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A NOTE ON THE FOUNDATION OF THE NUCLEAR FIELD THEORY

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К обоснованию теории ядерных полей

На основе трактовки остального взаимодействия в рамках диаграмм Фейнмана получен соответствующий ядерно-полевой подход на примере простой модели. Доказано, что определенные классы диаграмм в этом подходе отбрасываются, что как раз является причиной появления духовых состояний.

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A Note on the Foundation of the Nuclear Field Theory

Starting from the Feynman diagrammatic perturbation treatment of the residual interaction we derive for a schematic model the corresponding nuclear field treatment. We show that some definite classes of diagrams are neglected in the nuclear field theory. It is just this neglect that leads to the appearance of spurious states. Nevertheless, in any case the nuclear field theory yields also the exact eigensolution of the nuclear many-body problem.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. In the last years the nuclear field theory (NFT) has proved a powerful method for evaluating all the different coupling and anharmonic effects when superimposing single particle and collective (phonon) degrees of freedom. The rules for the diagrammatic nuclear field treatment have been empirically found in ref. [1]. Further, in refs. [2,3] the equivalence between the Feynman diagrammatic expansion involving only fermion degrees of freedom and the corresponding nuclear field treatment of the residual interaction has been shown for processes connecting intermediate states. However, there still remained questions concerning the equivalence between the two treatments for processes connecting initial and final states. To give an answer to these questions is the aim of the present note.

According to the rules of the nuclear field treatment (see ref. [1]) initial and final states of proper diagrams may involve both collective (phonon) modes and particle modes, but not any particle configuration that can be replaced by a combination of collective modes. However, by this replacement prescription the NFT yields more basic states than the fermion treatment which uses properly antisymmetrized states. Therefore some spurious states remain in the spectrum obtained from the nuclear field approach. Recently[4] this problem has been dealt with only in a heuristic way. For a schematic model it has been shown there that the spurious components present in the initial or final states are eliminated through the field treatment giving rise to states which have zero matrix elements for any processes connecting them with physical states.

In the present paper, by using the schematic model of ref [4] we show how the nuclear field treatment can be derived from the fermion treatment. Thereat it comes out that in the NFT some definite classes of diagrams are always neglected. This neglect results in the appearance of spurious states in the NFT.

2. The model under consideration consists of Ω single particle levels in which the fermions interact pairwise through a monopole force

$$H = H_{sp} + H_{tb} ,$$

where

$$H_{sp} = \frac{1}{2} \sum_m \epsilon_m (a^+_{m,1} a_{m,1} - a^+_{m,-1} a_{m,-1})$$

and

$$H_{tb} = -VA^+ A$$

with

$$A^+ = \sum_{m=1}^{\Omega} a^+_{m,1} a_{m,1} .$$

Here V denotes the interaction strength and ϵ_m is the energy to excite a particle from the state $|m,-1\rangle$ to its counter part $|m,1\rangle$. In the vacuum state $|0\rangle$ all the $|m,-1\rangle$ states are filled and all the $|m,1\rangle$ states are empty.

The poles of the particle-hole amplitude (see fig. 1(a))

$$\langle m,1;m',-1|T|m'',1;m''',-1\rangle = T(\omega)\delta_{mm'}\delta_{m''m'''},$$
$$T(\omega) = [\sum_m \frac{1}{\epsilon_m - \omega} - \frac{1}{V}]^{-1} \qquad (1)$$

define the frequencies of the phonons, ω_μ. The NFT representation of the particle-hole amplitude reads

$$T(\omega) = -V + T^c(\omega), \qquad (2)$$

$$T(\omega) = -V + \sum_\mu \Lambda_\mu D_\mu(\omega)\Lambda_\mu, \quad D_\mu(\omega) = \frac{1}{\omega - \omega_\mu},$$

where the particle-phonon coupling strength is given by

$$\Lambda_\mu = [\sum_m \frac{1}{(\omega_\mu - \epsilon_m)^2}]^{-1/2}.$$

We restrict the considerations to the set of two-particle-one-hole states* $|\bar{m};(m)\rangle \equiv a^+_{\bar{m},1} a^+_{m,1} a_{m,-1}|0\rangle$. This is the simplest system which in the corresponding NFT description (one-particle-one-phonon states) displays spurious states. Let us consider the total two-particle-one-hole Green function

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*The residual interaction H_{tb} does not mix states with a different number of particles and holes.

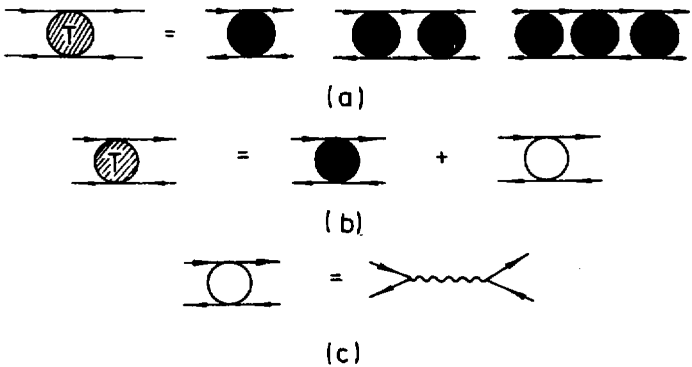


Fig. 1. Diagrammatic representation of the particle-hole interaction part T, (a) in the fermion, (b) the nuclear field treatment of the residual interaction; (c) shows the collective interaction part T^c. A full line represents a fermion, a wavy line, a phonon. A full circle stands for the two-body interaction (−V).

$$G(E) = G_0(E) + G_0(E)\,\mathcal{T}(E)\,G_0(E)$$

$$G(E) = \frac{1}{E-H}, \qquad G_0(E) = \frac{1}{E-H_{sp}}. \tag{3}$$

The exact excitation energies are obtained from the poles of G(E) or equivalently of the interaction part $\mathcal{T}(E)$, which is given by the diagrams of fig. 2(a)

$$\langle \bar{m};(m_1)|\mathcal{T}|E\rangle)|\bar{m}';(m_2)\rangle = \delta_{\bar{m}\bar{m}'}\,\mathcal{T}(\bar{m},E),$$

$$\mathcal{T}(\bar{m},E) = T(z)\sum_{n=0}(g_{\bar{m}}(z)\,T(z))^n \tag{4}$$

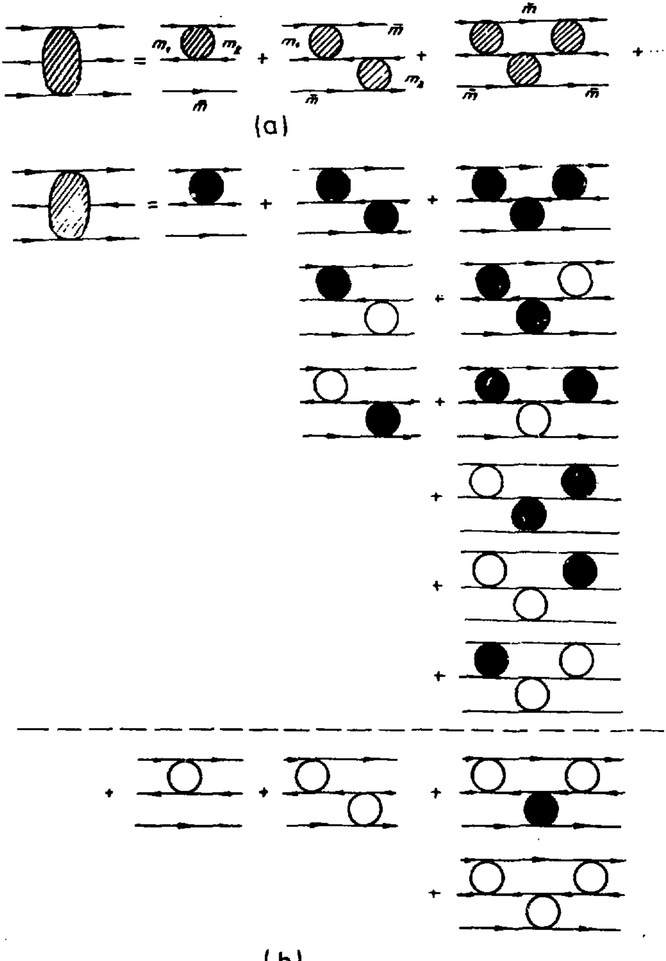
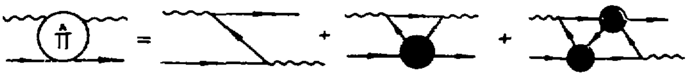


Fig. 2(a),(b).



(c)

Fig. 2. Diagrammatic representation of the total two-particle-one-hole interaction part $\mathcal{T}$ (a) in terms of the full particle hole amplitude T. In (b) T has been redrawn in its NFT representation eq. (2). Only the diagrams below the dash line are taken into account in the nuclear field treatment. Their sum is given by the amplitude $\mathcal{T}^f$. The self-energy part $\hat{\Pi}(m,z)$ is displayed in (c).

where

$$g_{\bar{m}}(z)=\frac{1}{\epsilon_{\bar{m}}-z}; \qquad z=E-\frac{1}{2}\epsilon_{\bar{m}}.$$

The intermediate states of the diagrams higher than first order in T(z) contain two particles in the state $|\bar{m}\rangle$. Thus they violate the Pauli principle. They represent exchange diagrams to the graph of first order in T(z) which involves also a Pauli principle violating contribution, as the summation in eq. (1) includes also the intermediate state $m=\bar{m}$. Just this component is cancelled by the higher order (exchange) diagrams. Indeed we find

$$\mathcal{T}(\bar{m},E)=\Big[\sum_{m\neq\bar{m}}\frac{1}{\epsilon_m-z}-\frac{1}{V}\Big]^{-1}, \qquad (5)$$

where now the $|\bar{m};(m=\bar{m})\rangle$ configuration is excluded.

3. Our aim is to get a description of the system considered in terms of collective fields. For this purpose we insert the particle-hole amplitude $T=-V+T^c$ given in its NFT representation (eq. (2)) into the expression for the total interaction part $\mathcal{T}(\bar{m},E)$ (eq. (4)). $\mathcal{T}(\bar{m},E)$ is then expressed by the diagrams shown in fig. 2(b) which involve the phonons via $T^c(z)$. To arrive at the NFT description we have to neglect in $\mathcal{T}(\bar{m},E)$ all such diagrams which do not begin or end with the collective interaction part T^c, i.e., with the phonon image*. The processes left are given by

$$\mathcal{T}_f(m,E)=T^c(z)+T^c(z)\,g_{\bar{m}}(z)\,T^c(z)+ \\ +T^c(z)\,g_{\bar{m}}(z)\,\mathcal{T}(m,E)\,g_{\bar{m}}(z)\,\mathcal{T}^c(z). \tag{6}$$

Inserting here the value for the total amplitude $\mathcal{T}(\bar{m},E)$ given by eq. (5), after some straightforward calculations we find

$$\mathcal{T}_f(\bar{m},E)=T^c(z)+T^c(z)\frac{1}{1-\Sigma(\bar{m},z)T^c(z)}\Sigma(\bar{m},z)T^c(z), \tag{7}$$

where $\Sigma(\bar{m},z)=\dfrac{1}{\epsilon_{\bar{m}}-z+V}$.

Defining the total one-phonon-one-particle (NFT) Green function $D^f(\bar{m},E)$ by

$$\mathcal{T}_f(\bar{m},E)=\sum_{\mu\nu}\Lambda_\nu D^f_{\nu\mu}(\bar{m},E)\Lambda_\mu$$

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*By definition the external fermion lines are not included in the interaction parts.

from eq. (7) we get

$$\hat{D}^f(m,E)=\hat{D}_0(z)+\hat{D}_0(z)\frac{1}{1-\hat{\Pi}(\bar{m},z)\hat{D}_0(z)}\hat{\Pi}(m,z)\hat{D}_0(z),$$

where

$$(\hat{D}_0)_{\nu\mu}=\delta_{\nu\mu}D_\mu(z),\quad D_\mu(z)=\frac{1}{z-\omega_\mu},$$

$$(\hat{\Pi}(\bar{m},z))_{\nu\mu}=\Lambda_\nu\Sigma(\bar{m},z)\Lambda_\mu .$$

Equation (7) represents a Dyson-type equation

$$\hat{D}^f(\bar{m},E)=\hat{D}_0(z)+\hat{D}_0(z)\hat{\Pi}(\bar{m},z)\hat{D}^f(\bar{m},E)$$

The (perturbed) energies of the one-phonon-one-particle system are obtained from the poles of $\hat{D}^f(\bar{m},E)$

$$\det|\hat{D}_0^{-1}(z)-\hat{\Pi}(\bar{m},z)|=0.$$

This leads to the dispersion relation

$$(\epsilon_{\bar{m}}-z+V)=\sum_\mu\frac{\Lambda_\mu^2}{z-\omega_\mu} \tag{9}$$

As stated in ref.[4] this equation yields the exact energies of the two-particle-one-hole system (i.e., the same energies as one gets in the fermion treatment from the poles of $\mathcal{G}(\bar{m},E)$ (see eq. (5)). Besides, the dispersion relation (9) yields a double root at the unperturbed energies $z=\epsilon_{\bar{m}}$. Let z_a denote the eigenvalue of eq. (9). Then the amplitudes of the eigenstates $|a,\bar{m}>$ in the unperturbed basis $|\mu,\bar{m}>$ are given by

$$|<\mu,\bar{m}|a\bar{m}>|^2=\lim_{z\to z_a}[(z-z_a)D^f_{\mu\mu}(\bar{m},z)].$$

Only for $z_a\neq\epsilon_{\bar{m}}$ we obtain finite (normalizable) amplitudes

$$|\langle\mu,\bar{m}|\alpha,\bar{m}\rangle|^2 = \left(\frac{\Lambda_\mu}{z_\alpha-\omega_\mu}\right)\left[\sum_\mu \frac{\Lambda_\mu^2}{(z_\alpha-\omega_\mu)^2} - 1\right]^{-1},$$

as the second factor in the last equation becomes infinite for $z_\alpha \to \epsilon_{\bar{m}}$.

4. Already from eq. (6) it can be read off that the nuclear field treatment which uses the amplitude $\mathfrak{I}^f$ instead of the full interaction part $\mathfrak{I}$, will yield (i) the same (exact) eigenvalues as the fermion treatment, (ii) additional (spurious) solution (double roots) at the unperturbed energies $z = \epsilon_{\bar{m}}$*. Thus from eq. (6) it is clear by now that the spurious solutions obtained in the nuclear field treatment are a consequence of the neglect of such diagrams (present in the fermion treatment) which do not begin or end with bubble diagrams, i.e., such diagrams in which two fermion lines of an external state interact only once with each other. (see fig. 2(b)). It is obvious that this result is not restricted to the schematic model considered here. Also in the general case a relation similar to eq. (6) holds.

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*Using eq. (5) one easily shows that the full Green function of the fermion treatment, G(E) (see eq. (3)), has no poles at the unperturbed energies.

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