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Упругое и неупругое рассеяние протонов с энергией 1 ГэВ на изотопах Ca, ^{48}Ti и ^{58}Ni

Проведен анализ экспериментальных данных по упругому и неупругому дифференциальным сечениям рассеяния протонов на ядрах $^{40-48}$Ca и ^{48}Ti в рамках теории Глаубера с использованием феноменологических плотностей. Для ядра ^{58}Ni рассчитаны микроскопические переходные плотности, и соответствующее им сечение сравнивается с экспериментом.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Elastic and Inelastic Scattering of 1 GeV Protons on Ca Isotopes, ^{48}Ti, ^{58}Ni

Using the Glauber formalism with phenomenological densities we analyse proton elastic and inelastic scattering differential cross sections for nuclei ^{40}Ca - ^{48}Ca and ^{48}Ti. The neutron phenomenological distribution parameters are determined for the ground state density and for the transition one. For nucleus ^{58}Ni the microscopic densities are calculated and the corresponding cross section is compared with experiment.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

It is the aim of the present paper to analyse the experimental data[1] on elastic and inelastic 1 GeV proton scattering on nuclei ^{40}Ca - ^{48}Ca and ^{48}Ti in the framework of the Glauber theory. The final goal, however, is to determine the parameters of the neutron density distribution $\rho_n(r)$ and neutron transition density $\rho_n^{tr}(r)$ and to study their sensitivity to the choice of the charge distribution parameters found from the different experiments on the electron scattering.

The possibility of defining the parameters of the neutron distribution is based on the fact that protons being strong-interacting particles "feel" both the proton and neutron components of the nuclear density while electrons interacting electromagnetically feel mainly the proton component.

2. FORMALISM

Following paper[2] one obtains the differential cross section of the elastic scattering

$$\frac{d\sigma_{el}}{d\Omega} = |f_C(q) + ik\int bdb\, J_0(qb)\exp(i\chi_P)\{1-\exp[i(\chi_C+\chi_N)]\}|^2$$

and for the excitation of a level with angular momentum L

$$\frac{d\sigma_{inel}}{d\Omega} = \sum_{\substack{M=-L \\ L+M=even}}^{L} \left| k \int bdb\, J_M(qb)\, g_{LM}(b) \exp[i(\chi_C+\chi_P+\chi_N)]\right|^2 .$$

Here $f_C(q)$ is the point charge Coulomb amplitude, $\chi_P(b)$ is the point charge Coulomb phase, $\chi_C(b)$ is the phase for the extended charge distribution of the nuclear target, $\chi_N(b)$ is the nuclear phase which in the first order of the Glauber theory can be written as

$$\chi_N(b) = \frac{A}{k} \int qdq\, J_0(qb)\, f(q)\, F_0(q)$$

$$g_{LM}(b) = (-1)^M \frac{A}{k} \frac{1}{\sqrt{4\pi}} \frac{[(L-M)!(L+M)!]^{1/2}}{(L-M)!!\,(L+M)!!} \int qdq J_M(qb) f(q) F_L($$

$$F_0(q) = 4\pi \int r^2 dr\, j_0(qr)\, \rho_0(r)$$

$$F_L(q) = 4\pi \int r^2 dr\, j_L(qr)\, \rho^{tr}(r)$$

$$f(q) = \frac{k\sigma_{PN}}{4\pi}(i + \alpha_{PN}) \exp(-\frac{1}{2}\beta_{PN}^2 q^2),$$

$f(q)$ is the proton-nucleon amplitide parametrized like in ref. /1/: $\sigma_{pp} = 4.75\,fm^2$, $\sigma_{pn} = 4.04\,fm^2$,

$\alpha_{pp} = -0.1$, $\alpha_{pn} = -0.45$, $\beta_{pp}^2 = 0.24\,fm^2$, $\beta_{pn}^2 = 0.17\,fm^2$,

$\rho_0(r)$ and $\rho^{tr}(r)$ are the ground state and the transition density for the excitation of the nuclear level with angular momentum L. To avoid complications the presented formulae (but not calculations) do not distinguish between neutrons and protons of a nucleus.

The whole information on nucleus comes from the densities $\rho_0(r)$ and $\rho^{tr}(r)$. We use phenomenological densities of the neutron and proton distributions, therefore no correction to the motion of the center mass is needed.

3. ELASTIC SCATTERING

The charge distribution in intermediate and heavy nuclei obtained from experiments on electron scattering is described rather well by the 3-parameter Fermi distribution

$$\rho_{ch}(r) = \rho_0 \left(1 + W\frac{r^2}{R^2}\right) / \left[1 + \exp\left(\frac{r-R}{a}\right)\right] . \qquad (3.1)$$

On the other hand, the Glauber formula contains the distribution of the nucleon centers. The form factor of this distribution equals to the form factor corresponding to density (3.1) divided by the mass proton form factor (which approximately coincides with the charge one). If the neutron density is chosen in the same form, then the form factor of the neutron centers distribution is the ratio of the form factor corresponding to (3.1) to the neutron mass form factor (equal approximately to the proton charge form factor).

It is important to know the sensitivity of the neutron distribution parameters to the proton distributions found from different experiments on the electron scattering on nuclei.For this purpose we made use of the parameters W_p, R_p and a_p of the proton distribution found in papers /3,4/, and for each set of these parameters we varied the parameters R_n and a_n of the neutron distribution (taken in the same functional form (3.1)) in order to fit experimental data on the proton scattering. As the parameter W_n little affects the differential cross sections it was fixed equal to W_p and was not varied. The fitting was performed for angles $\theta \leq 16°$. The quantity

$$\chi^2 = \frac{1}{N} \sum_{i=1}^{N} \frac{[\sigma_{TH}(\theta_i) - I\sigma_{EXP}(\theta_i)]^2}{[\Delta\sigma_{EXP}(\theta_i)]^2}$$

was minimized. Parameter I normalized the experimental cross section and equals to (1.0 ± 0.1)/1/. In preliminary calculations we considered this parameter to be free. Final results correspond to the averaged value I = 0.92.

The results of calculations are given in Table 1. The letter F denotes the fit of the neutron distribution parameters when one uses the charge distribution parameters from ref./3/ and H those from ref./4/. The results indicate that the calculated parameters of the neutron distribution essentially depend on the used input parameters of the charge distribution. For instance, for ^{44}Ca the use of parameters of ref./3/ results in the neutron skin, whereas from ref./4/ the proton skin. There are two further reasons for possible modification of the results obtained. First, the electron scattering experiments measure not the static density of the charge distribution but the effective one which includes the virtual excitations of a target nucleus. Second, due to the internal charge structure of neutrons

Table 1

Parameters of the ground state density obtained in this paper. The charge distribution parameters are from refs.

Nuclei		R_p	a_p	R_n	a_n	w	$\langle r^2\rangle_p^{1/2}$	$\langle r^2\rangle_n^{1/2}$	χ^2
Ca^{40}	F	3.676	.585	3.720	.557	-.I02	3.486	3.453	2.9
	H	3.777	.589	3.704	.564	-.I58	3.497	3.406	3.5
Ca^{42}	F	3.728	.59I	3.645	.63I	-.II6	3.5I6	3.538	I.0
	H	3.830	.595	3.622	.645	-.I72	3.525	3.466	I.2
Ca^{44}	F	3.748	.572	3.66I	.624	-.095	3.5I5	3.556	0.9
	H	3.850	.575	3.644	.63I	-.I5I	3.53I	3.490	I.3
Ca^{48}	F	3.744	.526	3.954	.53I	-.03	3.476	3.623	2.0
Ti^{48}	F	3.855	.563	3.826	.60I	-.076	3.585	3.639	I.4
	H	3.956	.566	3.8I7	.60I	-.I32	3.605	3.573	2.I

the charge density also depends (though rather weakly as the neutron charge form factor is rather less than the proton one) on the neutron distribution in nucleus /5/. Hence it is impossible to determine reliably and uniquely the proton and neutron centers distribution (it is just contained in the Glauber formula) out of the effective charge distribution. These effects, renormalizing the proton and neutron densities are not very important /5-7/ if one is interested in such absolute values as the differential cross sections. However, the difference of the mean square radii of the proton and neutron distributions is much more sensitive to the above corrections. So, no definite and unconditional conclusion can be made as to the absence or presence of the neutron skin without careful inspections of these corrections. One may expect, however, that these corrections are almost the same for neighbouring nuclei. For this reason and due to a smaller sensitivity to systematic experimental errors, the relative changes of the parameters of the neutron and proton distributions are determined with smaller ambiguities than the absolute ones.

Table 2 demonstrates the isotopic variation of the neutron parameters relative

Table 2

The parameters of proton and neutron densities of Ca isotopes and ^{48}Ti relative to ^{40}Ca

Nuclei		ΔR_n	Δa_n	$\Delta \langle r^2 \rangle_n^{1/2}$	$\Delta \langle r^2 \rangle_p^{1/2}$	ΔR_p	Δa_p
$Ca^{42} - Ca^{40}$	F	-.075	.074	.075	.030	.052	.006
	H	-.082	.08I	.060	.028	.053	.006
$Ca^{44} - Ca^{40}$	F	-.059	.067	.093	.029	.072	-.0I3
	H	-.060	.067	.084	.034	.073	-.0I4
$Ca^{48} - Ca^{40}$	F	.234	-.026	.I60	-.0I	.068	-.059
$Ti^{48} - Ca^{40}$	F	.I06	.044	.I76	.099	.I79	-.022
	H	.II3	.037	.I67	.I09	.I79	-.023

to ^{40}Ca. In the right columns we give also similar variations of the charge distribution parameters. An optimistically minded person may see a correlation between two these variations. As an example we present in Fig. 1 the experimental and calculated cross sections

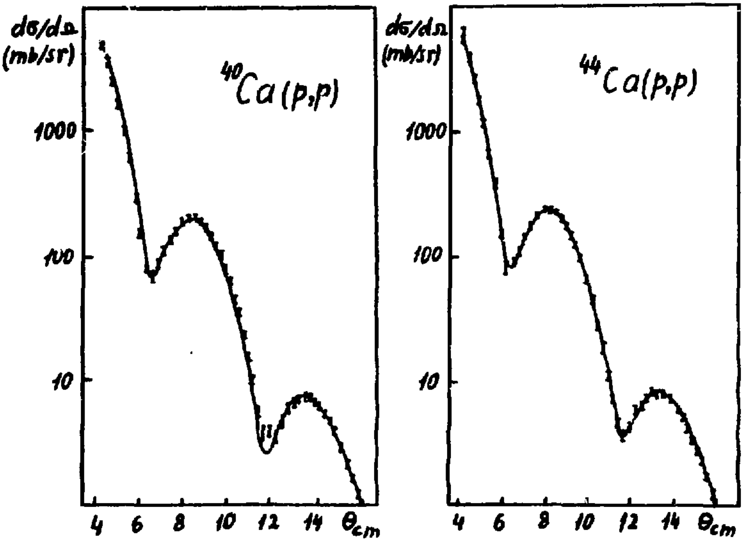

Fig. 1. Differential cross sections cf the 1.044 GeV proton elastic scattering on ^{40}Ca and ^{44}Ca, calculated with the charge distribution parameters of ref./3/ and the neutron ones from Table 1. Experimental data are from ref./1/.

of the proton elastic scattering on ^{40}Ca and ^{44}Ca. The charge density parameters are taken from ref. /3/. Though of the first sight the calculated cross sections are rather consistent with the experimental ones in both cases, these are essentially different with respect to χ^2 criterion:

$$\chi^2 = 2.98 \text{ for } ^{40}\text{Ca} \quad \text{and } \chi^2 = 0.86 \text{ for } ^{44}\text{Ca}.$$

4. INELASTIC SCATTERING

Within the formalism presented the proton inelastic scattering cross section is defined by the transition density $\rho_L(r)$. In ref. /4/ from the analysis of the electron inelastic scattering on the treated nuclei the Gaussian form of the charge transition density

$$\rho_L^{ch}(r) \sim \exp\left[-\left(\frac{r-R}{a}\right)^2\right]$$

was adopted which we will use below. The normalization factor is defined by the reduced electromagnetic transition probability

$$\int \rho_L^{ch}(r)\, r^{L+2}\, dr = \frac{\sqrt{B(EL)}}{Z}.$$

We assume that the charge transition density completely determines the proton transition density $\rho_L^p(r)$. The neutron transition density is taken in the same form as the proton one and normalized by the condition

$$\int \rho_L^n(r)\, r^{L+2}\, dr = \frac{\sqrt{B(EL)}}{N},$$

where N is the number of neutrons. This normalization implies that the square root of the transition probability per one nucleon is the same for protons and neutrons. The common factor B(EL) for proton and neutron transition densities and parameters R_n and a_n are fitting parameters. The resulting parameters of the neutron transition density are given in Table 3. Parameters of the proton ground state density are taken from ref.[4], the ones for neutrons are obtained by the variation procedure described in sec. 3 and presented in Table 1 in lines marked by H. From Table 3 there is evident the correlation between the ground state density parameters are those of transition density. For instance, for ^{42}Ca the radius of the neutron distribution is smaller than that of proton one both for the transition and ground state densities.

The parameter B(EL) was found to be close to the electromagnetic transition probability $B(EL)_{el}$, obtained from the electron inelastic scattering for ^{42}Ca and ^{44}Ca and this justifies the normalization used for these nuclei. The difference of this parameter for ^{48}Ti from the electron one indicates that the relative contribution of neutrons into the excitation of the considered collective

Table 3

Parameters of the neutron transition density obtained in this paper. The charge distribution parameters are from paper[4]

[illegible]	I^{π}	R_p	a_p	R_n	a_n	$\frac{B(EL)}{B(EL)_{el}}$	χ^2
Ca^{40}	3^-	3.536	I.483	3.5II	I.2[illegible]3	.772	20.
Ca^{42}	2^+	3.54I	I.459	3.207	I.679	I.09	2.4
Ca^{44}	2^+	3.67I	I.428	3.I92	I.779	I.	5.2
Ti^{48}	2^+	3.777	I.340	3.280	I.698	I.[illegible]2	5.7

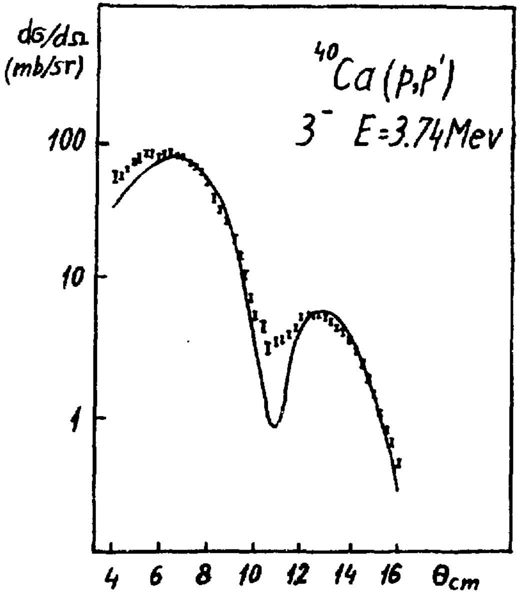

Fig. 2. The experimental and calculated differential cross sections of the 1.044 GeV proton inelastic scattering on ^{40}Ca with the parameters of the transition density from Table 3.

state of this nucleus noticeably differs from that of protons.

The large value $\chi^2 = 20$ should be noted for the excitation of the octupole state in ^{40}Ca (Fig. 2). The calculated cross section differs from the experimental one at small angles and near the diffraction minima. The difference at small angles can be attributed to the Gaussian asymptotics of the transition density. To check this we have performed an analogous calculation with the transition density exponentially decreasing at large distances:

$$\rho^{ch}_{L} \sim r^{L-1} \frac{d}{dr} [1 + \exp (\frac{r-R}{a})]^{-1}$$

and consistent with experimental data on the electron inelastic scattering on the same nucleus/8/. As is expected, the agreement at small angles becomes rather better, while in the region of minima the discrepancy remains almost the same ($\chi^2 = 13$). In an attempt to account for that discrepancy, it is natural to assume an auxiliary structure in the transition density. So we made calculations with the following transition density

$$\rho^{ch}_{L}(r) \sim y^3 (1 - \alpha y^2) \exp(-y^2) \qquad y = \frac{r}{b}$$

simulating the transition densities of microscopic models/9,10/ and used in paper/8/ for description of the electron scattering. In this way we obtained almost the same cross sections as in the first case ($\chi^2 = 19.8$), that indicates the weak sensitivity to the internal (but not to the external) structure of the nuclear density. Thus, the reason for the mentioned anomaly in the case of ^{40}Ca remains still unclear to us.

A satisfactory description of the excitation cross sections of quadrupole states with phenomenological transition densities makes one hope that the proton inelastic scattering will be a useful tool for testing the microscopic models. To demonstrate this we have calculated proton and neutron transition densities within the model with pairing and multipole-multipole inter-

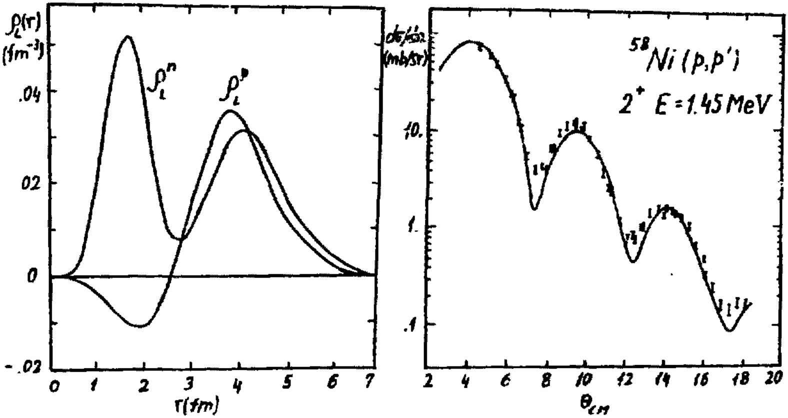

Fig. 3. Neutron and proton transition densities and differential cross section of the proton inelastic scattering on ^{58}Ni. Experimental data are from paper /13/.

action/10/ for nucleus ^{58}Ni. The use of the isovector multipole forces in addition to the isoscalar ones and of a great amount of single-particle levels makes the electromagnetic transition probabilities close to the experimental data for nuclei with the closed proton shells /11/. The results are shown in Fig. 3. The parameters of the proton and neutron ground state distribution are chosen to be equal and taken from the analysis of the electron elastic scattering /12/. The c.m. motion is taken into account by multiplying the calculated inelastic scattering differential cross section by the following factor

$$R(q) = \exp\left(\frac{q^2\langle r^2\rangle}{6A}\right).$$

Here we have used the experimental data on proton scattering from ref.[13]. Note that contrary to paper[14] the differential cross section is calculated correctly without multiplying it by the normalization factor. This is an argument in favor of the right behaviour of the calculated transition densities in the external region of nucleus.

5. CONCLUSION

The previous discussion tells us that experiments on proton elastic and inelastic scattering at intermediate energies are useful tool for studying nuclear characteristics such as mean square mass radii, proton and neutron distributions in nuclei, the location of collective motions inside the nucleus. However, our conclusions about the possibility of the phenomenological description of such subtle effects as the neutron or proton skin are rather pessimistic for the following reasons:

1) the proton component of the density cannot reliably be separated from the neutron one; both of them contribute to the proton and electron scattering;

2) one must not forget that the functional structure of the proton and neutron densities can be different in microscopic models, thus the concept of the neutron skin becomes rather meaningless;

3) and finally, the conclusion about the presence or absence of neutron skin depends on the input proton density parameters, found from the electron scattering.

REFERENCES

1. Alkhazov G.D., Bauer T., Beutrey R. et al. Preprint LINP, No. 218, Leningrad, 1976.
2. Ahmad I. Nucl.Phys., 1975, A247, 418.
3. Frosch R.F.,Hofstadter R., McCarty J.S. et al. Phys.Rev., 1968, 174, 1380.
4. Heisenberg J.H., McCarthy J.S., Sick I. Nucl.Phys., 1971, A164, 353.
5. Bertozzi W., Friar J., Heisenberg J., Negele J.W. Phys.Lett., 1972, 41B, 408.
6. Afanasiev G.N., Shilov V.M., Eliseev S.M. JINR, P2-9415, Dubna, 1975.
7. Onley D.S. Nucl.Phys., 1968, A118, 436.
8. Jtoh K., Oyamada M., Torizuka Y. Phys. Rev., 1970, C2, 2181.
9. Gillet V., Green A.M., Sanderson E.A. Nucl. Phys., 1966, 88, 321.
10. Soloviev V.G. Theory of Complex Nuclei, Pergamon Press, Oxf. 1976.
11. Afanasiev G.N., Shilov V.M. JINR, P4-10497, Dubna, 1977.
12. Afanasiev V.D., Afanasiev N.G., Gulkarov I.S. et al. Sov. J. Nucl. Phys., 1969, 10, 33.
13. Bertini R., Beutrey R., Brochard F. et al. Phys. Lett., 1973, 45B, 119.
14. Brissaud I., Bimbot L., LeBornes V. et al. Phys. Lett., 1974, 48B, 319.

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