215	-0.45	
^{193}Fr	0.325 (a)	-0.235	1.03	0.097
^{195}Fr	0.31	-0.231	0.10	0.082
^{197}Fr	0.145	-0.23	-0.44	-
^{199}Fr	0.12	-0.225	-0.27	

(a): $\epsilon_4 = -0.02$.

of two shape isomeric bands is predicted for $I \approx 8$ at about 2 MeV.

We would like to emphasize that in the specific case of ^{196}Rn the total spin I seems to be small enough to justify the application of the perturbation approach. However, it should be noted that for more reliable predictions of the properties of yrast states with $I > 8$ as well as the value of the angular momentum for which the crossing of the shape isomeric bands is taking place the non-perturbative treatment of rotation should be involved. Further,it seems to us to be very important to complete these investigations by the inclusion of the γ -deformation degree of freedom and, moreover, by taking into account the different types of dynamics and the interplay between them. Nevertheless, the results discussed here may stimulate studies to obtain evidence for the predicted shape transition effects in the lead region.

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