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T.I.Kopaleishvili, A.I.Machavariani,
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THE STUDY OF πd -SCATTERING IN THE (3.3) RESONANCE REGION ON THE BASIS OF THREE-BODY RELATIVISTIC EQUATIONS

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* Tbilisi State University, USSR

Копалейшвили Т.И., Мачавариани А.И., Емельяненко Г.А.

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Исследование πd – рассеяния в области (3.3)-резонанса на основе релятивистских уравнений

Исследуется πd -рассеяние в области (3.3)-резонанса на основе частичных релятивистских уравнений. Рассчитаны дифференциальное и интегральное сечения упругого πd -рассеяния и полное сечение с использованием матриц πN -столкновения, определенных при помощи фаз рассеяния до 300 МэВ [9] и на основе решения обратной задачи [20]. Показано, что: 1) эффект полного учета релятивистской кинематики пиона того же порядка, что и эффект многократного рассеяния; 2) πd -рассеяние достаточно чувствительно к внеэнергетическому поведению матрицы πN -столкновения; 3) основной вклад от многократного рассеяния в сечения вносят члены с NN -перерассеянием.

Проведено сравнение с экспериментальными данными.

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Kopaleishvili T.I., Machavariani A.I., Emelyanenko G.A.

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The Study of πd -Scattering in the (3.3) Resonance Region on the Basis of Three-Body Relativistic Equations

The pion-deuteron scattering in the (3.3) resonance region is studied on the basis of the three-body relativistic equations [6]. The differential and integral cross sections for the πd elastic scattering and total cross section are calculated using the πN collision matrices defined by fit with phase shifts up to 300 MeV [9], on the one hand, and the solution of the inverse πN scattering problem [20]. It is shown that i) the effect of the full account of the relativistic pion kinematics is of the same order as the multiple scattering effect, ii) the pion-deuteron scattering is rather sensitive to the off-shell behaviour of πN scattering matrix and iii) the main contribution from the multiple scattering into the cross sections comes from the terms with NN rescattering. The comparison with the experimental data is carried out.

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1. INTRODUCTION

The investigation of the pion-deuteron scattering is the first and necessary step for understanding pion scattering processes on more complicated nuclei. At present there is no universally accepted dynamical framework within which to describe the pion-nuclear system. For the pion-deuteron scattering in the pion kinetic energy region $T_\pi \leq$ ≤ 300 MeV the Faddeev equations on their relativistic generalizations which are solvable (numerically) may serve as such a basis.

In the past few years many studies [1-12] were devoted to the investigations of the πd -scattering in the framework of three-body equations. Some of them use the non-relativistic three-body equations not only at low pion energy (in the calculations of the πd -scattering length [1-3]), but also at the pion energy in the (3.3) resonance region [5,11]. Making use of three-body relativistic equations obtained in ref. [10] we have shown that the relativistic kinematics has to be taken into account even in the calculations of the πd -scattering length. In studies [6-9] some aspects of relativity (in the main the kinematical one) are included into consideration. For example, in ref. [7] one of the versions of the three-body relativistic equations obtained in ref.[13] with the help of Blankenbecler-Sugar (BS) procedure[14] is used, but the relativistic transformation between two- and three-body c.m.f. is ignored. In ref.[9] the relativistic kinematics is taken into account only in the propagator by using the relativistic expression for the pion kinetic energy. In ref.[8] there are used Lorentz invariant three-body equations obtained[15] using the BS procedure[14] and the idea of isobar dominance. These equations are generalized to particles with spin and isospin in ref.[12].

The relativistic equations for the πd scattering problem derived in ref.[6] on the basis of relativistic three-body equations[16] in the quasi-potential Logunov-Tavkhelidze[17] approach take into account the Lorentz-transformation between two- and three-body c.m.f. and involve the propagators with linear energy-dependence contrary to the non-linear propagator in the equations of refs.[7,8,12]. As a result, in the equations[6] the so-called "cluster property" of the Faddeev equations has been conserved. What is more the equations[6] are a natural generalization of the corresponding non-relativistic equations and allow a simple non-relativistic reduction. Here we would like to mention another approach to the πd-scattering problem based on the application of the boundary condition method taking into account the relativity developed in the studies[18]. The basic equations of ref.[6] are used here for the πd -scattering study in the (3.3) resonance region. The main aim of the present investigation is to clarify the importance of some effects of relativistic kinematics and the dependence

of the πd -scattering cross section on the different choice of πN -interactions. The role of nucleon-nucleon rescattering for the πd -scattering processes is considered as well.

2. THREE-BODY RELATIVISTIC EQUATIONS AND THE SOLUTION PROCEDURE

The system of integral equations for partial transition matrices with given total angular momentum and isospin of the πNN -system taking into account the identity of nucleons which are obtained in the separable model for two-particle interaction have the following schematic forms[6]

$$T_{11} = K_{12} D_2^{-1} T_{2+3,1},$$
$$T_{2+3,1} = 2K_{21} + 2K_{21} D_1^{-1} T_{11} - K_{23} D_2^{-1} T_{2+3,1}, \qquad (1)$$

where T_{11} is the πd -elastic scattering matrix, $T_{2+3,1}$ the auxiliary transition matrix, D_1^{-1} and D_2^{-1} the propagators for NN and πN system in the separable (isobar) model, $K_{12}(x,y) = K_{21}(y,x)$ and $K_{23}(x,y)$ are the kernels of the integral equations the radial parts including the relativistic kinematics for pion (particle 1) and nucleon (particles 2 and 3) has the form[6] (with equality $\phi_{ki} = \chi_{ki}$ to be easily checked)

$$h_{ij}^{\lambda_i \lambda_j \mathcal{L}}(q_i, q_j, \mathcal{P}_0) = q_i^{\lambda_i + \lambda_j} q_j^{L_i + L_j - \lambda_i - \lambda_j} \int_{-1}^{1} p_{\mathcal{L}}(x)\, dx\, a_i(q_{jk}, q_i, x) \times$$

$$\times a_j(q_{ki},q_j,\mathcal{P}_0)\frac{[\phi_{jk}(q_i,q_j,x)]^{\lambda_i}[\phi_{ki}(q_i,q_j,x)]^{L_j-\lambda_j}}{q_{jk}^{L_i}q_{ki}^{L_j}[\mathcal{P}_0+iO-\omega_i(q_i)-\omega_j(q_j)-\omega_k(q_k)]}\times \tag{2}$$

$$\times g^{S_iJ_iI_i}_{L_i}(q_{jk})g^{S_jJ_jI_j}_{L_j}(q_{ki})\left[\frac{\omega_i(q_{jk})\omega_k(q_{jk})\omega_k(q_{ki})\,\omega_j(q_{ki})}{\omega_j(q_j)\omega_k(q_k)\omega_k(q_k)\,\omega_i(q_i)}\right]^{1/2}.$$

Here $g^{S_1J_1I_1}_{L_1}(q_{23})$, $g^{J_2I_2}_{L_2}(q_{31})$ and $g^{J_3I_3}_{L_3}(q_{12})$ represent the form factors for NN and πN interactions

$$q_{jk}=|-\vec{q}_j-\phi_{jk}(q_i,q_j,x)\vec{q}_i|;\quad q_{ki}=|-\vec{q}_i-\phi_{ki}(q_i,q_j,x)\vec{q}_j|, \tag{3}$$

$$\phi_{jk}(q_i,q_j,x)=\frac{\omega_j(q_j)}{s^{1/2}_{jk}(q)}+ \\ +\frac{}{s^{1/2}_{jk}(q)[\omega_j(q_j)+\omega_k(q_k)+s^{1/2}_{jk}(q)]}, \tag{4}$$

$$s_{jk}(q)=[\omega_j(q_j)+\omega_k(q_k)]^2-q_i^2\ ;\quad \omega_i(q)=\sqrt{m_i^2+q^2}, \tag{5}$$

$$q_k=|\vec{q}_i+\vec{q}_j|;\quad x=\frac{(\vec{q}_i\cdot\vec{q}_j)}{q_iq_j}.$$

The expression for the relative momentum of the jk pair, $\vec{q}_{jk}$ (3) is obtained using the Lorentz transformation from three-body c.m.f. with relevant Jacob momenta $\vec{p}_{jk}=\dfrac{m_k\vec{p}_j-m_j\vec{p}_k}{m_j+m_k}$ $\vec{\mathcal{P}}_i=\vec{p}_j+\vec{p}_k=\vec{q}_i$ to the two-body c.m.f. then for the corresponding transformation for two-body t-matrix we have [6]

$$\begin{aligned}\tilde{T}_i(\vec{p}\,'_{jk},\vec{p}_{jk},\mathcal{P}_{0i},\vec{\mathcal{P}}_i)&=\\ &=a_i(q'_{jk},q'_i,\mathcal{P}_0)\sqrt{\omega_j(q'_{jk})\omega_k(q'_{jk})}\times\\ &\times T_i(\vec{q}\,'_{jk},\vec{q}_{jk},\mathcal{P}_i^2)\sqrt{\omega_j(q_{jk})\omega_k(q_{jk})}a_i(q_{jk},q_i,\mathcal{P}_0),\\ \mathcal{P}^2&=\mathcal{P}_{0i}^2-\vec{\mathcal{P}}_i^2=[\mathcal{P}_0-\omega_i(q_i)]^2-\vec{q}_i^{\,2}\end{aligned} \tag{6}$$

where $\mathcal{P}_0$ is the total energy of the πd system.

The functions a_i and a_j in eqs. (2) and (6) are defined in ref. [6] and are equal to unity if the collision matrix is on the energy shell $\mathcal{P}_0=\omega_i(q_i)+\omega_j(q_j)+\omega_k(q_k)$. Notice that the relativistic relative momenta used in ref. [12] for the case (a) are the same as $\vec{q}_{jk}$ and $\vec{q}_{ki}$ as to the case (b) the function $\sqrt{\sigma_{\vec{q}_i}}=\sqrt{\mathcal{P}_i^2}$ appeared in some formulas in ref. [12] needs to be redetermined as it was done, for example, in ref. [6] because it becomes complex at high momentum q_i.

In the non-relativistic limit for the nucleons $(p/M\to0)$ and for the pion with fully relativistic kinematics (case "FRPK") we have

$$a_1 = a_2 = 1 \qquad \omega_2(q) = \omega_3(q) = M\,,$$

$$\phi_{23} \simeq \frac{1}{2};\ \phi_{31} \simeq \frac{\omega_\pi(q_1)}{M+\omega_\pi(q_1)};\ \phi_{21} = \phi_{13} \simeq \frac{M}{M+\omega_\pi(q_1)}\,, \tag{7}$$

where M is the nucleon mass. If the relativistic kinematics is taken into account only in propagators via the relativistic expression for the pion kinetic energy semi-classical relativistic pion kinematic, case "SRPK" then in (7) it should be put $\omega_\pi(q) = m_\pi$, where m_π is the pion mass.

The function $\mathfrak{T}_1^{110}(E_1)$ corresponding to the NN interaction with ${}^3S_1 + {}^3\mathfrak{D}_1$ bound state dominance (e.g., deuteron) has the form

$$\mathfrak{T}_1^{110}(E_1) = (M_d - E_1) \sum_{L_1=0,2} \int_0^\infty \frac{[g_{L_1}^{110}(q)]^2 q^2\, dq}{[E_1 - E_1(q) + i0][M_d - E_1(q)]}. \tag{8}$$

Here M_d is the deuteron mass and E_1 the total energy for NN in the c.m.f. For the form factor $g_{L_1}^{110}(q)$ we use the expression for a rank-one potential /19/ which gives a good fit to the phase up to 100 MeV (c.m.f.) with non-relativistic kinematics. For the relativistic case we use the same form factor /19/ since the functions $\mathfrak{T}_{L_1}^{110}(E_1)$ (8) with relativistic kinematics in the energy region under consideration do not practically differ from the non-relativistic ones as our calculations show. For pion-nucleon interactions with non-relativistic nucleon we use /9/

$$\mathfrak{T}_2^{L_2J_2I_2}(E_2) = \lambda^{-1}_{L_2J_2I_2} - \int_0^\infty \frac{[g_{L_2}^{J_2I_2}(q)]^2 q^2\, dq}{E - m_1 - q^2/2M - \omega_1(q_1) + i0}\,, \tag{9}$$

where the form factor $g_{L_2}^{J_2I_2}(q)$ is taken to fit the selected data for the phase shifts up to 300 MeV, E_2 is the total πN energy in the c.m.f.

For pion-nucleon interaction with relativistic pion and nucleon use is made of

$$\mathfrak{T}_2^{L_2J_2I_2}(E_2) = \frac{t_{L_2}^{J_2I_2}(E_2)}{[v_{L_2}^{J_2I_2}(k_{E_2})]^2}\,, \tag{10}$$

where $t_{L_2}^{J_2I_2}(E_2)$ is the on-shell πN collision matrix and the form factor $v_{L_2}^{J_2I_2}$ is defined in the momentum region ≤ 3 GeV /20/ on the basis of the solution of the inverse scattering problem making use of the Lippman-Schwinger equation with relativistic kinematics *.

As far as the πd scattering problem is considered in the (3.3) resonance region and we are mainly interested in the relative effects we have neglected the others than the P_{33} -wave in the πN interactions. As to the NN interactions we are limited by the ${}^3S_1$ -wave. The system of the integral equations (1) has been solved for partial transition matrices with total angular momenta $J \leq 6$. For the pion-deuteron relative orbital angular momentum ℓ_1 all the allowed values were included in the calculations.

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*The authors would like to thank Dr. E.Moniz who kindly sent us all the data on πd collision matrix necessary for our calculations.

The πd elastic scattering amplitudes with $\ell_1' \neq \ell_1$ are found to be negligibly small.

The calculation of the radial part h_{ij} of the kernels of the integral equations was carried out with a given (L%) relative accuracy by the contour rotation method [21]. The system of inhomogeneous linear integral equations was reduced to an equivalent system of algebraic complex equations which was solved by the iteration method. The number of the approximation procedure points was taken under the condition of the approximation of a given accuracy and stability of the first iteration as well as a stable convergence of the iteration series. For the approximation of the potential [20] tabulated the Lagrange interpolation polynomials of the second and third order and the splines [22] of the third order have been used. The results are found to be independent of the method of integration. The calculations were performed on the computer CDC-6500 and BESM-6 of the JINR (Dubna).

3. RESULTS AND DISCUSSION

The differential and integral πd elastic scattering cross sections have been calculated making use of their expressions derived in ref. [6] The total cross section was, as usual, calculated with the help of the optical theorem. The results of the numerical calculations are given in figs. 1-8 with the available experimental data. In figs. 1-4 we compare the results obtained using the πN collision matrix with propagator (9) but in the two different approximations: 1) the relativistic pion kinematics is taken

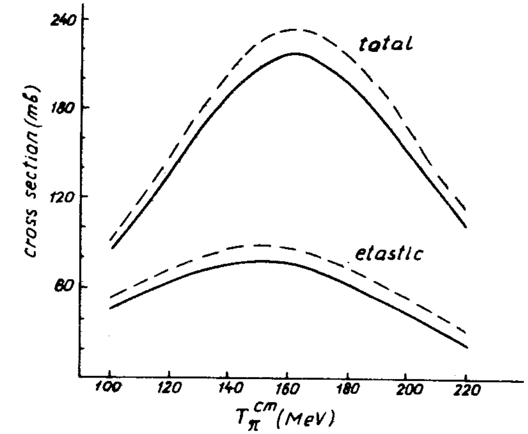


Fig. 1. The πd scattering cross sections in the single scattering approximation. The solid lines correspond to "FRPK", the dashed ones to "SRPK".

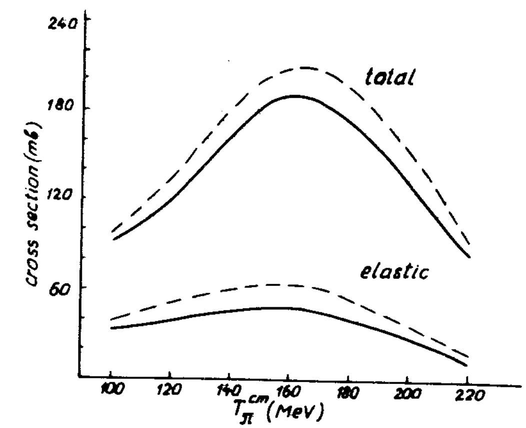
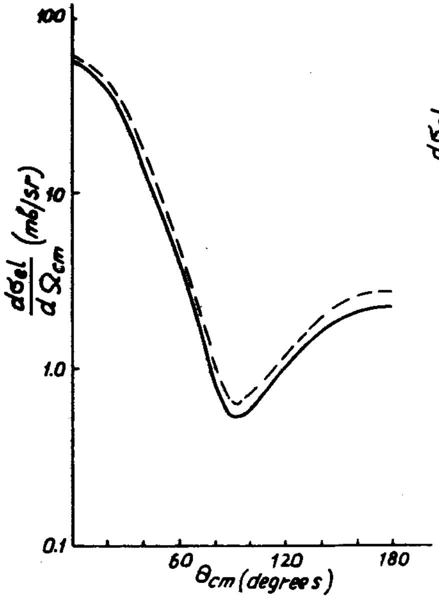


Fig. 2. The same as in fig. 1, but with the exact solution of the three-body equations.



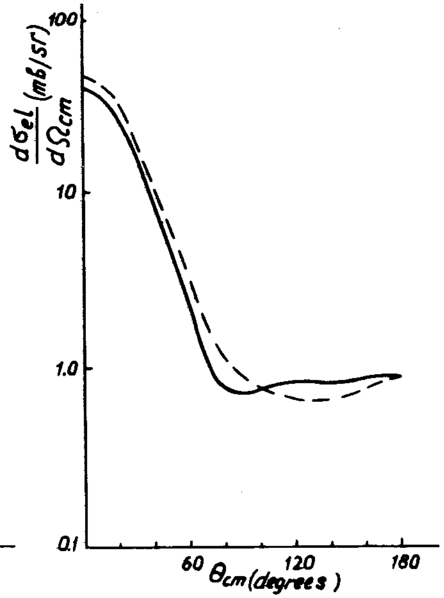


Fig. 3. The differential πd elastic cross section at $T_{\pi}^{cm} = 160$ MeV. The solid lines correspond to "FRPK", the dashed ones to "SRPK".

Fig. 4. The same as in fig. 3, but with the exact solution of the three-body equations.

into account only in a semi-relativistic manner (case "SRPK") and 2) the relativistic pion kinematics is included in full in the calculations (case "FRPK"). We see that the inclusion in full of the pion relativistic kinematics leads to a noticeable decrease of the value of the cross sections and this effect is of the same order as the multiple scattering effect. This result seems to be

important for better understanding of the pion scattering processes on more complicated nuclei which at the present stage of the development of nuclear physics are treated non-relativistically and therefore in studying the pion-nucleus scattering it is more consistent to consider the nucleus non-relativistically and the pion, if necessary, relativistically. Here it is worth while noting that our approximation "SRPK" coincides with "RPK"[12] but our "FRPK" differs from "RPKI"[12]. Namely, in the case "RPKI" the relativistic kinematics for the pion and nucleon are included in the form factors $g^{S_1 J_1 I_1}_{L_1}$ and $g^{J_2 I_2}_{L_2}$ making use of the relativistic expression (3) for the relative momenta q' and q instead of the non-relativistic ones in the "RPK" where in addition the nucleon is treated non-relativistically in the propagators. In our approximation "FRPK" the nucleon is considered to be non-relativistic and the pion relativistic kinematics is taken into account not only in the propagators and form factors but in full in the expression (2) according to the transition to the limit (7). Our calculations show that the main contribution from the multiple scattering into the cross sections comes from the terms with nucleon-nucleon rescattering between two pion-nucleon interactions. This result is in a qualitative agreement with an analogous results of refs.[5,8,12] and means that the term $K_{23}\mathcal{T}_2^{-1}T_{2+3,1}$ can be neglected in eq. (1) at least in the energy region under study. This fact has been used in the calculations the results of which are discussed below.

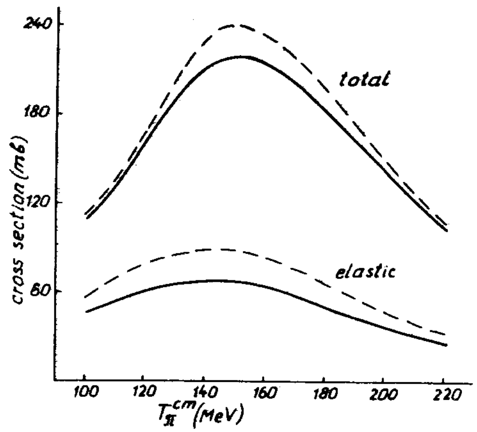


Fig. 5. The πd scattering cross sections ("FRK"). The solid lines correspond to the exact solution of the three-body equations, the dashed lines to the single scattering approximation.

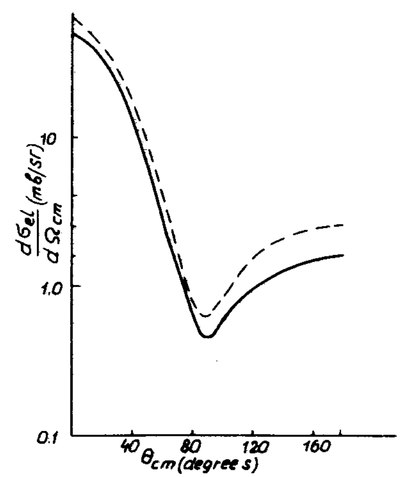


Fig. 6. The differential πd elastic cross section at T_{π}^{cm}= 160 MeV. The solid line corresponds to the exact solution of the three-body equations, the dashed one to the single scattering approximation.

The results of our calculations in which the inverse problem solution for the πN collision matrix is used are given in figs. 5 and 6. Comparing these results with the results of figs. 1-4 in the case "FRPK" one should bear in mind that the case "FRK" includes two additional effects: that of the nucleon relativistic kinematics and of the different off-shell behaviour of the πN collision matrix. The latter is caused by the fact that in the "FRPK" case use is made of the πN phase shifts in an energy region up to 300 MeV and in the "RFK" case in a much wider energy region [20]. The main contribution to the integration of eq. (1) as the calculations show, comes from the momentum region up to 300÷400 MeV/c, where the nucleon may be considered to be nonrelativistic. Therefore the difference between the cross sections in figs. 1-4 ("FRPK") and in figs.5,6 is caused by different choice of the πN collision matrix. As a result, we come to the conclusion that in the (3.3) resonance region the πd scattering is rather sensitive to the off-shell behaviour of the πN t-matrix (a similar conclusion is made in refs. [23,24] on the basis of calculation of the single scattering term for the πd elastic scattering). Here it is worth while to mention the work [25] where it is shown that the two solutions of the inverse πN scattering problem with different sets of the phase shifts in the energy region E_{cm} > > 1900 MeV have led practically to the same results for the $\pi\,^{12}C$ scattering. The different sensitivity of the πd and $\pi\,^{12}C$ scattering to the off-shell behaviour is probably due to two reasons: the approximation of the

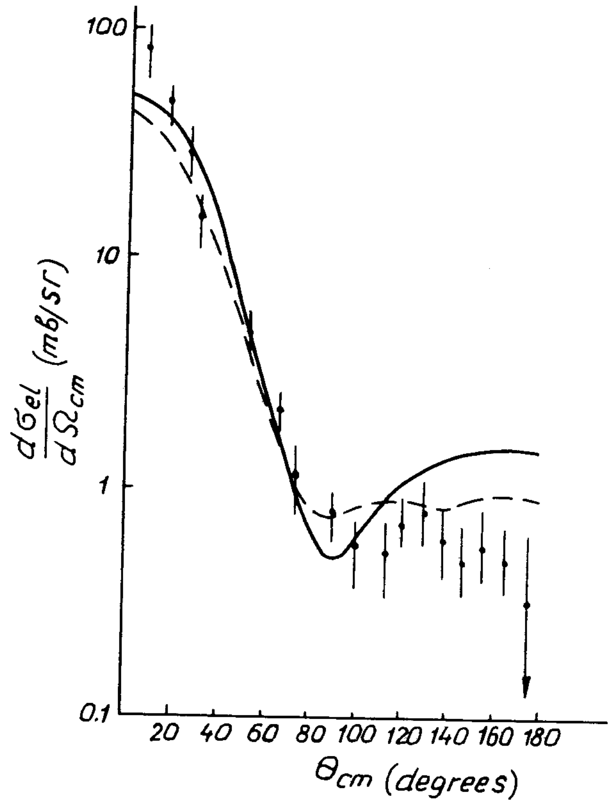


Fig. 7. The differential πd elastic cross section at $T_{\pi}^{cm} = 160$ MeV. The solid line corresponds to "FRK", the dashed one to "FRPK". The experimental data are taken from ref. /27/.

first order optical potential constructed in static or factorized impulse approximation used in ref. /25/ and a considerable difference between the deuteron and ^{12}C nucleus form factors.

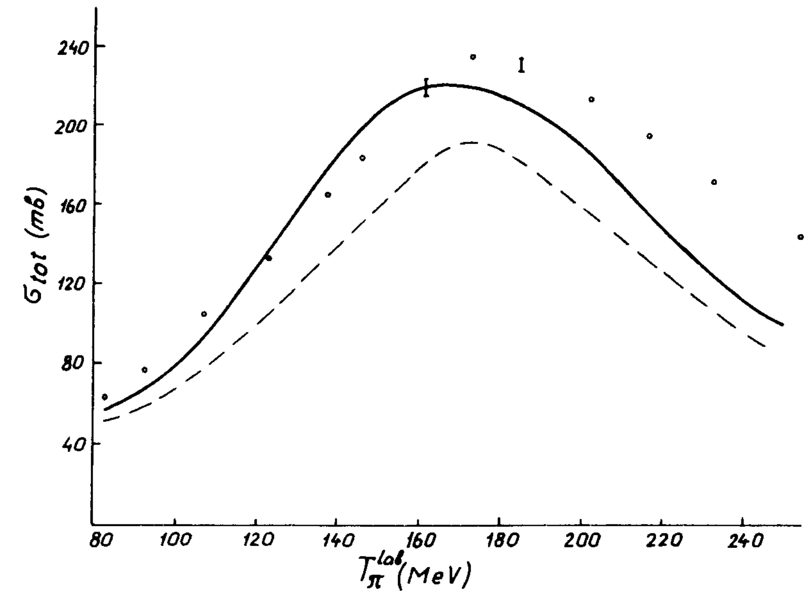


Fig. 8. The lab. total πd scattering cross sections. The solid line corresponds to "FRK", dashed one to "FRPK". The experimental data are taken from ref. /27/.

In figs. 7 and 8 we compare the results of calculations with experimental data. The differential cross section at different energy and for both the cases "FRK" and "FRPK" we are interested in, are shown to be in satisfactory agreement with experiment data in the angle region $\theta_{cm} < 80°$. While in the region $\theta_{cm} > 80°$ it is impossible to make a detailed comparison, since in this region, according to refs. /5,7,8,12/, a considerable contribution comes from the deuteron $\mathfrak{D}$-state, which is neglected in our calculations. In fig. 7 the differential cross section at $T_{\pi}^{cm} = 1600$ MeV is given for the sake of illustration. From fig. 8 we see that the total cross section obtained in the case "FRK" gives a somewhat better description of the experimental data [27] than in the "FRPK" case, though there is a considerable discrepancy beyond the resonance. One of the possible reasons for this discrepancy may be the neglect of the other than P_{33}-wave in the πN interactions, for example, $\mathfrak{T}$-waves the relative role of which increases with increasing pion energy. Notice that, the discrepancy at high energy (beyond the resonance) with the experimental data is also seen in the calculations of ref. [12].

In conclusion we would like to note that the three-body relativistic equations of ref. [6] allow us to study the other than elastic channel of the πd interactions, such as pion scattering with deuteron disintegration and the charge exchange processes which are at present being studied.

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