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Новое распределение плотности нуклонов в атомных ядрах

Предлагается новая двухпараметрическая форма распределения плотности нуклонов, хорошо описывающая ряд экспериментальных данных как по рассеянию электронов, так и по рассеянию протонов на ядрах. Приведены расчеты дифференциальных сечений упругого рассеяния протонов и электронов на ^{16}O и ^{40}Ca.

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A New Phenomenological Density Distribution of Atomic Nuclei

A new two-parameter nucleon density distribution (NDD) is proposed in the present paper. It accounts for a set of experimental data, deduced from electron and proton scattering on nuclei. Calculations of differential cross sections for elastic electron scattering on ^{16}O and ^{40}Ca are carried out. The NDD parameters that follow from electron scattering analysis are used for the calculation of proton elastic scattering on the same nuclei.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. The nucleon density distribution (NDD) is an essential quantity closely connected to fundamental nuclear properties. Therefore, it has been thoroughly studied during the last few years [1]. The attention paid to the NDD is due to the fact that it is related to various geometrical and dynamical properties of nuclei, such as, r.m.s. radii, surface thickness, binding energies, etc. The NDD seems to be a good test of any theoretical approach, which deals with single-particle wave functions, or if it is based on the NDD as a variable, which determines both dynamical and static properties of nuclei. The theoretical significance of the NDD explains the further detailed study of the NDD, especially, by analysing experiments of particle scattering on nuclei. The well known Fermi-type of the NDD $\rho_F = \rho_0(1+\exp(r-R)/b)^{-1}$ has no clear theoretical ground and disagrees with electron-scattering data for $^{40-48}$Ca at energies 250, 500 and 750 *MeV*. It has been necessary to introduce additional parameters and to supplement an oscillating term to $\rho_F(r)$ [2] in order to obtain a better agreement between theory and experiment. A tentative attempt, based on the possible existence of a π -condensate in nuclei, has been, recently, made to explain these oscillations [3].

It is shown in the present paper, that a new two-parameter NDD can be successfully used for the simultaneous analysis of electron and proton scattering data without an introduction of an oscillating density term.

2. The proposed NDD is of the following type:

$$\rho(r) = \frac{\rho_0}{\sqrt{\pi}} \int_{(r-R)/s\sqrt{2}}^{(r+R)/s\sqrt{2}} \exp(-y^2)\,dy, \quad \rho_0 = \frac{3}{4\pi[R^3+3Rs^2]}. \qquad (1)$$

The density parameters R and s determine the r.m.s. radius:

$$\langle r^2 \rangle^{1/2} = \left(\frac{\frac{3}{5}R^4 + 6R^2s^2 + 9s^4}{R^2+3s^2} \right)^{1/2} \qquad (2)$$

and the surface diffuseness, defined as the distance of 90% to 10% density decrease: $t = 2.569s$.

Here the NDD is introduced phenomenologically. Nevertheless, it should be noted that (1) can be formally obtained /4/ from the expectation value for the density operator $\hat{\rho} = \rho_0\,\Theta(R(\alpha)-r)$ making use of the ground state wave-function of the Hamiltonian $H = \sum_{\lambda\mu} \hbar\omega_{\lambda}(b^{+}_{\lambda\mu} b_{\lambda\mu} + \frac{1}{2})$, where $\{\alpha\}$ are the deformations parameters $\alpha = (h/2B_{\lambda}\omega_{\lambda})^{1/2}(b_{\lambda\mu} + (-1)^{\mu} b^{+}_{\lambda,-\mu})$.

Besides the obvious physical meaning of the NDD parameters, formula (1) allows an exact Fourier transformation that yields an analytic expression for the nuclear form factor:

$$F(q) = \frac{3}{q^2[R^3+3Rs^2]}\left[\frac{\sin(qR)}{q} - R\cos(qR) + q s^2 \sin(qR)\right]\exp\left(-\frac{q^2s^2}{2}\right) \qquad (3)$$

The NDD parameters are determined by comparison of the differential cross section:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} |F|^2 \qquad (4)$$

with the electron scattering experimental data for ^{16}O /5/ and ^{40}Ca /2,6,7/. The optimal set of parameters of the NDD thus obtained, the r.m.s. radius, and the values of the surface thickness are given in *Table 1.*

Table 1

Nuclei	R *(fm)*	s *(fm)*	$\langle r^2\rangle^{1/2}$ *(fm)*	t *(fm)*
^{16}O	2.40	0.88	2.476	2.260
^{40}Ca	3.55	0.92	3.310	2.350

The calculated electron cross sections for ^{40}Ca and ^{16}O are shown in *figs. 1,2*. The satisfactory agreement of the calculated cross sections at energies 500 *MeV* and 750 *MeV* with the experimental data for ^{40}Ca is an independent test of the proposed NDD. Such a test, in the case of ^{16}O is the comparison between theoretical and experimental results of 1 *GeV* proton-scattering.

3. The proton elastic scattering amplitude at high energies, employed here, is the "optical limit" of the amplitude in the Glauber multiple scattering theory /8/ :

$$f(q)=\exp\left(\frac{q^2R^2}{4A}\right)\frac{ik}{2\pi}\int d^2b\, e^{i\vec{q}\vec{b}}\{1-e^{i\chi(\vec{b})}\}, \quad (5)$$

where the phase function $\chi(\vec{b})$ is determined through the NDD (1):

$$i\chi(\vec{b})=-A\int d^2s\,\rho(\vec{s})\,\gamma(\vec{b}-\vec{s});\ \ \rho(\vec{s})=\int_{-\infty}^{+\infty}dz\,\rho[(s^2+z^2)^{1/2}]. \quad (6)$$

Here

$$\gamma(\vec{b}-\vec{s})=\frac{(1-i\alpha)\,\sigma}{4\pi a}\exp\left(-\frac{(\vec{b}-\vec{s})^2}{2a}\right) \quad (7)$$

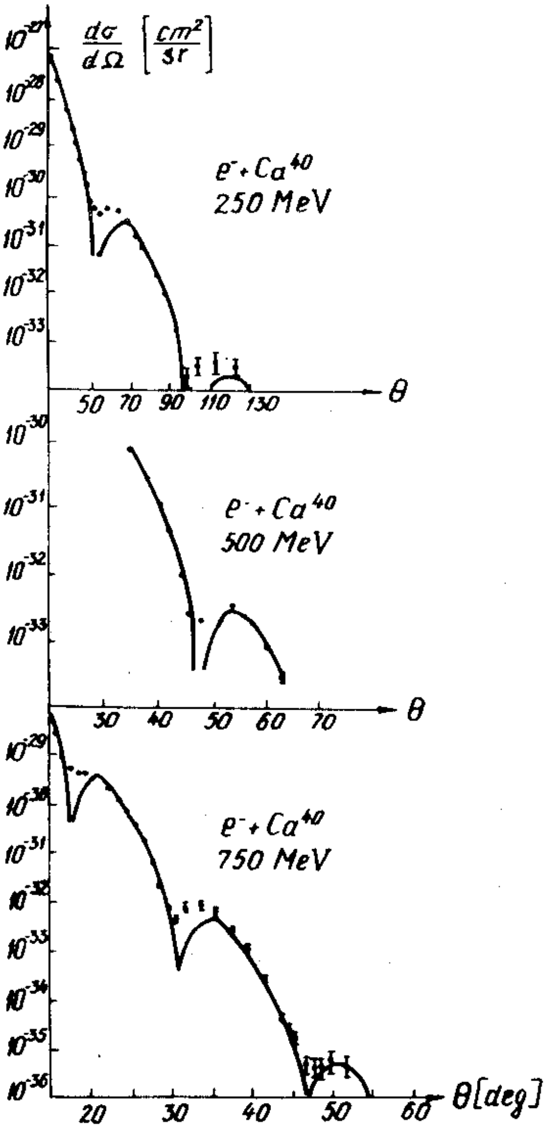

Fig. 1. Differential cross sections for elastic electron scattering on ^{40}Ca at energies 250, 500 and 750 MeV. The solid curves represent the calculated cross sections.

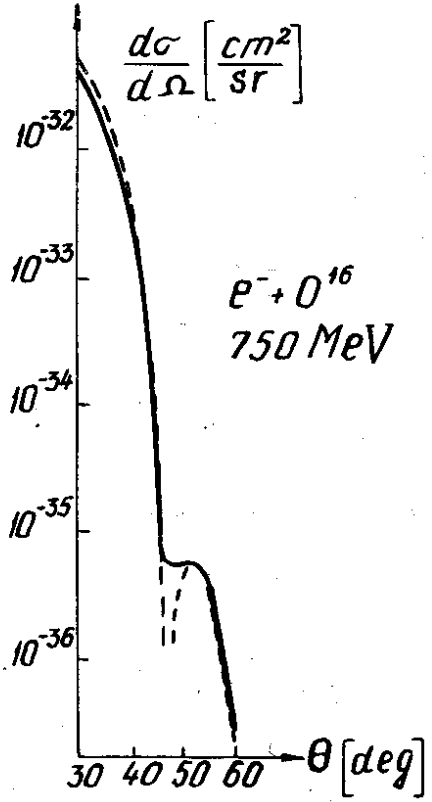

Fig. 2. Differential cross section for elastic electron scattering on ^{16}O at energy 750 MeV. The solid line represents the experimental data. The dashed line is the calculated cross section.

is the "profile" function of the nucleon. The nucleon-nucleon scattering amplitude:

$$f_{NN}(\delta) = \frac{(i+\alpha)}{4\pi} k\sigma \exp\left(-\frac{a\delta^2}{2}\right) \qquad (8)$$

which determines (7) has the following parameters[8]:

$\sigma = 4.377\ \mathrm{fm}^2$; $a = 0.181\ \mathrm{fm}^2$, $\alpha = -0.40$.

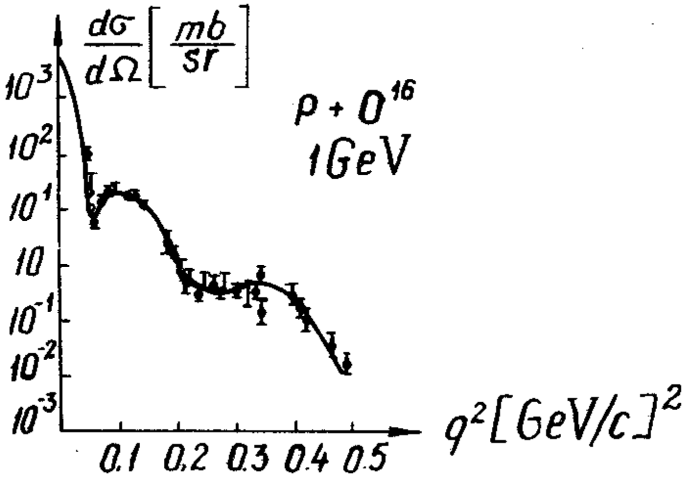

Fig. 3. Differential cross section for 1 GeV elastic proton scattering on ^{16}O. *The solid line represents the calculated cross section.*

The calculated cross sections and the experimental data for 1 *GeV* elastic proton scattering on ^{16}O /9/ and ^{40}Ca /10/ are given in *figs. 3,4*. The parameters of the NDD have been already fixed from electron experiments (i.e., the same as in *Table 1*).

4. It is seen, that the proposed NDD (1) gives a satisfactory explanation of the set of experimental data of electron and proton scattering on ^{16}O and ^{40}Ca at different energies. It should be emphasized that the large discrepancy between theory and experiment in the region of the third maximum for ^{40}Ca at E = 750 *MeV* in other approaches, disappears in our case. Using the new NDD (1), a good agreement is reached even in the first approximation. Hence, one can argue, that the oscillating density introduced in refs. /2,3/ does not necessarily follow from experimental data of particle-scattering on nuclei. Besides, the satisfactory simulta-

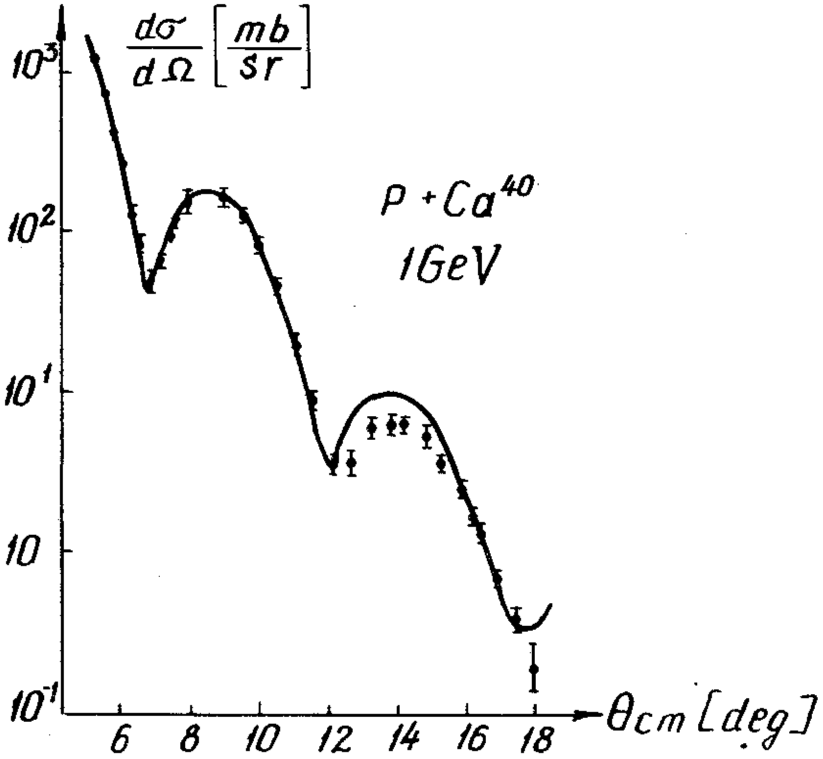

Fig. 4. Differential cross section for 1 GeV elastic proton scattering on ^{40}Ca. *The solid line is the calculated cross section.*

neous analysis of electron and proton scattering data with one and the same NDD shows, that elastic proton scattering at high energies is a rather effective method of an independent determination of the NDD parameters.

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