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REFRACTION OF POLARIZED NEUTRONS  
IN A MAGNETICALLY NON-COLLINEAR MEDIUM

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## 1. Introduction

Until recently investigations of magnetic layers using neutron reflection or transmission have been restricted to the case when the magnetization vector is aligned with an external magnetic field and the non-polarized neutron beam splits into two beams as a result of the refraction at the vacuum - magnetic medium boundary[1].

Owing to the internal anisotropy and the shape anisotropy of the sample under investigation the magnetization vector can be non-collinear to the external magnetic field (magnetically non-collinear layer). In the magnetically non-collinear layer the effective magnetization vector  $M_{eff} = M - Q(MQ)$ , where  $M$  is the magnetization vector, and  $Q$  is the momentum transferred to the neutron or by the neutron - is non-collinear to the external magnetic field, and the probability of the transition of the neutron from one spin state to another (i.e., from one Zeeman sublevel in a magnetic field to another) is non-zero [2]. As a result, a number of effects take place [3,4].

First, when the neutron changes from one spin state to another, the neutron potential energy varies, resulting in the generation of the second neutron beam spatially separate from the initial one (effect of polarized neutron beam splitting). Second, two neutron beams with different spin states should interfere, since the phase difference occurs, which is caused by the difference in the velocities of neutrons in the layer. As the velocity of the neutron in a spin state directed along the magnetic field approaches zero, the interference pattern becomes more complicated and appears to be something other than ordinary spin precession around the magnetic field vector [3].

At present, the beam splitting effect has been detected [5,6] when studying the neutron reflection from a magnetic film placed in a magnetic field at an angle. In the transmission

geometry this effect has been used to determine the width of domain walls [7] in the Fe (4 at. % Si) crystal.

In the present paper the refraction of polarized neutrons in a magnetic layer in relation to the neutron wavelength, the magnitude and direction of the external magnetic field, is investigated. Particular emphasis is placed upon the effects of neutron beam splitting and the interference of neutron beams with different spin states.

## 2. Interface between two magnetic-nuclear media

Figure 1 presents a scheme of reflection and transmission of the polarized neutron beams at the interface between two (marked Roman numbers I and II) magnetic-nuclear media. In the general case, the magnetic-nuclear medium as regards the processes of coherent propagation of neutrons in media, is characterized by the complex interaction potential  $U = V + iW$ , where the real part of the potential,  $V$ , consists of the part  $N$  due to the nuclear interaction of the neutron with the medium, and the part  $M$  resulting from the interaction of the neutron magnetic moment with the induction of a magnetic field in the medium,  $B$ . The imaginary part of the potential  $W$  is due to the neutron capture processes, and inelastic and diffusion elastic neutron scattering. Now suppose that the neutron is incident on the interface at a grazing angle  $\theta$  from the vacuum ( $N_1 = 0$ ,  $W_1 = 0$ ,  $M_{1+} = -\mu H$ ,  $M_{1-} = \mu H$ , where  $\mu$  is the neutron magnetic moment,  $\mu < 0$ ). In the magnetic field  $H$  the neutron has the states "+" and "-" corresponding to the direction of the neutron spin along and opposite to the direction of the magnetic field and characterized by the interaction potentials  $M_{1+} = -\mu H$  and  $M_{1-} = \mu H$ , respectively. For the neutrons with a fixed wavelength  $\lambda$ ,

the kinetic energy of the motion of the neutron in the vacuum in the direction perpendicular to the interface  $K_{\perp}$  is determined by the chosen grazing angle  $\theta$ , and therefore is equal for both spin states ( $K_{\perp+} = K_{\perp-} = K_{\perp}$ ). The total neutron energy in the direction perpendicular to the interface ( $E = K_{\perp} + U$ ) is different for two spin states ( $E_{+} \neq E_{-}$ ) because of the difference in the values for the potential energy. Since the total energy is conserved ( $E = \text{const}$ ), the change in the potential energy  $\Delta U$  in passing from one medium to another leads to an equivalent, but with different sign, change in the kinetic energy  $\Delta K_{\perp} = -\Delta U$ . As a result, for the difference of the kinetic energies of the neutron, which does not change the "+" and "-" spin states (let us mark the beams of these neutrons by the indices "+ +" and "--", where the first sign indicates the neutron state prior to its incidence on the interface, and the second sign signifies the neutron state following the reflection from the interface ("r" index) or its transmission through the interface ("tr" index)) we have:

$$\begin{aligned} \Delta K_{r,+,+} &= K_{r,+,+} - K_{r,+,-} = 0 \\ \Delta K_{tr,+,+} &= K_{tr,+,+} - K_{tr,+,-} = 2\mu (B-H). \end{aligned} \quad (1)$$

It can be seen from (1) that the neutrons with both spin states are reflected at one and the same grazing angle  $\theta_{r,+} = \theta_{r,-}$  (the grazing angle is determined with the required accuracy by the ratio of the perpendicular velocity component changing at the interface to the unchanging longitudinal component) which is equal to the grazing angle  $\theta$  of the neutron (beam) incident on the interface. At the same time, in case of transmission through the interface there exists a non-zero angle between the "+ +" and "--" beams which is determined by  $\Delta K_{tr,+,+}$ .

In the case that the vectors B and H are non-collinear, the additional beams of the neutrons which underwent the transition from one spin state in a magnetic field to another (from one Zeeman sublevel to another) arise. In this instance, the effective magnetization vector  $M_{eff} = I - Q(IQ)$  is non-collinear to the P polarization vector, and in case of reflection from the interface or transmission through it, there exists the non-zero probability of the neutron transition from one spin state  $+(-)$  to another  $-(+)$  [2]. For the neutrons which underwent the transition, the additional change in the potential energy will be observed:

$$\begin{aligned}
 \Delta U_{r,+} &= \Delta U_{r,-} + 2\mu H \\
 \Delta U_{r,-} &= \Delta U_{r,+} - 2\mu H \\
 \Delta U_{t,+} &= \Delta U_{t,-} = 0 \\
 \Delta U_{t,+} &= \Delta U_{t,-} + 2\mu B = \mu(B+H) + N_{II} \\
 \Delta U_{t,-} &= \Delta U_{t,+} - 2\mu B = -\mu(B+H) + N_{II} \\
 \Delta U_{r,+} &= -\mu(B-H) + N_{II} \\
 \Delta U_{r,-} &= \mu(B-H) + N_{II}
 \end{aligned}
 \tag{2}$$

From (2) for the difference of the kinetic energies of the neutrons which experienced the  $+ -$  and  $- +$  transitions, it follows:

$$\begin{aligned}
 \Delta K_{r,1,2} &= K_{r,1,-} - K_{r,1,+} = -4\mu H, \\
 \Delta K_{t,1,2} &= K_{t,1,-} - K_{t,1,+} = -2\mu(B+H).
 \end{aligned}
 \tag{3}$$

So (Fig.1), in the case that the B and H vectors are non-collinear, for the initially non-polarized neutron beam there are three reflected beams and four beams which passed through the interface (fourfold refraction). In the reflection geometry

the middle beam is formed by the neutrons of both spin states and which did not experience the transition on reflection, the beam with lesser grazing angle is formed by the  $- +$  transition, and the neutron beam with greater grazing angle is formed by the  $+ -$  transition. Among the beams which passed through the interface, two beams are in one spin state, and the other two are in another spin state, and one beam from each pair of beams consists of the neutrons which did not undergo the transition, and the other one - of the neutrons which underwent it. It should be pointed out that the presented relationships (1-3) allow us to determine the directions of the beams and their coordinates. The intensities of the neutron beams which passed the interface (assuming that at the interface the direction of the magnetic field changes instantly) can be determined by the formulas given in [2,3]. In the case that the magnetic field changes its direction and magnitude in the lengthy region, the probabilities of the transition can be calculated using the recursion method for solving the Schroedinger equation.

### 3. Magnetic layer

Let us dwell on the case of a magnetic layer on a nuclear substrate (Fig.2), which can be realized experimentally. We shall consider the pattern of transmission and reflection of neutrons for an infinitely narrow non-polarized neutron beam. The neutrons from the vacuum (medium I) are incident on the magnetic layer (medium II) at a grazing angle  $\theta$ . A number of neutrons are reflected back into the vacuum, and the rest of them penetrate the layer. In the layer, as we have seen when considered the problem of transition through the interface,

four beams propagate. At the interface between the layer and the substrate (medium III) two beams which were reflected into the layer and two beams which penetrated the substrate are formed from each beam, in accordance with two possible processes with and without transition of the neutron from one spin state to another. As a result, eight beams of the first order propagate in the substrate. The beams reflected from the "magnetic layer - substrate" interface, in turn, are partially reflected from the "magnetic layer - vacuum" interface. As a result, sixteen beams of the second order will come to the "magnetic layer - substrate" border, and thirty two beams will propagate in the substrate. So, the number of beams in the next order will increase four times as compared with the number of beams in the previous order. In a similar way, in the flux of reflected neutrons there will be four beams of the first order, sixteen beams of the second, and so on. The density of the neutrons leaving the layer and reflected from it will be damped in the direction of increasing order of neutron transmission (reflection).

It should be emphasized that depending on the magnitude of the interaction potential, there will be three types of neutron beams with the magnetic interaction potentials  $-2\mu H$ ,  $0$  and  $2\mu H$  in the substrate (it can be referred to as triple refraction from the vacuum into the substrate). As a result, in registering neutrons in the plane perpendicular to the plane of the sample and spaced at the distance, which far exceeds the length of the sample along the beam, the position distribution of neutrons will be determined by three values of the interaction potential, and not by the neutron density distribution along the beam at the interface "magnetic layer - substrate". In other words, the beams which are characterized

by one potential, but coming from different points of the interface "magnetic layer - substrate" will be inseparable.

To this point we considered the case for the neutrons with a fixed wavelength. We assumed that to observe the splitting effect, it is necessary to register the refracted neutron beam in relation to the coordinate in the direction perpendicular to the plane of the sample. The splitting effect, however, is a manifestation of the neutron transition from one Zeeman sublevel to another in space. Another manifestation of the occurrence of this transition is the existence of the intensity maxima of the "ij" beams at a fixed point at different, corresponding to the types of the beams, wavelengths. Assuming that the refracted beam propagates in the non-magnetic substrate, we obtain for the neutron kinetic energy in the refracted beam:

$$\begin{aligned} K_{cr,l,\dots(-)} &= (K_{in,l} - N_{III}), \\ K_{cr,l,\dots} &= K_{cr,l,\dots(-)} + 2\mu H, \\ K_{cr,l,\dots} &= K_{cr,l,\dots} - 4\mu H. \end{aligned} \quad (4)$$

Then, since the grazing angles of the incident and refracted beams are fixed, the following relationships are true:

$$(K_{cr,l}/K_{in})^{1/2} = \sin(\theta_{cr}), \quad (K_{in,l}/K_{in})^{1/2} = \sin(\theta). \quad (5)$$

By combining (4) and (5), we obtain:

$$\begin{aligned} \Delta\lambda^{-2} = \lambda_{cr}^{-2} - \lambda_{in}^{-2} &= 4\mu H(2m/h^2) / (\theta^2 - \theta_{cr}^2) \\ &= \alpha H / (\theta^2 - \theta_{cr}^2), \end{aligned} \quad (6)$$

where  $\alpha = 8\mu m/h^2 = 3.4 \text{ mrad}^2 \text{ \AA}^{-2}/\text{kOe}$ .

It could be seen from (4-6) that there are characteristic wavelengths for each beam and the difference of inverse values of the squares of these characteristic wavelengths (or which is equivalent, the difference of the total neutron kinetic energies) for the "- +" and "+ -" beams is proportional to the magnetic field intensity.

The real neutron beam cross-section has finite dimensions. This determines the possibility of beam interference. Double refraction of the polarized neutron beam at the interface results in the interference of neutron waves of different intermediate spin states (Fig.3). In this case, the phase difference (for instance for neutron in "+" initial spin-state) occurs due to the difference in the velocities of the neutrons of two spin states inside the layer:

$$\Delta\varphi = 2\pi(2m)^{1/2} l/h((\mu(B-H)+N_{it})/(K_1-\mu(B-H)-N_{it}))^{1/2} + (\mu(B+H)-N_{it})/(K_1+\mu(B+H)-N_{it}))^{1/2}, \quad (7)$$

where  $l$  is the layer width.

Then, from the condition  $\delta\Delta\varphi = 2\pi$  at  $K_1 \rightarrow (\mu(B-H)+N_{it})$  we obtain for the oscillation period:

$$\delta\lambda = \sin(\theta) (\lambda_{11a}^2/l) (1 - (\lambda/(\sin(\theta)\lambda_{11a}))^2)^{1/2}, \quad (8)$$

where  $\lambda_{11a} = h/(2m(\mu(B-H)+N_{it}))^{1/2}$ .

It can be seen from (9) that as  $\lambda$  approaches  $\lambda_{11a}$  the oscillation period  $\delta\lambda$  tends to zero.

#### 4. Experimental details

Figure 4 presents the measuring scheme of the polarized neutron spectrometer. Here 1 is the neutron polarizer, 2 is the sample, 3 is the polarization analyzer, and 4 is the neutron detector. The spin-flippers 5 and 6 are placed between the polarizer and the sample, and between the sample and the analyzer, respectively. This scheme corresponds to the so-called complete polarization analysis scheme and makes it possible to measure the probabilities of the transitions (marked "+ -", "- +", "+ +" and "- -") from the "+" (" -") spin states at the point of entry of the beam (the beam incident on the sample) into the "+" (" -") spin states at exit of the beam (the beam which passed through the sample or reflected from it). The magnetic field at the sample was established using the electromagnet 7. By rotating the electromagnet about the sample, it was set up so that the angle  $\beta$  between the direction of the magnetic field and the plane of the sample could be chosen within 0-90°.

The divergence of the neutron beam at the sample in the horizontal plane was  $\pm 0.1$  mrad, and the grazing angle of the neutron beam incident on the sample was  $\theta = 3.17$  mrad. The neutron detector was placed at a distance of 2615 mm from the sample, the cadmium diaphragm at the detector inlet measured 0.5 mm (horizontally)  $\times$  20 mm (vertically). The detector was positioned in the direction perpendicular to the neutron beam with an accuracy better than 0.1 mm.

The sample measured 1 mm (width)  $\times$  10 mm (vertically)  $\times$  20 mm (horizontally, along the beam). The area exposed to neutrons measured 8 mm (vertically)  $\times$  20 mm (along the beam). The magnetic layer 5 $\mu$ m thick was comprised of iron (86%), aluminium

(9.6%) and silicon (4.4%). The face of the sample was shielded by cadmium. The substrate was ceramics with a low nuclear density.

## 5. Results of the measurements and discussion

Figure 5 presents the angular distribution pattern of the polarized neutron beam which passed through the sample under study for the cases when the spin-flipper was switched on ("- beam) and off ("+" beam) and the angle between the direction of the magnetic field and the plane of the sample was  $\beta=0^\circ$  (curves 1 and 3) and  $\beta=70^\circ$  (curves 2 and 4). The curves at  $\beta=0^\circ$  distinctly show two peaks. One of them (at the right) corresponds to the beam entering and leaving the sample through the front and rear faces. This beam practically does not change its direction in passing through the sample, and this is a direct beam. The second peak (at the left) corresponds to the refracted beam. The curves at  $\beta=70^\circ$  also show two peaks. The distribution pattern, however, changes significantly. The peaks in the region of the direct beam differ in amplitude, and the peaks at the left are less in size. This change can be interpreted as a shift of the distribution of the "+" beam to the right, and the distribution of the "-" beam to the left. This could be presumably connected with the occurrence of the neutron beams on the right and left of the refracted beam, which are caused by the "+ -" and "- +" transitions (let us indicate the newly formed beams by the symbols "+ -" and "- +", respectively, and the neutron beams with the unchanged spin states by the symbols "+ +" and "- -"). But to make sure that this assumption is true, it is necessary to perform the

polarization analysis of the neutron beam which has passed through.

The results of the similar measurements but conducted along with the neutron polarization analysis are given in Fig.6. Here, it can be seen that for  $\beta=70^\circ$ , as compared to  $\beta=0^\circ$ , a decrease of the peaks at the left is observed for the beams "- -" and "+ +". At the same time the beams "+ -" and "- +" arise, one of them going on the right of the "- -" beam and closer to the direct beam and the other is on the left of the "+ +" beam.

The intensities presented in Fig.5 and 6 are integral in wavelength and correspond in essence to some mean wavelength of order 1.4 Å. From these data we can conclude that the "- +" and "+ -" neutron beams do not coincide in direction. The distance between these beams at the detector is 1.4 mm (0.535 mrad).

Figure 7 presents the spectral dependence of the difference of the squares of the grazing angles of the "+ -" and "- +" beams  $\Delta\theta^2 = \theta_{-}^2 - \theta_{+}^2$  on the wavelength squared  $\lambda^2$  for the magnetic field intensities 6.8 kOe (experimental data are indicated by triangles). Theoretically, the expected dependence for the difference is  $\Delta\theta^2(\text{mrad}^2) = 0.294H(\text{kOe})\lambda^2(\text{Å}^2)$  (indicated by straight line in the figures). It can be seen that this dependence is not realized.

We now turn to the description of the spectral dependence of the neutron intensity results obtained at a fixed detector position.

The spectral dependence of neutron transmittance on the wavelength at the point spaced 0.9 mm (0.33 mrad) from the direct beam for different values of the external magnetic field is given in Fig.8. From this figure we notice that the transmittance has a maximum at certain wavelength values. And

as  $H$  rises, the distance (in terms of wavelength) between the peaks increases so that  $\Delta\lambda^{-2}$  is proportional to  $H$  (see Fig.9). At the same time one can see from Fig.8 that as the magnetic field intensity rises, the transmittance of the "+ +" and "- -" beams increases, and the transmittance of the "+ -" and "- +" beams decreases. From this it follows that as the magnetic field increases, the probability of neutron transition from one spin state to another decreases (beam depolarization decreases). This can be explained by a decrease of the permeability of the magnetic layer with increasing magnetic field, which results in a decrease in the angle between the induction vector in the layer and the magnetic field vector outside the layer. The spectral dependence of the transmittance of the neutron beam for different values of the angle  $\beta$  is given in Fig.10. The dependence of the maximum value of transmittance on the angle  $\beta$  at  $H=4.6$  kOe is presented in Fig.11. It can be seen that as the angle  $\beta$  grows, the transmittance increases for the beams of neutrons with changing spin state and decreases for the beams of neutrons with unchanging spin state. Thus, with decreasing  $\beta$ , the probability of transition diminishes, which can be explained by the same reasons as in the case of increasing magnetic field at a fixed  $\beta$ . Figures 12-13 present the results demonstrating the interference effects. The spectral dependence of the transmittance of the "+ +" beam at  $H=6.8$  kOe,  $\beta=70^\circ$  and  $\theta_{cr}=2.8$  mrad is given in Fig.12. The oscillations with a period that decreases as the wavelength increases can be seen. This corresponds to the dependence described by (8). However the observed period of oscillations is in order 2 times larger than the predicted one. The dependence of the intensity in the finite wavelength range of  $0.22 \text{ \AA}$  on the angle  $\theta_{cr}$  at  $H=6.8$  kOe

and  $\beta=70^\circ$  is shown in Fig.13. It can be seen that the oscillations are observed for the "+ +" and "- +" beams. They are explained by the interference dependence on the wavelength, which manifests itself due to the dependence of the refraction angle in the substrate on the wavelength. This result demonstrates the possibility to investigate interference when measuring the distribution of the neutron flux in relation to coordinates.

## 6. Conclusions

In the investigations of the polarized neutron transmission through the polycrystalline sample with the FeSiAl magnetic layer conducted under conditions when the magnetic field vector makes an angle  $\beta \neq 0$  with the plane of the magnetic layer, the double refraction of the polarized neutron beam has been detected. At the same time the observed interference manifests itself in the dependencies of the neutron transmission on the wavelength and the coordinate in the direction perpendicular to the plane of the layer. On the basis of these investigations we may state that the use of the complete polarization analysis along with the registration of the spatial distribution of the flux of reflected and transmitted neutrons will allow us to study magnetic non-collinear structures with a high degree of precision.



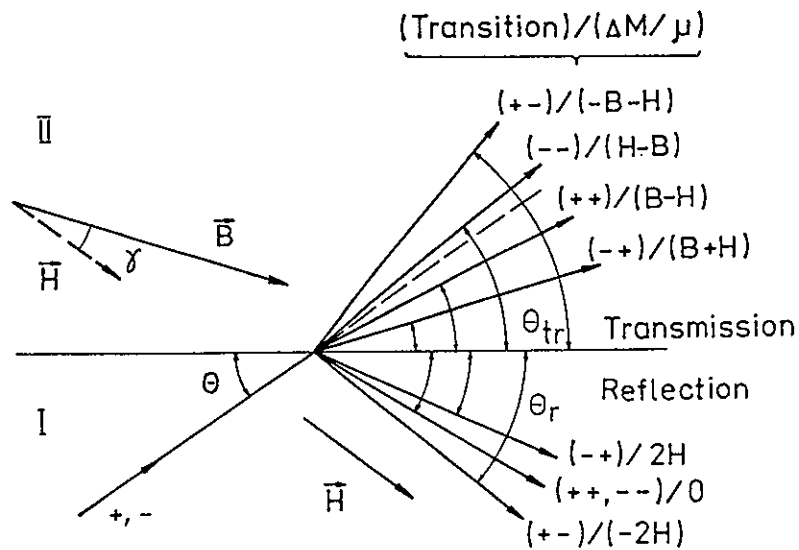


Fig.1. Scheme of the processes of reflection and transmission of the polarized neutron beam at the interface of two magnetic media.

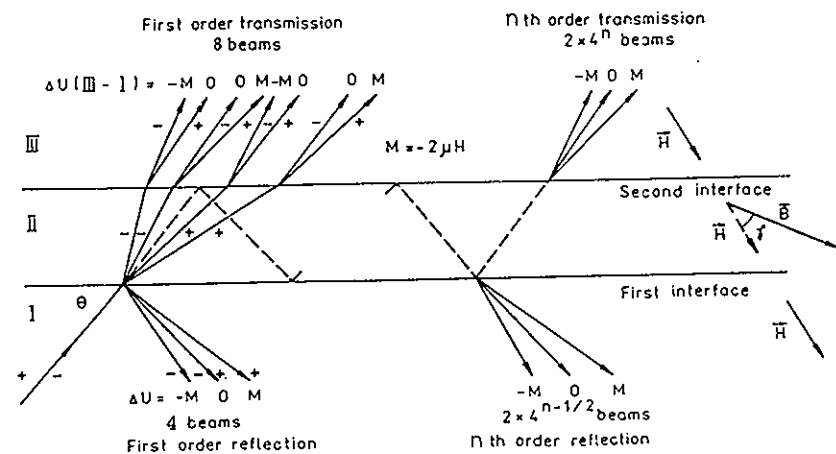


Fig.2. Scheme of the processes of reflection and transmission of the polarized neutron beam through a magnetic layer.

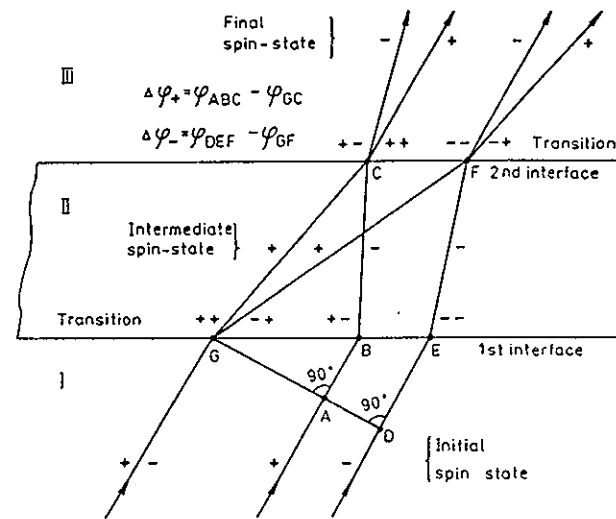


Fig.3. Interference of waves of neutrons with "+" and "-" intermediate spin states.

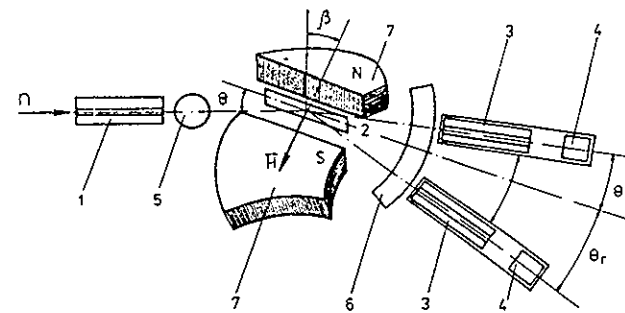


Fig.4. Scheme of the measurement: 1-neutron polarizer, 2-sample, 3-polarization analyzer, 4-neutron detector, 5,6-spin-flippers, 7-electromagnet.

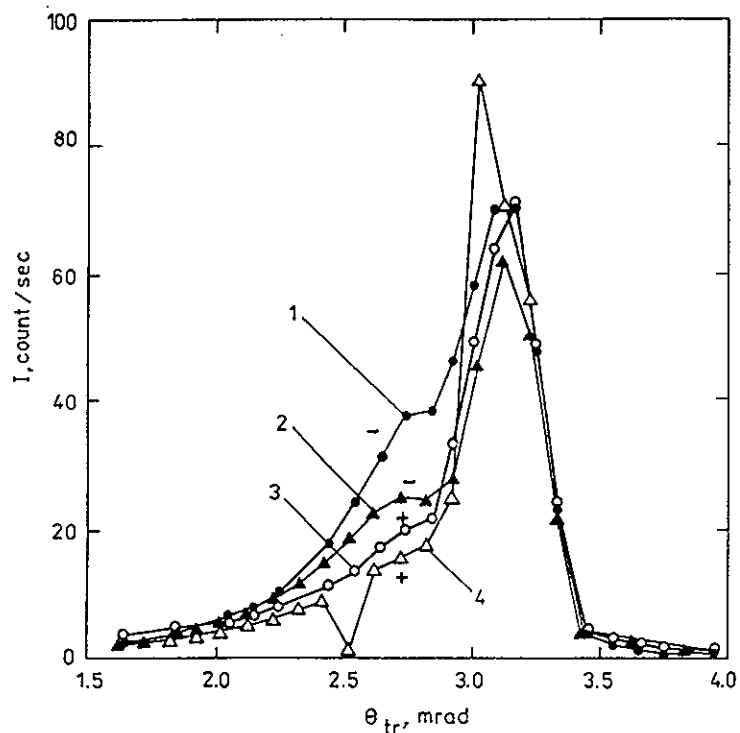


Fig. 5. Dependence of the intensity  $I$  (count/sec) of the neutron beam transmitted through the sample at the grazing angle  $\theta_{tr}$  (mrad) of the refracted neutron beam, measured without resort to the polarization analyzer: curves 1,3 -  $H=4.6$  kOe and  $\beta=0^\circ$ ; curves 2,4 -  $H=6.8$  kOe and  $\beta=70^\circ$ ; curves 1,2 - polarization of the incident neutron beam "-"; curves 3,4 - polarization "+".

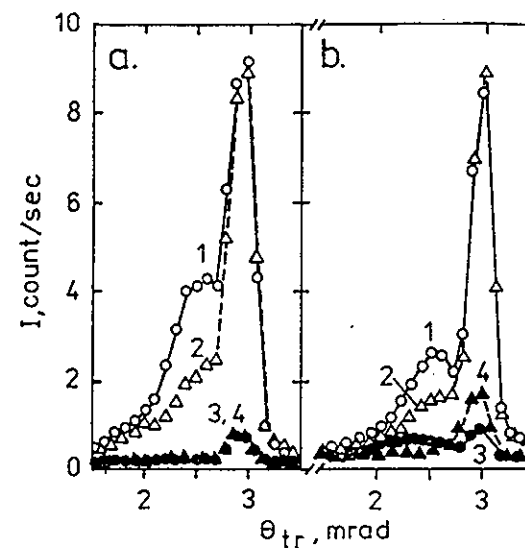


Fig. 6. Dependence of the intensity  $I$  (count/sec) of the neutron beam transmitted through the sample at the grazing angle  $\theta_{tr}$  in the magnetic field  $H=6.8$  kOe, measured using the polarization analyzer: a)  $\beta=0^\circ$ ; b)  $\beta=70^\circ$ ; curve 1 - beam "- -", curve 2 - beam "+ +", curve 3 - beam "- +", curve 4 - beam "+ -".

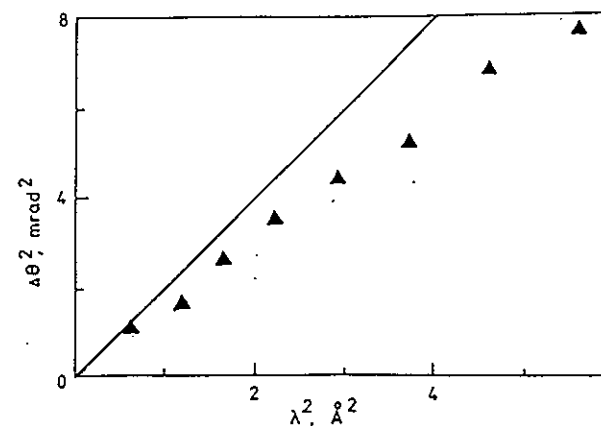


Fig. 7. Dependence of the difference of the squares of the grazing angles  $\Delta\theta^2$  (mrad<sup>2</sup>) of the "+ -" and "- +" beams on the wavelength squared  $\lambda^2$  (Å<sup>2</sup>) for external magnetic field  $H=6.8$  kOe: triangles - experimental data, straight line - the theory.

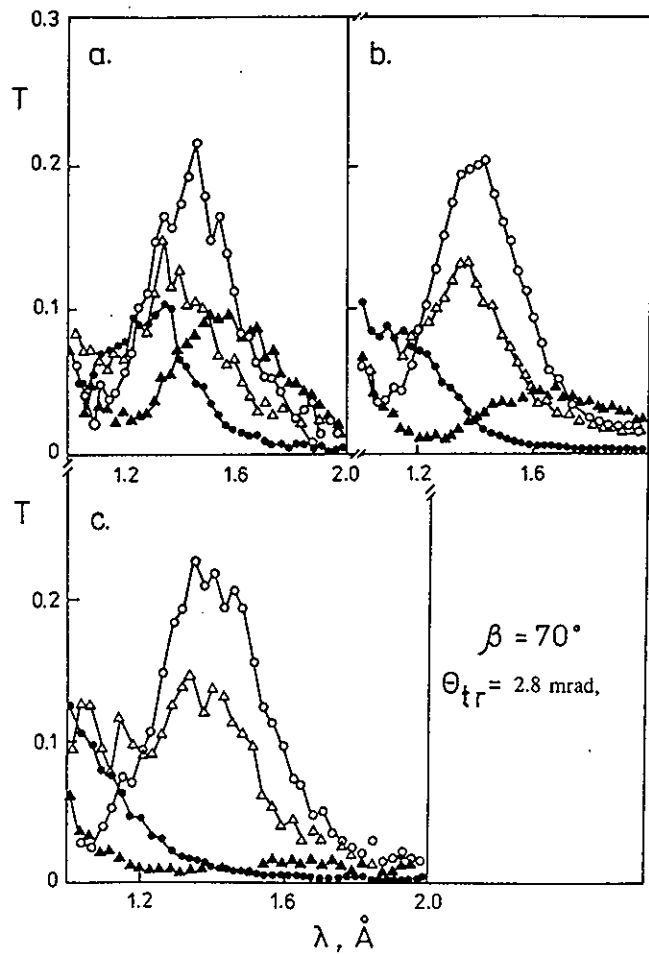


Fig.8. Dependence of the transmittance of neutrons by the sample  $T(\lambda)$  on the wavelength at the point spaced 0.9 mm ( $\theta_{tr} = 2.8$  mrad) from the direct beam, at  $\beta = 70^\circ$  and for different values of the external magnetic field intensities  $H$ : a) - 2.3 kOe, b) - 4.6 kOe, c) - 6.8 kOe; open circles - beam "- -", open triangles - beam "+ +", closed circles - beam "- +", closed triangles - beam "+ -".

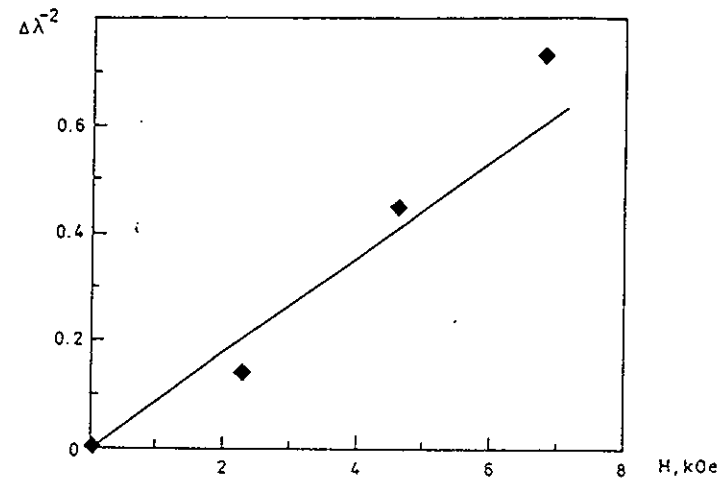


Fig.9 Dependence of  $\Delta\lambda^{-2}$  on the magnetic field intensity at  $\beta = 70^\circ$ : diamonds - experimental data, straight line - the theory.

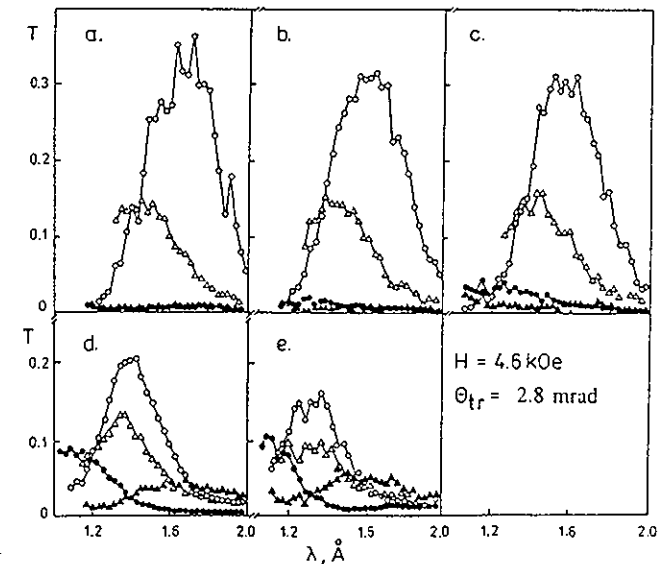


Fig.10. Spectral dependence of the transmittance of the neutron beam  $T(\lambda)$  at a point  $\theta_{tr} = 2.8$  mrad at  $H = 4.6$  kOe and different values of an angle  $\beta$ : a) -  $0^\circ$ , b) -  $20^\circ$ , c) -  $40^\circ$ , d) -  $70^\circ$ , e) -  $90^\circ$ ; closed circles - beam "- +", open triangles - beam "+ +", open circles - beam "- -", closed triangles - beam "+ -".

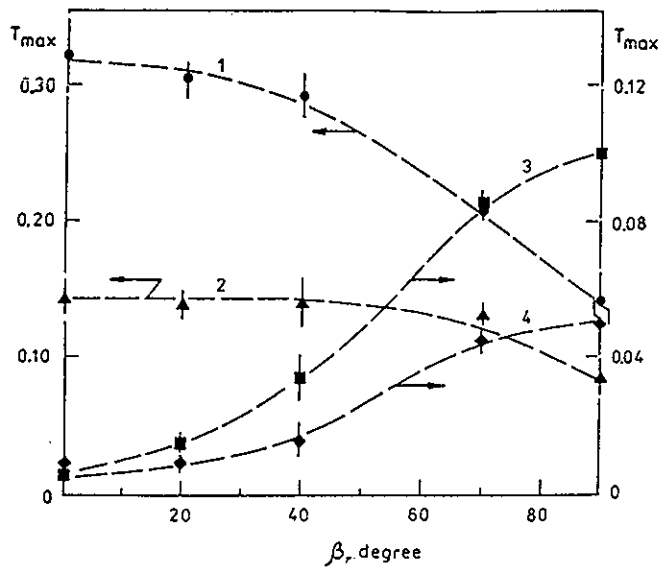


Fig.11. Dependence of the maximum neutron transmittance value  $T_{max}$  on the angle  $\beta$  at  $H=4.6$  kOe: curve 1 - beam "--", curve 2 - beam "+ +", curve 3 - beam "+ -", curve 4 - beam "- +".

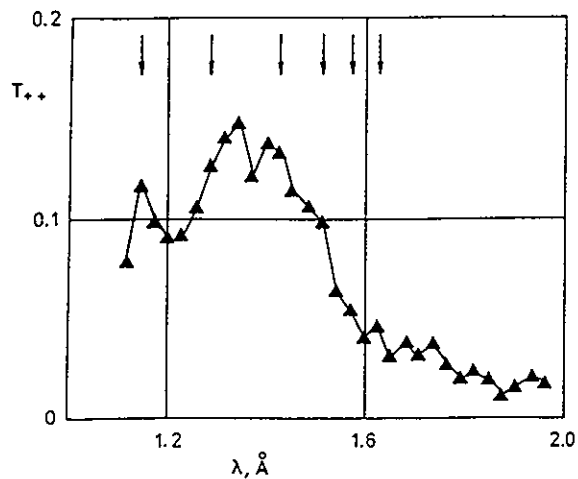


Fig.12. Spectral dependence of the transmittance  $T_{++}(\lambda)$  at  $\theta_{tr}=2.8$  mrad,  $H=6.8$  kOe and  $\beta=70^\circ$ .

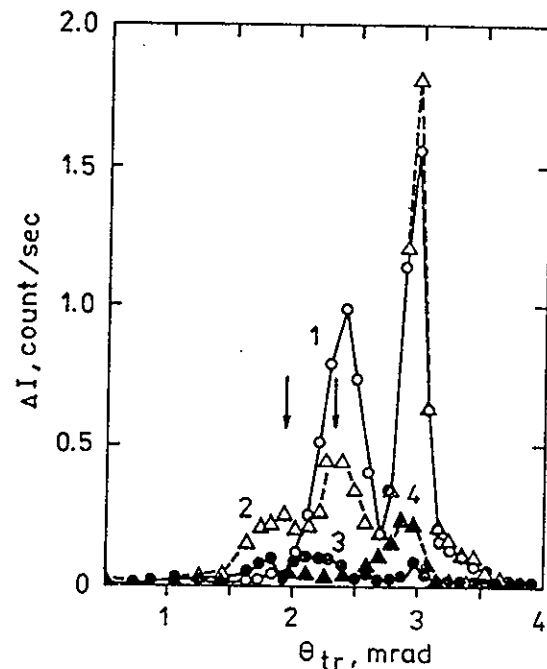


Fig.13. Dependence of the intensity of the transmitted beam  $\Delta I$  in the wavelength range  $1.6-1.82$   $\text{\AA}$  on  $\theta_{tr}$  at  $H=6.8$  kOe and  $\beta=70^\circ$ : curve 1 - beam "--", curve 2 - beam "+ +", curve 3 - beam "+ -", curve 4 - beam "- +".

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