СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

19/IX-77

E4 - 10691

E.Bang, M.I.Baznat, F.A.Gareev

PAIRING CORRELATIONS IN SPHERICAL NUCLEI

1977

E4 - 10691

E.Bang,[1] M.I.Baznat,[2] F.A.Gareev

PAIRING CORRELATIONS IN SPHERICAL NUCLEI

[1] The Niels Bohr Institute, Copenhagen, Denmark.

[2] Institute of Applied Physics, AN MSSR (Kishinev).

Банг Е., Базнат М.И., Гареев Ф.А.

Е4 -10691

Парные корреляции в сферических ядрах

Рассмотрено влияние учета остаточных взаимодействий типа δ -сил и взаимодействия Гаусса на сверхтекучие свойства сферических ядер. Сравнение с приближением G=const показало, что по мере удаления от поверхности Ферми учет остаточных взаимодействий указанного типа является необходимым. Это приводит к улучшению сходимости, т.е. уменьшается зависимость результатов от числа учитываемых в расчетах одночастичных состояний. Вычисления спектроскопических факторов и факторов усиления в α -распаде показало, что приближение G=const приводит к переоценке этих величин. В расчетах были использованы одночастичные состояния в потенциале Саксона-Вудса, максимальное число которых определялось высотой и проницаемостью потенциального барьера (проницаемость - 0,1,что соответствует шнрине уровня 50 кэВ).Анализ результатов вычислений показал, что δ -силы и взаимодействие Гаусса дают почти те же значения вычисляемых величин и поэтому для простоты расчетов можно ограничиться δ -силами.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1977

Bang E., Baznat M.I., Gareev F.A.

E4 - 10691

Pairing Correlations in Spherical Nuclei

The influence of the residual interactions of the δ -type and of the Gauss potential on the superfluid properties of spherical nuclei is considered.The comparison with the approximation G = const has shown that with increasing distance from the Fermi surface the above residual interactions become more important. This improves the convergence, i.e., the dependence of the results on the number of the calculated single-particle states.The calculation of the spectroscopic factors and α -strengths has shown that the approximation G=const results in the overestimation of these quantities.We have used the single-particle states of the Saxon-Woods potential the maximal number of which was defined by the height and the potential barrier penetration (penetration is 0.1, which corresponds to the width of 50 keV). The analysis of the calculated results has shown that the δ -interaction and the Gauss potential give almost the same values of the calculated quantities. Therefore, for the sake of simplicity we may restrict ourselves to the δ -interactions.

Communication of the Joint Institute for Nuclear Research. Dubna 1977

1. INTRODUCTION

The introduction of pairing correlations of superconductive type is an important step towards the understanding of the low-lying states of deformed and shperical nuclei, of level densities, electromagnetic transition probabilities and other effects, connected with the structure of these states.

In this method, as introduced in the works of Soloviev[1], Belyaev [2],Kisslinger, and Sorensen[3] the Hamiltonian is written in the form

$$H=\sum_{\nu\sigma}\epsilon_\nu a^+_{\nu\sigma}a_{\nu\sigma}-\frac{1}{2}\sum\langle\nu_1\nu_2\sigma_1\sigma_2|G|\nu_1'\nu_2'\sigma_1'\sigma_2'\rangle\times \quad (1)$$

$$\times a^+_{\nu_1\sigma_1}a^+_{\nu_2\sigma_2}a_{\nu_2'\sigma_2'}a_{\nu_1'\sigma_1'},$$

where ϵ_ν are the single particle energies and $a^+_s(a_s)$ are the creation (annihilation) operators of the corresponding states.

To describe a system of N particles it is convenient to use a variational method of solution, i.e., minimizing the energy with a wave function of the type given below, (3), changing the Hamiltonian to

$$H' = H - \lambda N, \quad (2)$$

where the Lagrangian multiplier λ allows for the possibility of fixing the average particle number to N.

The variational solution is according to the method of Bardeen, Cooper, and Schrieffer[4] written in the form

$$\Psi = \prod_{\nu} (u_\nu + v_\nu a^+_{\nu+} a^+_{\nu-}) |0>, \qquad (3)$$

where $|0>$ is the particle vacuum and u_ν and v_ν are the variational parameters subject to the condition $u^2_\nu + v^2_\nu = 1$.

Equivalently, the ground state of the form (3) may be described as the vacuum of the quasiparticle creation (annihilation) operators $a^+(a)$ defined by the Bogolubov transformation[5]

$$\begin{aligned} a_{\nu\sigma} &= u_\nu a_{\nu-\sigma} + \sigma v_\nu a^+_{\nu\sigma}, \\ a^+_{\nu\sigma} &= u_\nu a^+_{\nu-\sigma} + \sigma v_\nu a_{\nu\sigma}. \end{aligned} \qquad (4)$$

For the u_ν and v_ν, the variation with the Hamiltonian (1) leads to the system of equations[1,2]

$$\begin{aligned} \Delta_\nu &= \sum_{\nu'} G_{\nu\nu'} \mu_{\nu'} v_{\nu'}, \\ N &= \sum_{\nu} 2 v^2_\nu \end{aligned} \qquad (5)$$

with

$$v^2_\nu = \frac{1}{2}\left(1 - \frac{\tilde{\epsilon}_\nu - \lambda}{\sqrt{(\tilde{\epsilon}_\nu - \lambda)^2 + \Delta^2_\nu}}\right), \quad u^2 = 1 - v^2, \qquad (6)$$

$$\tilde{\epsilon}_\nu = \epsilon_\nu - \sum_{\nu'} G_{\nu\nu'} v^2_{\nu'}, \qquad (7)$$

where

$$G_{\nu\nu'} = <\nu + \nu - |G| \nu' - \nu' + > - <\nu + \nu - |G| \nu' + \nu' - > \qquad (8)$$

determine the pairing effects whereas the residual pairing term

$$\bar{G}_{\nu\nu'} = G_{\nu\nu'} + <\nu+\nu'+|G|\nu+\nu'+> - <\nu+\nu'+|G|\nu'+\nu+> \qquad (9)$$

contributes to the self-consistent nuclear field.

2. THE RESIDUAL INTERACTION

In the majority of the works, where the influence of pairing correlations on different nuclear properties is investigated, it is suggested that the short range part of the nuclear forces, which leads to the pairing correlations can be represented by δ-functions and that their matrix elements between the wave-functions of the average nuclear field can be approximated by a constant

$$G_{\nu\nu'} = G. \qquad (10)$$

This leads to great simplifications in the system of equations (5)-(7), i.e., the gap Δ and the chemical potential are determined by

$$\begin{aligned} \frac{2}{G} &= \sum_{\nu} \frac{1}{\sqrt{(\tilde{\epsilon} - \lambda)^2 + \Delta^2}}, \\ N &= \sum_{\nu} 2 v^2_\nu. \end{aligned} \qquad (11)$$

The properties of low-lying nuclear states, which are connected with pairing, are generally investigated by means of these equations[6], where, at least in medium heavy

and heavy nuclei, the pairing of neutrons and of protons are treated as two independent problems, and the corresponding coupling constants G_p and G_N are adjusted to fit the experimental data.

In this article we are investigating the influence of the choice of residual interaction on the pairing phenomena of spherical nuclei. We use interactions of the form

$$V_1 = V_{01}\,\delta(\vec{r}_1 - \vec{r}_2), \tag{12}$$

$$V_2 = V_{02}\exp\{-(\vec{r}_1 - \vec{r}_2)^2/r_0^2\}, \tag{13}$$

and the results of the calculations are compared with those obtained in a constant G.

When the simple pairing method of ref /1,2,3/ is replaced by more realistic calculations, a number of problems is met /7/. The expressions (11) will give coupling constants, which depend on the number of single particle states taken into account.

When this number is increased, and Δ is kept fixed, G converges to 0. A similar lack of real convergence is met with the δ-forces whereas any calculation with realistic forces obviously leads to a finite result. The root of the difficulty seems to be two-fold. Partly, the number of bound single particle states of real nuclei is finite. Even if the resonances should also be taken into account, the constant matrix element approximation should therefore lead to a finite result. Partly, a "realistic" interaction will correspond to finite energies, even if a basis with infinitely many bound states, like the harmonic oscillator, is used.

Another problem concerns the localization of the correlation effects. If we consider such reactions, which are apt to reveal the pair correlations, like two-particle transfer, the amplitude gets its main contributions from a region outside the nuclear radius. Since all bound states fall off exponentially in this region, admixtures of high lying single particle states ("unbound"), even if the corresponding v-coefficients may be small, can play a large role in these transfer amplitudes. However, a realistic estimate of the wave function is difficult with the harmonic oscillator basis where the states fall off in an unrealistic way with increasing distance from the nucleus. For this reason, the Sturmian basis functions were introduced in the calculations of transfer form factors for the case of two particles outside a closed shell /8/. Similar calculations are under preparation for the many body problem, but the complications connected with the weight-orthogonality of the Sturmian functions mean, that a comparison with simpler calculations is also needed, even if the asymptotic form of the wave function in this case is not completely correct.

From the above considerations, it is obvious, that it is desirable to investigate a model where both the potential and the residual interactions are finite, non-singular functions of the coordinates, and where continuum admixtures are taken into account approximately by using single particle resonances along with the bound states.

The residual interactions must in principle, of course, include tensor forces, etc., but since our aim is not so much a complete-

ly realistic calculation, but rather to see the effects of refining certain approximations, we shall limit ourselves in the two interactions

$$V_1 = V_{01}\,\delta(\vec{r}_1 - \vec{r}_2),$$

$$V_2 = V_{02}\exp\{-(\vec{r}_1 - \vec{r}_2)^2/r_0^2\}.$$

The results of the calculations are then compared to those obtained with a constant G. To solve the system of coupled equations

$$\Delta_j = \sum_{j'} G_{jj'} u_{j'} v_{j'} = \frac{1}{2}\sum_{j'} \frac{(j'+1/2)G_{jj'}\Delta_{j'}}{\sqrt{(\tilde{\epsilon}_{j'}-\lambda)^2 + \Delta_{j'}^2}}, \qquad (13a)$$

$$N = \sum_j (j+1/2)\left(1 - \frac{\tilde{\epsilon}_j - \lambda}{\sqrt{(\tilde{\epsilon}_j - \lambda)^2 + \Delta_j^2}}\right)$$

we start by calculating the matrix elements $G_{jj'}$.

3. CALCULATION OF THE MATRIX ELEMENTS OF THE RESIDUAL INTERACTION

Our basis states are the single particle states of a Woods- Saxon potential

$$V(r) = -\frac{V_0}{1 + \exp(r - R_0/a)} \qquad (14)$$

and of spin orbit term

$$V_{s.o.}(r) = -\kappa\frac{1}{r}\frac{dV}{dr}(\vec{\ell}\vec{\sigma}) \qquad (15)$$

for the Coulomb potential protons

$$V_c(r) = \frac{(Z-1)e^2}{r}\begin{cases}\frac{3}{2}\frac{r}{R_0} - \frac{1}{2}\left(\frac{r}{R_0}\right)^3, & r \le R_0\\ 1, & r > R_0.\end{cases} \qquad (16)$$

The parameters of the potentials are given in ref. /9/.

The basis functions are written

$$\Psi^{\Omega}_{n\ell j} = R_{n\ell j}(r)\,\mathcal{Y}^{\Omega}_{\ell j}(\theta,\phi). \qquad (17)$$

The residual interactions are now expanded in multipoles

$$V(r_{12}) = \sum_L f_L(r_1 r_2)\,P_L(\cos\theta_{12}) \qquad (18)$$

for the δ-force

$$f_L(r_1 r_2) = \frac{V_{01}(2L+1)}{4\pi}\,\frac{\delta(r_1 - r_2)}{r_1 r_2} \qquad (19)$$

and the matrix elements have the simple form

$$\langle (j_1 j_1)J=0 | V_{01}\delta(\vec{r}_1 - \vec{r}_2) | (j_2 j_2)J=0\rangle = \qquad (20)$$

$$= V_{01}\frac{\sqrt{(2j_1+1)(2j_2+1)}}{8\pi}\int R^2_{n_1\ell_1 j_1}(r) R^2_{n_2\ell_2 j_2}(r) r^2 dr$$

for the Gaussian interaction

$$f_L(r_1 r_2) = V_{02}\exp\{-\frac{1}{r_0^2}(r_1^2 + r_2^2)\}(2L+1)i^L j_L\left(i\frac{2r_1 r_2}{r_0^2}\right) \qquad (21)$$

and the matrix elements are given by the expression

$$\langle (j_1 j_1)J=0 | V_{02} e^{-(\vec{r}_1 - \vec{r}_2)^2/r_0^2} | (j_2 j_2)J=0\rangle = \sum_L F_L V_L, \qquad (22)$$

where the radial part is

$$F_L = \iint r_1^2 dr_1 r_2^2 dr_2 R_{n_1\ell_1 j_1}(r_1) R_{n_1\ell_1 j_1}(r_2) f_L(r_1 r_2) R_{n_2\ell_2 j_2}(r_1) \times \qquad (23)$$
$$\times R_{n_2\ell_2 j_2}(r_2)$$

and the angular part is

$$V_L = (-1)^L \left\{ \frac{(2j_2+1)}{(2j_1+1)} \right\}^{\frac{1}{2}} <j_2\ \tfrac{1}{2} L 0 | j_1 \tfrac{1}{2}>^2 . \qquad (24)$$

In contrast to the δ-force, where the radial part of the matrix elements does not depend on L, the radial and angular part of the matrix elements of the Gaussian force do not separate. The Klebsch-Gordan coefficient shows, that the maximal L which contributes to a certain matrix element is determined by

$$L_{max} = \ell_{1\,max} + \ell_{2\,max} .$$

The matrix elements calculated for a number of single particle states are shown in tables 1 and 2. Obviously, both for the δ-force and for the finite range force, the diagonal elements are the larger. The non-diagonal elements going to quasistationary states are 5-10 times smaller than the diagonal ones.

To compare the matrix elements of the δ-force and the Gaussian with the constant G approximation, we take for these interaction a G^{eff} (average matrix element), determined by

$$G^{eff} = \frac{V_0 \sum_{jj'} <(jj)J=0|G_{jj'}|(j'j')J=0>}{\sum_{jj'} (j+\tfrac{1}{2})(j'+\tfrac{1}{2})} .$$

The calculated pairing properties (Δ, λ, u, v) for the δ-force and the finite range force will differ from those for a constant G mainly for such states where the matrix elements differ strongly from G^{eff}.

Table 1

Matrix elements $\frac{<(jj)J=0|V_{01}\delta(\vec{r}_1-\vec{r}_2)|(j'j')J=0>}{\sqrt{(j+\frac{1}{2})(j'+\frac{1}{2}}.}$ (MeV)

V_0 = 250 MeV/F^3

	2p3/2	Ig9/2	2d5/2	IhII/2	3sI/2	2f7/2	Ii13/2
2p3/2	.333	.083	.232	.078	.213	.143	.075
1g9/2		.163	.080	.153	.068	.080	.138
2d5/2			.223	.073	.130	.175	.070
IhII/2				.148	.063	.070	.140
3sI/2					.748	.113	.060
2f7/2						.168	.065
Ii13/2							.138

Table 2

Matrix elements $\frac{<(jj)J=0|V_{02}\, e^{-\{\frac{\vec{r}_1-\vec{r}_2}{r_0}\}^2}|(j'j')J=0>}{\sqrt{(j+\frac{1}{2})(j'+\frac{1}{2}.)}}$ (MeV)

r_0 = 1.5 fm, V_{02} = 25 MeV

	2p3/2	Ig9/2	2d5/2	IhII/2	3sI/2	2f7/2	Ii13/2
2p3/2	.378	.073	.228	.068	.115	.105	.060
Ig9/2		.180	.063	.158	.105	.058	.130
2d5/2			.228	.058	.093	.158	.053
IhII/2				.148	.040	.048	.130
3sI/2					.720	.090	.038
2f7/2						.155	.048
Ii13/2							.123

4. THE DEPENDENCE OF THE COUPLING CONSTANT ON THE NUMBER OF CONFIGURATIONS TAKEN INTO ACCOUNT

As mentioned above, in the case of constant matrix elements, coupling constant determined from a given Δ converges to zero, when the number of single particle states taken into account grows beyond all limits.

We have, for a number of nuclei with different single particle schemes, calculated the coupling constants for the case of constant matrix elements, of δ-force, and of Gaussian force, taking into account 10, 15, 20 and 25 single particle states. The unbound states were limited by the requirement, that the width should be below 50 keV, corresponding to a penetration .01 . The results of the calculations are shown in figure 1 and in table 3.

It is seen, that the coupling constants for the δ-force and the Gaussian force depend much less strongly on the number of levels, than the constant G. Nevertheless, the dependence is not negligible, and it is even not clear from the figure, whether a convergence is completely reached. The similarity between the Gaussian and the δ-force curves shows, that the convergence problem with the δ-force which is due to admixtures of high-lying states are of an importance here, and that as far as energies are concerned, the Gaussian force can be well approximated by the δ-force with properly adjusted coupling constant.

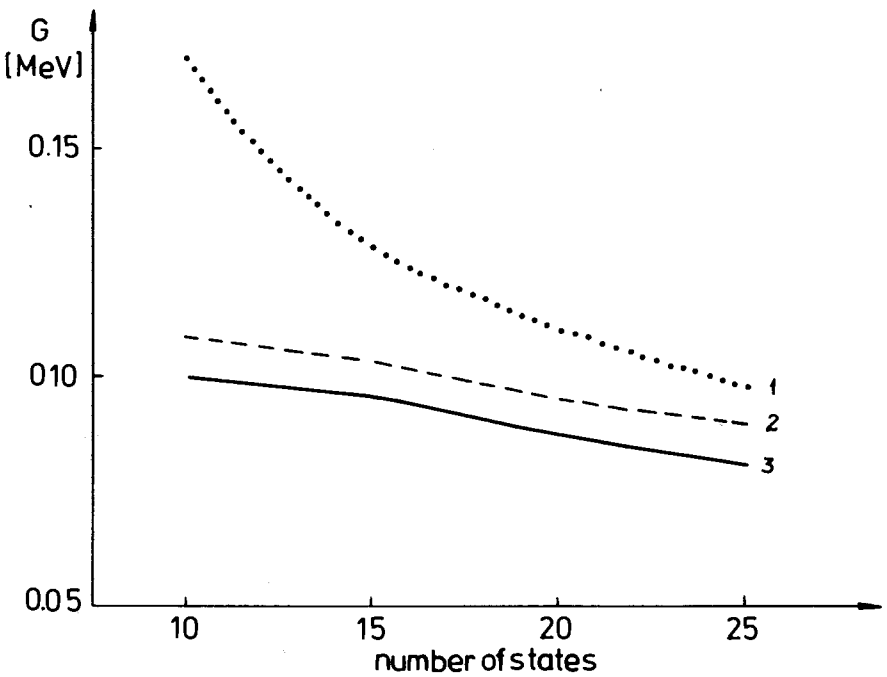

Fig. 1. The dependence of G^{eff} on the number of states taken into account. Dotted curve: G = const. Dashed curve: δ-force, full curve: Gaussian force.

5. THE PAIRING PROPERTIES OF THE CONFIGURATIONS

The Δ, u and v for configurations in the neighbourhood of the Fermi surface, for the three different types of residual interactions are shown in tables 4-6. It is seen that the Δ calculated with the different interactions are very different, and that Δ_j also differs strongly from state to state. The maximal magnitude Δ_{max} is 0.830 in the Gaussian case and 0.842 in the δ-force case,

Table 3

The dependence of Δ (in MeV) on the number of levels taken into account with fixed $G^{eff} = 0.1$ MeV (for the δ-force (II) and the Gaussian force (III)). In the cases II and III we give the values of Δ for the nearest level below and above the Fermi level, as well as for the latter (marked with *).

	Number of levels taken into account			
	12	17	22	27
G-const.	.509	.583	.739	.853
δ-force	.415	.487	.617	.680
	.603*	.674*	.847*	.946*
	.524	.582	.737	.829
Gaussian interactions	.442	.525	.659	.738
	.672*	.729*	.861*	.927*
	.443	.532	.692	.778

Table 4

The value of Δ, u and v for the proton system with Z=86 with G = const.

State	Δ	u_s	v_j
1f7/2		.093	.996
2d5/2		.106	.994
1h11/2		.140	.990
2d3/2		.165	.986
3s1/2	.95	.848	.531
1h9/2		.960	.280
2g7/2		.972	.234
1i13/2		.994	.109
2g7/2		.994	.106
3p3/2		.996	.086

Table 5

The values of Δ, u and v as well as the leading matrix element for the proton system with Z = 86 with δ-force and $V_0 = 202$ MeV/F^3

State	Δ_j	u_j	v_j	$G_{jj'}$
1f7/2	.842	.059	.998	.137
2d5/2	.619	.058	.998	.060
1h11/2	.765	.082	.997	.123
2d3/2	.635	.088	.996	.063
3s1/2	.553	.078	.997	.052
1h9/2	.821	.822	.569	.135
2g7/2	.584	.978	.207	.061
1i13/2	.708	.981	.194	.112
2g9/2	.602	.997	.077	.063
3p3/2	.478	.998	.057	.048

whereas the corresponding minimal magnitudes are 0.270 and 0.478. In tables 5,6 are also shown the matrix elements of the interaction between the Fermi level configuration and the other configurations. There is a clear correlation between the magnitudes of these matrix elements and Δ_j. This also explains, why the Δ_j of pisitive energy states are considerably smaller than those of the bound states. A comparison between the tables shows, that the approximation of constant matrix element is very good, when we are mainly interested in properties of configurations in the neighbourhood of the Fermi level. The further we come from the Fermi level, the more the results of the calculations with constant G with δ-forces and with forces of finite range differ from each other. This

Table 6

The values of Δ, u and v as well as the leading matrix element for the proton system with Z = 86 with a Gaussian residual interaction.
$V_0 = 28.8$ MeV, $r_0 = 1.5$ fm.

State	Δ_j	u_j	v_j	$G_{jj'}$
1f7/2	.83	.057	.998	.293
2d5/2	.43	.040	.999	.054
1h11/2	.51	.053	.999	.092
2d3/2	.47	.063	.998	.096
3s1/2	.38	.051	.998	.054
1h9/2	.76	.800	.606	.277
2g7/2	.36	.990	.141	.039
1i13/2	.43	.992	.127	.070
2g9/2	.41	.999	.051	.088
3p3/2	.27	.999	.033	.039

can lead to large differences in determination of spectroscopical factors from transfer reactions, and of α-decay strengths, etc.

To illustrate this claim, we give two examples: 1) Calculation of α-decay strength. The inclusion of pair correlations in the description of α-decay leads to a change of the decay probability by a factor /6,10/

$$W = (D_p \cdot D_n)^2,$$

where

$$D_{n,p} = \sum_\nu u_\nu v_\nu .$$

We get the following results for D_p and D_n in our 3 cases

	G_{const}	δ-force	Gaussian force
D_p	8.49	6.92	5.11
D_n	4.8	3.94	3.22

It is seen, that the results may change by a factor 3-5.

2) Spectroscopic factors in one particle transfer reaction. When the interaction between quasiparticles is neglected, the spectroscopic factor is determined by the quantity $v^2(u^2)$ for each state. v^2 is given below for the states near the Fermi surface.

G_{const}	δ-force	Gaussian force
0.98	0.98	0.98
0.28	0.33	0.38
0.079	0.045	0.020
0.053	0.036	0.017
0.012	0.0064	0.0025

The differences are obviously very large.

6. CONCLUSION

The results of the calculations of the pairing properties with residual interactions of δ-type with finite range show small differences from the cases of constant G for states near the Fermi surface. The differences are particularly large for states far from the Fermi surface. Particularly, the constant matrix element approximation seems to lead to an overestimate of spectroscopic factors and α-strengths contributions from

high lying configurations. In some cases, the pairing gaps of neighbour nuclei calculated with G=const are very different, whereas the experimental gaps vary in a smooth way[11]. In such cases the estimation may be improved by introducing the more defined methods.

The variational method of Bogolubov used here is of course in itself an approximation. Particularly when other properties than energies, like spectroscopic factors, etc., are calculated, it may be in need for radical improvements. These will, again, mainly affect the configurations far from the Fermi level[12], however not in the same way as those mentioned here.

REFERENCES

1. Soloviev V.G. JETP, 1958, 35, 823:, 1959, 36, p.1869.
2. Belyaev S.T. Mat.Fys.Medd.Dan.Vid.Selsk., 1959, 31, No.11.
3. Kisslinger L.S., Sorensen R.A. Mat.Fys. Medd.Dan.Vid.Selsk., 1960, 32, No.9.
4. Bardeen J., Cooper L.N., Schrieffer J.R. Phys.Rev., 1957, 108, p.1175.
5. Bogolubov N.N. Nuovo Cim., 1958, 7, p.794.
6. Soloviev V.G. Struktura Slozhnykh Jader, Atomizdat Moscow, 1966.
7. Arvieu R. Ann.Phys., 1963, 8, p.407.
8. Bang E., et al. Particles and Nucleus, 1974, No. 2, p.263.
9. Chepurnov V.A. Soviet Journ. of Nucl. Phys., 1967, 6, p.955.
10. Mang H.J., Pogzenburg J.K., Rasmussen J.O. Nucl.Phys., 1965, 64, p.359.

11. Malov L.A., Soloviev V.G., Christov J.D. Yad.Fiz., 1967, 6, p.1186.
12. Bang J., Krumlinde J. Nucl.Phys., 1970, A141, p.18.

Received by Publishing Department
on May 25, 1977.