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Однофононные состояния в деформированных ядрах для изоскалярного и изовекторного взаимодействий

Проведено обобщение формул, описывающих в рамках RPA однофононные состояния сложных четно-четных ядер с одновременным учетом изоскалярных и изовекторных мультиполь-мультипольных сил. Изложенный формализм позволяет дать единое описание низколежащих и высоковозбужденных состояний и ряда характеристик гигантских мультипольных резонансов.

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One-Phonon States in Deformed Nuclei for Isoscalar and Isovector Interactions

The formulas describing one-phonon states of complex even-even deformed nuclei in the frame of RPA are generalized to simultaneous consideration of isoscalar and isovector multipole-multipole interaction. The formalism reported gives a united description of low-lying states and of several characteristics of giant multipole resonances. Here we present a procedure of expressing the reduced probability $B(E\lambda; 0^{+}0 \to I_f^{\pi_f} K_f)$ and energy weighted sum rule (EWSR) by means of strength functions averaged over some energy interval. This procedure makes calculations much easier allowing not to solve the seqular equation for the energies of one-phonon states.

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In recent years much interest arises in experimental and theoretical study of giant multipole resonances in atomic nuclei. The investigations beyond the frame of the phenomenological approach[1,2] are important. In the description of the structure of low lying collective states the phonons are used with characteristics calculated taking into account the multipole-multipole interaction[3,4]. For describing the region of giant multipole resonances one needs phonons which characteristics are calculated considering both isoscalar and isovector multipole interactions simultaneously.

In the present paper the formulas written for one-phonon states are generalized to the simultaneous consideration of isoscalar and isovector multipole-multipole interactions.

1. SECULAR EQUATIONS AND WAVE FUNCTIONS OF THE ONE-PHONON STATES

To describe the giant multipole resonances we introduce the phonons with isotopic spin $T=0$ and $T=1$ with $T_z=0$. In complex nuclei the collective vibrations with $T=0$ and $T=1$ are not independent and, hence, one should consider them simultaneously.

Following[4] the Hamiltonian of the system has the form:

$$H = H_{av} + H_{pair} + H_Q , \qquad (1)$$

where H_{av} describes the average field, H_{pair} is the interaction resulting in the pairing correlations of superconducting type, H_Q has the form

$$H_Q = -\frac{1}{2}\sum_{\lambda,\mu>0} \{(\kappa_0^{(\lambda)} + \kappa_1^{(\lambda)})[Q^+_{\lambda\mu}(n)Q_{\lambda\mu}(n) + Q^+_{\lambda\mu}(p)Q_{\lambda\mu}(p)] + (\kappa_0^{(\lambda)} - \kappa_1^{(\lambda)})[Q^+_{\lambda\mu}(n)Q_{\lambda\mu}(p) + Q^+_{\lambda\mu}(p)Q_{\lambda\mu}(n)]\}, \qquad (2)$$

here $Q^+_{\lambda\mu}(n)$ is the multipole momentum operator of the neutron system (see (3.63) in ref. [4]). Isoscalar $\kappa_0^{(\lambda)}$ and isovector $\kappa_1^{(\lambda)}$ constants of multipole-multipole interaction are connected with constants $\kappa_{nn}^{(\lambda)}$, $\kappa_{pp}^{(\lambda)}$ and $\kappa_{np}^{(\lambda)}$ in the following way[4]:

$$\kappa_{nn}^{(\lambda)} = \kappa_{pp}^{(\lambda)} = \kappa_0^{(\lambda)} + \kappa_1^{(\lambda)} , \qquad \kappa_{np}^{(\lambda)} = \kappa_0^{(\lambda)} - \kappa_1^{(\lambda)} . \qquad (3)$$

Let us introduce the phonon creation operator

$$Q^+_g = \frac{1}{2}\sum_{qq'} \{\psi^g_{qq'} A^+(qq') - \phi^g_{qq'} A(qq')\}, \qquad (4)$$

where

$$A(qq') = \frac{1}{\sqrt{2}}\sum_\sigma \sigma a_{q'\sigma} a_{q-\sigma} \quad \text{or} \quad \frac{1}{\sqrt{2}}\sum_\sigma a_{q\sigma} a_{q'\sigma} ,$$

$a_{q\sigma}$ is the operator of quasiparticle absorption, $g=\lambda\mu j$, j being the number of collective states with given $\lambda\mu$, $q\sigma$ denotes the set of quantum numbers for a single-particle state of the neutron and proton system; $s\sigma$ is that for the neutron system, and $r\sigma$ is for the proton system; $\sigma=\pm 1$. The part of the Hamiltonian (1) for calculating one-phonon states can be written as

$$\begin{aligned} H_v = {} & \sum_q \epsilon(q) B(qq) - \\ & -\frac{1}{4}\delta_{\lambda\mu,20}\sum_{jj'}\{G_N \sum_{ss'}[(u_s^2 - v_s^2)(u_{s'}^2 - v_{s'}^2)g^{20j}_{ss}g^{20j'}_{s's'} + w^{20j}_{ss}w^{20j'}_{s's'}] + \\ & + G_Z \sum_{rr'}[(u_r^2 - v_r^2)(u_{r'}^2 - v_{r'}^2)g^{20j}_{rr}g^{20j'}_{r'r'} + w^{20j}_{rr}w^{20j'}_{r'r'}]\}Q^+_{20j}Q_{20j'} - \qquad (5) \\ & -\frac{1}{2}\sum_{gg'}\{(\kappa_0^{(\lambda)} + \kappa_1^{(\lambda)})[\sum_{\substack{ss' \\ s_2s_2'}} u_{ss'}u_{s_2s_2'} f^g(ss')g^g_{ss'} f^{g'}(s_2s_2')g^{g'}_{s_2s_2'} + \\ & + \sum_{\substack{rr' \\ r_2r_2'}} u_{rr'}u_{r_2r_2'} f^g(rr')g^g_{rr'} f^{g'}(r_2r_2')g^{g'}_{r_2r_2'}] + \\ & + (\kappa_0^{(\lambda)} - \kappa_1^{(\lambda)})\sum_{\substack{ss' \\ rr'}} u_{ss'}u_{rr'}[f^g(ss')g^g_{ss'}f^{g'}(rr')g^{g'}_{rr'} + f^{g'}(ss')g^{g'}_{ss'} \times \\ & \times f^g(rr')g^g_{rr'}]\}Q^+_g Q_{g'} , \end{aligned}$$

where $\epsilon(q) = \sqrt{C^2 + (E(q)-\lambda)^2}$, $u_{qq'} = u_q v_{q'} + u_{q'} v_q$, $f^g(qq') = f^{\lambda\mu}(qq')$ is the matrix element from $\frac{r^\lambda}{\sqrt{2(1+\delta_{\mu,0})}}(Y_{\lambda\mu} + (-1)^\mu Y_{\lambda-\mu})$, $g^g_{qq'} = \psi^g_{qq'} + \phi^g_{qq'}$, $w^g_{qq'} = \psi^g_{qq'} - \phi^g_{qq'}$, G_N and G_Z are the pairing constants.

The wave function of a one-phonon state has the form:

$$Q^+_g \Psi , \qquad (6)$$

where Ψ denotes the phonon vacuum. According to the orthonormalization condition of eq. (6) we have:

$$\sum_{qq'} g^{g'}_{qq'} w^{g}_{qq'} = 2\delta_{g,g'} \qquad (7)$$

Following ref.[4] we shall find the energy ω_g of a one-phonon state with fixed $\lambda\mu$ or K^π by variational principle defined as:

$$\delta\{<Q_g H_v Q^+_g> - \frac{\omega_g}{2}[\sum_{qq'} g^g_{qq'} w^g_{qq'} - 2]\} = 0. \qquad (8)$$

After some transformation one obtains the following secular equation

$$\begin{vmatrix} (\kappa_0^{(\lambda)}+\kappa_1^{(\lambda)})X^g(n)-1 & (\kappa_0^{(\lambda)}-\kappa_1^{(\lambda)})X^g(n) \\ (\kappa_0^{(\lambda)}-\kappa_1^{(\lambda)})X^g(p) & (\kappa_0^{(\lambda)}+\kappa_1^{(\lambda)})X^g(p)-1 \end{vmatrix} = 0, \qquad (9)$$

which form coincides with that of eq.(8.134) in ref.[4]. Here

$$X^g(n) = 2\sum_{ss'} \frac{f^g(ss')\tilde f^g(ss')u^2_{ss'}\epsilon(ss')}{\epsilon^2(ss')-\omega_g^2}, \qquad (10)$$

where

$$\tilde f^g(ss') = f^g(ss') - \frac{\Gamma^g_n(s)}{\gamma^g_n}\delta_{s,s'}, \quad \epsilon(ss') = \epsilon(s)+\epsilon(s'), \qquad (10')$$

$$\gamma^g_n = \sum_{ss'} \frac{4C_n^2 - \omega_g^2 + 4\in(s)\in(s')}{\epsilon(s)(4\epsilon^2(s)-\omega_g^2)\epsilon(s')(4\epsilon^2(s')-\omega_g^2)}, \qquad (10'')$$

$$\Gamma^g_n(s) = \sum_{s_2 s_2'} \frac{f^g(s_2 s_2')[4C_n^2-\omega_g^2+4\in(s_2)\in(s_2')-4\in(s)\in(s_2)+4\in(s)\in(s_2')]}{\epsilon(s_2)[4\epsilon^2(s_2)-\omega_g^2]\epsilon(s_2')[4\epsilon^2(s_2')-\omega_g^2]}, \qquad (10''')$$

$\in(s) = E(s)-\lambda_n$. As can be seen from eq. (10′) $\tilde f^g(ss')$ coincides with $f^g(ss')$, if $\lambda\mu \neq 20$.

Eq. (9) can be written as

$$\mathcal{F}(\omega) = \kappa_0^{(\lambda)}\kappa_1^{(\lambda)}(X^g(n)-X^g(p))^2 - (1-\kappa_0^{(\lambda)}X^g)(1-\kappa_1^{(\lambda)}X^g) = 0, \qquad (9')$$

where $X^g = X^g(n)+X^g(p)$. Note, the influence of the values of constant $\kappa_1^{(\lambda)}$ on the first one-phonon states was investigated in ref.[3] where the introduction of $\kappa_1^{(\lambda)}$ was shown to cause the renormalization of the constant $\kappa_0^{(\lambda)}$ without noticeable change of the state structure. Therefore only isoscalar strength component $\kappa_0^{(\lambda)}$ was used, in further calculations of the properties of low lying states in isovector constant $\kappa_1^{(\lambda)}$ being assumed to be zero.

In finding the $g^g_{qq'}$, and $w^g_{qq'}$ functions one makes use of the normalization condition of eq. (7) and after cumber transformations obtains:

$$g^g_{rr'} = \sqrt{\frac{2}{Y_g}}\, y^g_p \frac{\tilde f^g(rr')u_{rr'}\epsilon(rr')}{\epsilon^2(rr')-\omega_g^2}, \qquad (11)$$

$$w^g_{rr'} = \sqrt{\frac{2}{Y_g}}\, y^g_p \{\frac{\tilde f^g(rr')u_{rr'}\omega_g}{\epsilon^2(rr')-\omega_g^2} - \delta_{r,r'}\frac{C_p \Xi^g_p}{\epsilon(r)\omega_g \gamma^g_p}\}, \qquad (12)$$

where

$$y^g_p = \frac{(\kappa_0^{(\lambda)}-\kappa_1^{(\lambda)})X^g(n)}{1-(\kappa_0^{(\lambda)}+\kappa_1^{(\lambda)})X^g(p)}, \qquad (12')$$

$$\Xi^g_p = \sum_{rr'} \frac{f^g(rr')}{\epsilon(r)(4\epsilon^2(r)-\omega_g^2)} \cdot \frac{4C_p^2-\omega_g^2+4\in(r)\in(r')}{\epsilon(r')(4\epsilon^2(r')-\omega_g^2)}. \qquad (12'')$$

Expressions for $g^g_{ss'}$, $w^g_{ss'}$ look similarly with replacement of $C_p, \Xi^g_p, \gamma^g_p, y^g_p$ by C_n, Ξ^g_n, γ^g_n, $y^g_n \equiv 1$, which correspond to neutrons.

Here

$$Y_g = Y_g(n) + (y^g_p)\, Y_g(p) = \frac{1}{4}\frac{y^g_p}{\kappa_0^{(\lambda)}-\kappa_1^{(\lambda)}}\frac{\partial F(\omega)}{\partial\omega}, \qquad (13)$$

$$Y_g(n) = \sum_{ss'} \frac{(\tilde{f}^g(ss')u_{ss'})^2 \epsilon(ss')\omega_g}{[\epsilon^2(ss') - \omega_g^2]^2} = \frac{1}{4} \frac{\partial X^g(n)}{\partial \omega}, \qquad (14)$$

expression for $Y_g(p)$ being analogous to exp. (14). If the isovector strength component is absent ($\kappa_1^{(\lambda)}=0$) then $y_p^g=1$ and exps. (11)-(14) have the form reported in chapter 8 of ref. [4].

2. $E\lambda$ -TRANSITION PROBABILITIES

We shall obtain formulas for reduced probabilities of $E\lambda$ -transitions from the ground states of even-even nuclei to the excited states with $I_f^{\pi f}\ K_f$. In case the multipolarity λ of the electrical transition coincides with that of a phonon of excited state, then following [4] we have

$$B(E\lambda; 0^+0 \to I_f^{\pi f} K_f) = (00\lambda\mu \mid I_f K_f)^2 M^2, \qquad (15)$$

$$M = \left(\frac{2-\delta_{\mu,0}}{2}\right)^{1/2} \{ e_{eff}^{(\lambda)}(p) \sum_{rr'} f^{\lambda\mu}(rr') g_{rr'}^g u_{rr'} + \qquad (16)$$

$$+ e_{eff}^{(\lambda)}(n) \sum_{ss'} f^{\lambda\mu}(ss') g_{ss'}^g u_{ss'} \}.$$

The effective electrical charges are

$$e_{eff}^{(\lambda)}(p) = e + e_{pol}^{(\lambda)}(p),$$

$$e_{eff}^{(\lambda)}(n) = e_{pol}^{(\lambda)}(n), \qquad (17)$$

where according to [1]

$$e_{pol}^{(\lambda)}(p) = e_{pol}^{(\lambda)}(T=0) - e_{pol}^{(\lambda)}(T=1),$$

$$e_{pol}^{(\lambda)}(n) = e_{pol}^{(\lambda)}(T=0) + e_{pol}^{(\lambda)}(T=1).$$

As is known for E1 -transitions the effective charges are

$$e_{eff}^{(1)}(p) = \frac{N}{A} e, \quad e_{eff}^{(1)}(n) = -\frac{Z}{A} e. \qquad (17')$$

Using formula (11) we get

$$M = \frac{1}{2}\left(\frac{2-\delta_{\mu,0}}{Y_g}\right)^{1/2} [e_{eff}^{(\lambda)}(p)\ X^g(p) y_p^g + e_{eff}^{(\lambda)}(n) X^g(n)]. \qquad (18)$$

We perform some transformations of this expression considering exps. (9′), (12′) and (13)

$$M^2 = \frac{2-\delta_{\mu,0}}{\frac{\partial F(\omega)}{\partial \omega}} \{ (e_{eff}^{(\lambda)}(p))^2 X^g(p)[1-(\kappa_0^{(\lambda)}+\kappa_1^{(\lambda)})X^g(n)] +$$

$$+ (e_{eff}^{(\lambda)}(n))^2 X^g(n)[1-(\kappa_0^{(\lambda)}+\kappa_1^{(\lambda)})X^g(p)] + \qquad (18')$$

$$+ 2e_{eff}^{(\lambda)}(n) e_{eff}^{(\lambda)}(p)(\kappa_0^{(\lambda)}-\kappa_1^{(\lambda)}) X^g(n) X^g(p) \}.$$

As a result the reduced probability of the $E\lambda$ -transition can be written as follows

$$B(E\lambda; 0^+0 \to I_f^{\pi f}\ K_f) \equiv B(E\lambda; 0 \to \omega_g) =$$

$$= (00\lambda\mu | I_f K_f) \frac{2-\delta_{\mu,0}}{\frac{\partial F(\omega_g)}{\partial \omega_g}} \{ (e_{eff}^{(\lambda)}(p))^2 X^g(p) + (e_{eff}^{(\lambda)}(n))^2 X^g(n) - \qquad (19)$$

$$- [\kappa_0^{(\lambda)}(e_{eff}^{(\lambda)}(p) - e_{eff}^{(\lambda)}(n))^2 + \kappa_1^{(\lambda)}(e_{eff}^{(\lambda)}(p) + e_{eff}^{(\lambda)}(n))^2] X^g(n) X^g(p) \} \equiv$$

$$\equiv \frac{P(\omega_g)}{\frac{\partial F(\omega_g)}{\partial \omega_g}}.$$

In [5] calculations have been made for even-even deformed nuclei in the frame of the superfluid model considering multipole-multipole interactions. The results are in satisfactory agreement with experimental data both on energy of low-lying states and on

$E\lambda$ -transition probabilities. The characteristics of states at intermediate and high excitation energy may also be investigated in the frame of this model. However, the two factors should be taken into account[6]: high density of highly excited states and a significant complication of their structure. Consequently, simple vibrational and quasiparticle states will be fragmentated over many nuclear levels and the anharmonic effects play a more important role. Therefore, highly excited states cannot be studied individually neither in experiment nor theoretically. However, in spite of the structure complexity, in the study of direct electromagnetic transitions into the ground nuclear state or photonuclear reactions one may consider that the one-phonon components of the wave function are mainly responcible for these phenomena. Thus, a simple harmonic consideration may give a correct qualitative and even quantitative description of these effects in calculating probability of electromagnetic transitions averaged over some energy interval. The interval of averaging is defined by the experimental resolution, the vibrational states fragmentation range, and by accuracy of calculations due to uncertainty of the model parameters. In principle, expression (19) for $B(E\lambda; 0 \to \omega_g)$ can be strictly used for such calculations, for example, for determining photoexcitation probabilities of giant resonances of different multipolarities. For this, having a solution of the secular equation (9′) one should determine the energies of all the excited states ω_g with given K^π in the energy interval Δ ,

their wave functions, find corresponding to them values of $B(E\lambda; 0 \to \omega_g)$ an do averaging over a chosen interval Δ. As a result we obtain the strength function $b(E\lambda,\omega)$ defining the averaged value of a reduced transition probability via energy ω:

$$b(E\lambda,\omega) = \frac{1}{\Delta} \sum_g{}_\Delta B(E\lambda; 0 \to \omega_g), \tag{20}$$

$\sum_g{}_\Delta$ means summation over all the states in the energy interval Δ with a middle point ω. However, the strength function $b(E\lambda, \omega)$ can be obtained much easier, i.e., without calculating in detail the wave functions and energy of each state. Let us define the strength function $b(E\lambda,\omega)$ as

$$b(E\lambda,\omega) = \sum_g B(E\lambda; 0 \to \omega_g)\rho(\omega-\omega_g). \tag{21}$$

Following[7] the weight function is taken as

$$\rho(\omega-\omega_g) = \frac{1}{2\pi} \frac{\Delta}{(\omega-\omega_g)^2 + (\Delta/2)^2}. \tag{22}$$

This function has maximum at $\omega = \omega_g$ and a fast tendency to zero at $|\omega-\omega_g| \to \infty$. It is normalized as follows

$$\int_{-\infty}^{\infty} \rho(\omega-\omega_g)d\omega = 1. \tag{23}$$

Different from (20) in (21) the summation is done over a total set of states g with given K^π. However due to the fast decrease of the function (22) when passing ω from ω_g the following relationship holds

$$\int_{\omega-\Delta/2}^{\omega+\Delta/2} b(E\lambda,\omega')d\omega' \simeq \sum_g{}_\Delta B(E\lambda; 0 \to \omega_g). \tag{24}$$

Using (19) and (22) the strength function (21) may be written as a contour integral:

$$b(E\lambda,\omega) = \frac{\Delta}{2\pi}\sum_g \frac{P(\omega_g)}{\frac{\partial F(\omega_g)}{\partial\omega_g}} \frac{1}{(\omega-\omega_g)^2+(\Delta/2)^2} = \tag{25}$$
$$= \frac{\Delta}{2\pi}\cdot\frac{1}{2\pi i}\int_{C_g}\frac{P(z)}{F(z)}\frac{dz}{(\omega-z)^2+(\Delta/2)^2},$$

the contour of integration C_g is shown in the figure. The equation (25) is obvious since function

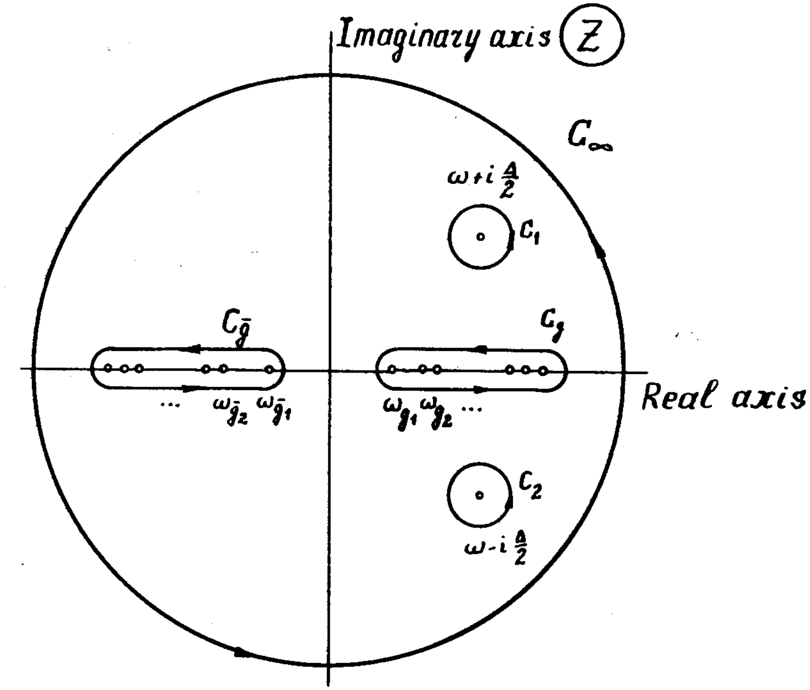
$$t(z) = \frac{P(z)}{F(z)}\rho(\omega-z) \tag{26}$$

in points $z=\omega_g$ has poles of first order (see eqs. (9′) and (13)). Taking into account that function $t(z)$ is meromorphic and consequently, the sum of residuals with respect to all its singularities including the point at infinity is equal to zero, then $b(E\lambda,\omega)$ may be written in the form:

$$b(E\lambda,\omega) \equiv \int_{C_g} = -\int_{C_\infty} - \int_{C_{\bar g}} - \int_{C_1+C_2} . \tag{27}$$

Here it is taken into consideration that the function $t(z)$ is holomorphic in points $z=\pm\epsilon(qq')$. The integral over contour C_∞ equals zero, since for $|z|\to\infty$ the integrant is proportional to z^{-4}. The strength function $b(E\lambda,\omega)$ has physical sense at $\omega>0$ only. With that one can easily see that

$$\int_{C_{\bar g}} = \frac{\Delta}{2\pi}\sum_g \frac{P(\omega_g)}{\frac{\partial F(\omega_g)}{\partial\omega_g}} \frac{1}{(\omega+\omega_g)^2+(\Delta/2)^2}, \tag{28}$$



Complex z-plane and integration contours for function $t(z)$ (26). Poles of $t(z)$ are shown on the real axis at $z=\omega_g$ and $\omega_{\bar g}$ and $z=\omega\pm i\frac{\Delta}{2}$.

i.e., it decreases monotonously $\sim\omega^{-2}$ and can be neglected at large enough values of ω. The equation (28) gives the greatest contribution when calculating the reduced probabilities of the lowest states. However in this case the number of levels is small and one may use an exact expression (19). Direct calculations have shown that the contribution of (28) to (27) is negligibly small even at ω equal to the energy of the lowest phonon.

In (27) $\int_{C_1+C_2}$ is the contour integral around points $z_{1,2}=\omega \pm i\Delta/2$ which are the poles of the first order of the function $\rho(\omega - z)$, and consequently of $t(z)$. Finding residuals directly in points $z_{1,2}$, finally, one obtains.

$$b(E\lambda,\omega)\approx -\int_{C_1+C_2} = \frac{\Delta}{2\pi}\sum_{1,2}\mathrm{res}\left[\frac{P(z)}{F(z)}\frac{1}{(\omega-z)^2+(\Delta/2)^2}\right]_{z_{1,2}=\omega\pm i\Delta/2} =$$

$$= \frac{1}{\pi}(00\lambda\mu \mid I_f K_f)^2(2-\delta_{\mu,0})\times \tag{29}$$

$$\times \mathrm{Im}\left\{\frac{1}{(1-\kappa_0^{(\lambda)}X^g)(1-\kappa_1^{(\lambda)}X^g)-\kappa_0\kappa_1(X^g(n)-X^g(p))^2}\times\right.$$

$$\times[(e_{eff}^{(\lambda)}(p))^2X^g(p)+(e_{eff}^{(\lambda)}(n))^2X^g(n)-$$

$$\left.-X^g(n)X^g(p)[\kappa_0^{(\lambda)}(e_{eff}^{(\lambda)}(p)-e_{eff}^{(\lambda)}(n))^2+\kappa_1^{(\lambda)}(e_{eff}^{(\lambda)}(p)+e_{eff}^{(\lambda)}(n))^2]]\right\}_{\omega\to\omega+i\Delta/2}$$

3. SUM RULES AND PHOTOABSORPTION CROSS SECTIONS

In the study of the giant multipole resonances different kinds of sum rules appear to be useful. They allow one to estimate the degree of collectiveness of giant resonances. It can be shown that if the operator of $E\lambda$ -transition is represented in the form

$$\hat{Q}=\sum_g M(Q_g+Q_g^+), \tag{30}$$

where Q_g^+ and M are defined by (4) and (16), then

$$S_\lambda=\sum_g \omega_g B(E\lambda;0\to\omega_g)=\frac{1}{2}<[\hat{Q},[H_v,\hat{Q}]]>=e^2(2-\delta_{\mu,0})\times \tag{31}$$
$$\times\{(e_{eff}^{(\lambda)}(p))^2\sum_{rr'}(u_{rr'}f^g(rr'))^2\epsilon(rr')+(e_{eff}^{(\lambda)}(n))^2\sum_{ss'}(u_{ss'}f^g(ss'))^2\epsilon(ss')\}.$$

This is the model dependent energy-weighted sum rule (EWSR). It permits to find the strength share for $E\lambda$ -transitions for states in a certain interval of excitation energy.

Especially interesting is a so-called model independent energy-weighted sum rule. Neglecting the exchange and dependent on velocity forces and performing summation over excited levels below meson-creation threshold one obtaines the following sum rule for $\lambda>1$, $T=0,1$ (see, for example, [8])

$$S_\lambda=\sum_g\omega_g B(E\lambda;0\to\omega_g)=3\frac{\lambda(2\lambda+1)}{4\pi}\frac{Ze^2\hbar^2}{2m}R_0^{2\lambda-2}=$$
$$=4.8\lambda(3+\lambda)^2\frac{Z}{A^{2/3}}B(E\lambda)_{s.p.}\ (\mathrm{MeV}), \tag{32}$$

where $R_0=r_0A^{1/3}$, $r_0=1.2\,\mathrm{fm}$, $B(E\lambda)_{s.p.}=\frac{2\lambda+1}{4\pi}\left(\frac{3}{3+\lambda}\right)^2R_0^{2\lambda}e^2$ is the single-particle probability of $E\lambda$ -transition. Here summation is done over all transitions to one-phonon states with all possible μ at a given λ. For example, for $\lambda=2$ the left hand part of (31) comprises the transitions to the states with $I_f^{\pi_f}K_f$ equal to 2^+0, 2^+1 and 2^+2 [9].

In the case of E1 -transition EWSR has the form

$$S_1=\sum_g\omega_g B(E1;0\to\omega_g)=\frac{9}{8\pi}\frac{e^2\hbar^2}{m}\frac{NZ}{A}, \tag{33}$$

which transforms into a well-known Tomas-Reihe-Künsum rule for the dipole photoabsorption cross section

$$\sigma_{E1}=\int_0^\infty\sigma_{E1}(\omega)d\omega=\sum_g\sigma_{E1}(\omega_g)=\frac{2\pi^2e^2\hbar}{mc}\frac{NZ}{A}=60\frac{NZ}{A}(\mathrm{MeV{\cdot}mb}), \tag{34}$$

if one minds the relationship between the photoabsorption cross section at g level and the probability of electrical transition $0 \to \omega_g$:

$$\sigma_{E\lambda}(\omega_g) = \pi^2 \left(\frac{\hbar c}{\omega_g}\right)^2 \hbar w(E\lambda; 0 \to \omega_g), \tag{35}$$

$$\hbar w(E\lambda; 0 \to \omega_g) = 8\pi \frac{\lambda+1}{\lambda[(2\lambda+1)!!]^2} \left(\frac{\omega_g}{c\hbar}\right)^{2\lambda+1} B(E\lambda; 0 \to \omega_g). \tag{36}$$

In some cases we may find useful the relationships between the photoabsorption cross sections and the probabilities of the corresponding electrical transitions for the lowest multipolarities:

$$\sigma_{E\lambda}(\omega_g) = \begin{cases} 0.282\,\omega_g B(E1;0 \to \omega_g)[e^2\cdot fm] = 4.04\,\omega_g B(E1;0 \to \omega_g)[MeV\cdot mb] \\ 0.217\cdot 10^{-6}\omega_g^3 B(E2;0 \to \omega_g)[e^2\cdot fm] = 3.12\cdot 10^{-6}\omega_g^3 B(E2;0 \to \omega_g)\ [MeV\cdot mb] \\ 0.101\cdot 10^{-12}\omega^5 B(E3;0 \to \omega_g)[e^2\cdot fm] = 1.44\cdot 10^{-12}\omega_g^5 B(E3;0 \to \omega_g)\ [MeV\cdot mb] \end{cases} \tag{37}$$

The above described procedure for determining the strength functions for reduced probabilities of electrical transitions $b(E\lambda,\omega)$, as is seen from formulas (31)-(37), may be applied for obtaining the energy-weighted sums S_λ (31) and photoabsorption cross sections $\sigma_{E\lambda}$ (35) in dependence on the energy. Corresponding formulas can be easily derived if the relationship between the values S_λ and $\sigma_{E\lambda}$, and $B(E\lambda; 0 \to \omega_g)$ defined by (31)-(37) is taken into account. For example, for the energy-weighted sum we get:

$$S_\lambda(\omega) = \sum_g \omega_g B(E\lambda;0 \to \omega_g)\rho(\omega - \omega_g) = \frac{1}{\pi}\,\mathrm{Im}\,\frac{z\cdot P(z)}{F(z)}\Big|_{z=\omega+i\Delta/2}. \tag{38}$$

In refs.[10] using eq. (29) the reduced probabilities photoexcitation cross sections of giant dipole, isoscalar and isovector E2- and E3 -resonances were calculated for a large number of deformed nuclei.

The present method[11] is more advantageous in application in comparison with convential method. The latter means a solution by a computer of a great number of complex equations in order to find directly the energy and wave functions of nuclear excited states. Our method takes several orders shorter computational time, final result being the same. This method can be used in the study of other processes taking place in spherical and deformed nuclei, in particular, in the study of excitation of giant resonances in reactions with different particles. The application of this method may give a possibility of investigating highly excited states of nuclei of a more complex nature[12].

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