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# ON THE SURFACE ENERGY OF NUCLEI

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### Поверхностная энергия ядер

Предлагается новое выражение для поверхностной энергии ядра в формализме плотности энергии. Разные N–N взаимодействия, которые согласуются с экспериментальными данными приводят к разумным значениям поверхностной энергии ядра. Однако ион-ионный потенциал сильно меняется.

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### On the Surface Energy of Nuclei

The surface energy of nuclei in the energy density formalism has been reconsidered and modified. A reasonable surface energy value of a separate nucleus comes out at any choice of the attractive part of the N–N interaction which agrees with experiment. However, the surface interaction potentials turn out to be sensitive to the specific choice of the two-body interaction and they display an additional attraction in the peripheral overlap region.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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The statistical theory of nuclei (the energy density formalism - EDF) has recently been very successfull in the description of bulk nuclear properties such as binding energies, average nuclear sizes and density distributions [1÷4]. The EDF might be also applied to the calculation of ion-ion interaction potentials. The latter are rather important and useful for the theoretical analysis of heavy-ion reactions [5,6]. The basic assumption of the theory concerns the choice of the energy density $\epsilon(\rho)$ that determines the total energy functional:

$$E = \int \epsilon(\rho)\, dr ,$$

where $\rho$ is the nucleon density distribution. The volume energy part of $\epsilon(\rho)$ is well established by nuclear matter calculations. A surface correction $\epsilon_s(\rho)$ added to the volume energy, accounts for the finite size of the nucleus:

$$\epsilon_s(\rho) = \eta \frac{h^2}{8M} (\nabla \rho)^2 .$$

The origin of this gradient correction is not completely clear, so $\eta$ is deliberately kept as a phenomenological parameter to be determined from experiment. Energy density potentials in the sudden collision approximation yield consistently lower values of critical angular momenta when set against experiment [7]. That comparison implies an inadequacy of the EDF interaction potentials, and it is possibly due to the surface correction, which is the least justified quantity of the statistical approach. Therefore, to study the origin and the behaviour of the surface energy in further detail is of great importance, and it is the purpose of the present paper as well.

The surface correction to the total energy can be formally obtained from the following approximate expression for the energy /1/:

$$E = \frac{1}{2} \int \rho(\vec{x})\, \rho(\vec{y})\, V(\vec{x}-\vec{y})\, d\vec{x}\, d\vec{y}\, . \tag{1}$$

If the density $\rho(\vec{y})$ is assumed to be a slowly varying function, one can perform a gradient expansion around $\vec{x} = \vec{y}$, and eq. (1) reduces to:

$$E = \frac{C_0}{2} \int \rho^2(\vec{x})\, d\tau - \frac{C_1}{2} \int (\nabla \rho(\vec{x}))^2 d\tau\ , \tag{2}$$

where

$$C_0 = \int V(r)\, d\vec{r}\ , \quad C_1 = \frac{1}{6} \int V(r) r^2\, d\vec{r}\, . \tag{3}$$

The first and second terms in eq. (2) correspond to the volume energy (proportional to the number of nucleons $A$) and to the $A^{2/3}$ term, respectively, in the Weizsacker formula. The result of the density gradient expansion (2) can be obtained, if we insert into eq. (1) the following separable nucleon-nucleon interactions:

$$V(\vec{x}-\vec{y}) = C_0\, \delta(\vec{x}-\vec{y}) + C_1 \Delta\delta(\vec{x}-\vec{y})\ , \tag{4}$$

where $\delta(\vec{x})$ is the vector delta-functions. It becomes clear that the gradient expansion leads to an exact expression for the total energy, if $V(\vec{x}-\vec{y})$ is given by eq. (4). Hence, to find an expression of the energy, which will allow for a separation of the surface inhomogeneity correction avoiding the density gradient expansion, is of great interest. Let us define, for this purpose, the function $\phi(\vec{u})$:

$$\Delta \phi(\vec{u}) = -V(\vec{u})\ , \tag{5}$$

where $V(\vec{u})$ is the nucleon-nucleon potential. The insertion of expression (5) into eq. (1) gives the following result:

$$E = \int \phi(\vec{u}) \nabla\rho(\vec{x}) \nabla \rho(\vec{x}+\vec{u})\, d\vec{x}\, d\vec{u} + \tag{6}$$
$$+ \frac{1}{2} \int \phi(\vec{u})\, \rho(\vec{x}+\vec{u}) \nabla \rho(\vec{x})\, d\vec{x}\, d\vec{u} .$$

It is easy to verify that, if $\phi(\vec{u})$ is of the type:

$$\phi(\vec{u}) = C_0 / 4\pi \frac{1}{u} - C_1 \delta(\vec{u}) , \tag{7}$$

expression (6) is identical to eq. (2). We should note that the first term in the right-hand side of eq. (7) generates the volume energy, while the second one is responsible for the surface energy in the sense of eq.(2).Obviously, either of the terms in expression (6) contain both volume and surface energy. An explicit separation of the surface energy requires separability of $\phi(\vec{u})$, respectively $V(\vec{u})$. We define, therefore, the surface energy correction in the following way:

$$E_s = 1/2 \int \nabla\rho(\vec{x}) \nabla\rho(\vec{x}+\vec{u})\, \phi_1(\vec{u})\, d\vec{x}\, d\vec{u} , \tag{8}$$

where $\phi_1 = \phi(u) - C_0 / 4\pi \frac{1}{u}$.
The above definition is a generalization of the surface correction following from a gradient expansion of the density, i.e., expression (2) is obtained from eq. (8) when $\phi(\vec{u})$ is given by (7).

The new definition of the surface energy provides with a possibility to avoid the density gradient expansion and, furthermore, it requires the determination of $\phi(\vec{u})$ on the basis of a more realistic N - N potential.

Numerical calculations of the surface energy $E_s$ of $^{16}O$ and the surface interaction potential $V_s(R)$ of the $^{16}O$- $^{16}O$ system are carried out as an illustration. A square potential well with a finite core is taken as an N-N interaction:

$$V(r) = \begin{cases} V_0 & 0 \leq r \leq r_c \\ V_1 & r_c \leq r \leq r_1 \\ 0 & r_1 \leq r \leq \infty . \end{cases} \tag{9}$$

The general solution of the differential equation (5) is:

$$\phi(r) = \int \frac{V(\vec{r}\,')}{4\pi\,|\vec{r} - \vec{r}\,'|}\, d\vec{r} .$$

When $V(r)$ is determined by eq. (9) $\phi(r)$ becomes:

$$\phi(r) = \begin{cases} -\frac{V_0}{6} r^2 + \frac{(V_0 - V_1)}{2} r_c^2 + \frac{V_1}{2} r_1 & 0 \leq r \leq r_c \\ -\frac{V_1}{6} r^2 + \frac{(V_0 - V_1)}{3} r_c^3 \frac{1}{r} + \frac{V_1}{2} r_1^2 & r_c \leq r \leq r_1 \\ \left[ \frac{V_0\, r_c^3}{3} + \frac{V_1 (r_1^3 - r_c^3)}{3} \right] \frac{1}{r} & r_1 < r < \infty . \end{cases} \tag{10}$$

In order to simplify the calculations, we use a one-parameter density distributions of $^{16}O$

$$\rho(r) = \rho_0\, e^{-\alpha^2 r^2} , \tag{11}$$

where $\alpha^2 \approx 0.21$. This value of $\alpha$ follows from the experimentally observed mean square radius of $^{16}O$ ( $r_{rms} \approx 2.7\,fm$ ).

Thus, we finally obtain:

$$E_s = \frac{\pi^{5/2}}{\sqrt{2}} \rho_0^2 \alpha \int_0^\infty \phi_1(r)\, r^2\, e^{-\frac{\alpha^2 r^2}{2}} \left( \frac{3}{\alpha^2} - r^2 \right) dr ,$$

$$V_s(R) = \frac{4}{\sqrt{2}} \pi^{5/2} \rho_0^2 \alpha\, e^{-\frac{\alpha^2 R^2}{2}} \int_0^\infty \phi_1(r)\, r^2 e^{-\frac{\alpha^2 r^2}{}} \times$$

$$\times \left[ \left( \frac{1}{2\alpha^4 R} - \frac{R}{2\alpha^2} \right) \frac{\mathrm{sh}(\alpha^2 rR)}{r} - \frac{r}{2\alpha^2 R}\, \mathrm{sh}(\alpha^2 rR) + \frac{1}{\alpha^2}\, \mathrm{ch}(\alpha^2 rR) \right] dr, \qquad (12)$$

where R is the relative distance between the centers of mass of the two nuclei.

The square well parameters are /8/ :

$$r_c = 0.5\,\mathrm{fm}, \quad r_1 = 2.2\,\mathrm{fm}, \quad -15\,\mathrm{MeV} \geq V_1 \geq -40\,\mathrm{MeV}.$$

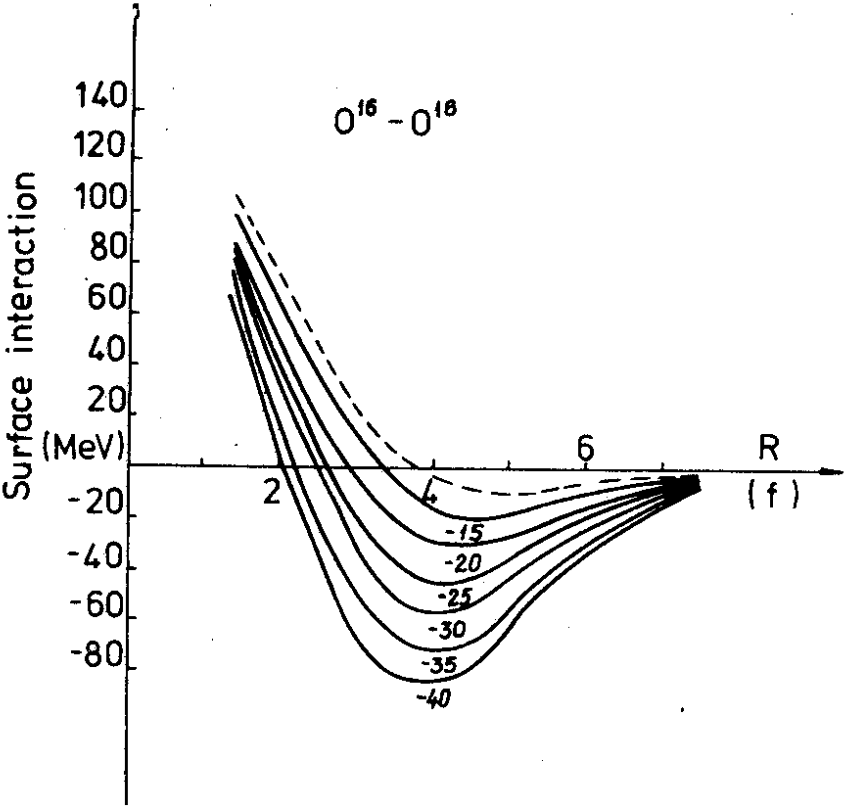
The above range of values of $V_1$ is in a satisfactory agreement with experimental data on scattering, binding energy of the deuteron and the nuclear matter saturation curve.

When $V_1 = -35$ *MeV* and $V_0 = 2300$ *MeV* the surface of $^{16}O$ is of the order of 75 *MeV* which agrees well with other estimates /1 ÷ 4/. *Table 1* contains a few sets of $V_0$ and $V_1$ values and they yield the same value of the $^{16}O$ surface energy. The surface interaction potential of the $^{16}O - {}^{16}O$ system is computed for each set of $V_0$ and $V_1$ values. The curves $V_s(R)$ are displayed in *fig. 1* together with the result of the gradient correction calculation

*Table 1*

Sets of $V_0$ and $V_1$ values used in the present paper for the calculation of the $^{16}O$ surface energy and the surface interaction potential of the $^{16}O - {}^{16}O$ system.

| $V_1$ *(MeV)* | $V_0$ *(MeV)* |
|---|---|
| -40 | 2700 |
| -35 | 2300 |
| -30 | 2000 |
| -25 | 1700 |
| -20 | 1400 |
| -15 | 1100 |



*Surface interaction energy of the* $^{16}O - ^{16}O$ *system. Dashed curve is the result of the standard gradient correction. Solid curves correspond to the present calculations at different values (indicated in the figure) of* $V_1$ .

$$(V_s(R) = \eta \frac{h^2}{8M} \int \nabla \rho_1(\vec{x}) \, \nabla \rho_2(\vec{x}) d\vec{x},$$

where $\eta \approx 15$ ).

It is seen, that while a reasonable value of the $^{16}O$ surface energy can be obtained for a large number of $V_0$ and $V_1$ values, the latter generate, however, quite different potenails $V_s$ (R). The surface ion-ion potential is from 2 to 10 times more attractive when compared with the result of the gradient expansion in the acceptable range of variation of $V_1$.

All medium and heavy nuclei have similar surface layers. This peculiarity together with the intensiveness of the surface energy to details in the N - N interaction explain the success of the previous parametrization

$$\eta \frac{h^2}{8M} (\nabla \rho)^2$$

applied to a wide range of nuclei with an universal value of $\eta$. That does not hold, however, for the case of the surface interaction potential. The latter displays a pronounced sensitivity to the specific choice of the N-N potential attractiveness. A stronger attraction of the N-N interaction leads to a deeper $V_s$ (R) curve. Thus, the new energy density potentials come in a qualitative agreement with the results of other phenomenological methods/9,10/.

It should be pointed out, in conclusion, that although the energy density formalism is very successful in the description of bulk nuclear properties, its surface correction term must be carefully reconsidered before calculating heavy-ion interaction potentials.

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