ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E3-96-79

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ON THE NEUTRON CHARGE RADIUS AND THE NEW EXPERIMENTS PROPOSED FOR THE PRECISE (n,e)-SCATTERING LENGTH MEASUREMENT

Submitted to «Ядерная физика»

1996

1. Introduction

Lately, an intense discussion has been developing concerning the discrepancies in the ne-scattering length data obtained from various experiments and, consequently, the controversial estimations of the neutron mean square charge radius based on b_{ne}-values. Having originated with Foldy [1] a long time ago, the experimental quantity b_{ne} is presumed to consist of two terms

$$b_{ne} = b_F + b_I \ ,$$

where b_F is the Foldy scattering length caused by the neutron anomalous magnetic moment μ interacting with the electron electrical field, and b_I is the scattering length — as though rendering an interaction between the neutron internal charge structure and an electrical field (strictly speaking, the charge density giving birth to this field). The following notations are used

$$b_I = \frac{1}{3}\frac{Me^2}{\hbar^2} < r_{in}^2 > \ ,$$

$$< r_{in}^2 >= \int r^2 \rho(\mathbf{r}) d\mathbf{r} \ ,$$

where M is the neutron mass. From this, the expression

$$< r_{in}^2 >= \frac{3\hbar^2}{Me^2} b_I = 86.4(b_{ne} - b_F) \ fm^2$$

appears. The experimental b_{ne} estimations existing in current literature are concentrated in the proximity of the two following values:

$$b_{ne} = (-1.59 \pm 0.04)10^{-3} \ fm \qquad [2],[3],$$

$$b_{ne} = (-1.31 \pm 0.03)10^{-3} \ fm \qquad [4],[5].$$

Eventually, two estimations of the quantity $< r_{in}^2 >$ need to be discussed:

$$< r_{in}^2 >= -0.010 \pm 0.003 \ fm^2 \ [2],[3],$$

$$< r_{in}^2 >= +0.014 \pm 0.003 \ fm^2 \ [4],[5] \ .$$

While no model of the nucleon structure yet leads to a positive neutron mean square charge radius, the Garching group [5] and the group that measured b_{ne} by means of liquid ^{208}Pb at Oak Ridge [6] have given up accounting for the Foldy-term (the first using

vague arguments [7] and the latter without any explanations at all), thereby reducing the definition of the neutron mean square charge radius to the expression

$$< r^2 >= \frac{3\hbar^2}{me^2} b_{ne} \ .$$

Under such circumstances, we are forced to face the primary sources of the problem and glance over the history to find the gist of the question.

2. Revision of Describing ne–Scattering

Here, from the very first, it is necessary to determine the genuine relation between the measured ne–scattering length and the nucleon structure. In our opinion, there is a certain misunderstanding of this question [8] and we believe it will be instructive to trace back how this relation is acquired in various approaches. As the ne–scattering length is obtained from slow neutron scattering in the field produced by atomic electrons, the neutron interaction with an electromagnetic field will be described first.

A free nucleon, as well as any free particle with spin 1/2, is described by the Dirac equation. Yet the "external field" concept is limited and, generally speaking, untenable. In fact, even for an electron itself, the Dirac equation in the face of an external electromagnetic field $A = (\Phi, \mathbf{A})$

$$i\hbar\frac{\partial\psi}{\partial t} = (c\alpha(\mathbf{p} - \frac{e}{c}\mathbf{A}) + \beta mc^2 + e\Phi)\psi,$$

$$i\hbar\frac{\partial\phi}{\partial t} = \mathcal{H}\phi,$$

$$\mathcal{H} \approx \frac{1}{2m}(\mathbf{p} - \frac{e}{c}\mathbf{A})^2 + e\Phi - \frac{\mathbf{p}^4}{8m^3c^2} -$$

$$-\frac{e\hbar}{2mc}\sigma\mathbf{H} - \frac{e\hbar}{4m^2c^2}\sigma[\mathbf{E}\times\mathbf{p}] - \frac{e\hbar^2}{8m^2c^2} div\mathbf{E} \qquad (1)$$

(where $\mathbf{E} = -(\nabla\Phi)$, $\mathbf{H} = [\nabla \times \mathbf{A}]$, $\frac{e\hbar}{2mc} = \mu_0$, α, β are Dirac matrixes, σ is the Pauli matrix, and ϕ is the large component of the bispinor ψ) does not enable to describe properly the properties of this point-like particle, because self-coupling to its own electromagnetic quantum field (radiative corrections) [9] is known to result in the additional terms in the equation to describe the behaviour of the electron in an external field A. The last three terms in $\mathcal{H}$ in eq. (1) are thus replaced by

$$-(\frac{e\hbar}{2mc} + \mu_e')\sigma\mathbf{H} - \frac{1}{2mc}(\frac{e\hbar}{2mc} + 2\mu_e')\sigma[\mathbf{E}\times\mathbf{p}] - \frac{\hbar}{4mc}(\frac{e\hbar}{2mc} + 2\mu_e') div\mathbf{E}, \qquad (1a)$$

where $\mu_e' = \frac{1}{2\pi}(\frac{e^2}{\hbar c})\frac{e\hbar}{2mc} + ...$ is the electron anomalous magnetic moment.

It is to emphasize here that in accounting for the radiative corrections, the coefficients prefixed to $\sigma\mathbf{H}$ and to $div\mathbf{E}$ and $\sigma[\mathbf{E}\times\mathbf{p}]$ are modified in different ways: the anomalous magnetic moment μ'_e has been added to the magnetic moment $\frac{e\hbar}{2mc}$ in the term containing $\mathbf{H}$, whereas a twofold quantity has been added to the coefficients in two latter terms. Thus, it is quite impossible for even an electron to obtain eq. (1a) from eq. (1), that is, to account for the electron interaction with quantum fields (the radiative corrections) merely by replacing the magnetic moment of the point-like particle in the Dirac equation by the total magnetic moment that incorporates the anomalous one as well.

In a general form, the behaviour of a particle with spin 1/2 can be described in a formal way by means of the relativistic and gauge invariant equation [1], [9]

$$[c(p_\mu\gamma^\mu - \frac{e}{c}A_\mu\gamma^\mu) + i\mu\frac{1}{2}\gamma_\mu\gamma_\nu F^{\mu\nu} + Mc^2 + \epsilon\Box(\gamma_\mu A^\mu)]\psi = 0,$$

$$i\hbar\frac{\partial\psi}{\partial t} = \mathcal{H}\psi = [c\boldsymbol{\alpha}(\mathbf{p} - \frac{e}{c}\mathbf{A}) - \mu\beta(\boldsymbol{\Sigma}\mathbf{H} - i\boldsymbol{\alpha}\mathbf{E}) + \beta Mc^2 + e\Phi - \epsilon\Box\Phi + \epsilon\boldsymbol{\alpha}\Box\mathbf{A}]\psi, \quad (2)$$

where we restrict ourselves to accounting for the first order terms in field A and the D'Alamber operator $\Box$ only; here $\boldsymbol{\Sigma}$ is the spin operator and $F^{\mu\nu}$ is the electromagnetic field tensor. The lowest approximation in $1/c$ runs as follows

$$i\hbar\frac{\partial\phi}{\partial t} = \mathcal{H}\phi,$$

$$\mathcal{H} \approx \frac{1}{2M}(\mathbf{p} - \frac{e}{c}\mathbf{A} + \frac{\epsilon}{c}\Box\mathbf{A})^2 + e\Phi - (\frac{e\hbar}{2Mc} + \mu)\boldsymbol{\sigma}\mathbf{H} -$$
$$\frac{1}{2Mc}(\frac{e\hbar}{2Mc} + 2\mu)\boldsymbol{\sigma}[\mathbf{E}\times\mathbf{p}] - [\epsilon + \frac{\hbar}{4Mc}(\frac{e\hbar}{2Mc} + 2\mu)]div\mathbf{E}. \quad (2a)$$

In such a purely phenomenological approach, the empirically introduced parameters μ, ϵ render both the internal structure of a particle and its interaction with a vacuum of the electromagnetic and other quantum fields, which are not taken into account in the Dirac equation (1). The physical meaning of the quantity μ emerges unambiguously as the coefficient prefixed to $\boldsymbol{\sigma}\mathbf{H}$ in eq. (2a). Consequently, it is understood to be the "nucleon anomalous magnetic moment", completing the magnetic moment $\frac{e\hbar}{2Mc}$ of a point-like particle with spin 1/2, mass M and charge e. Yet, according (2a), this empirically introduced quantity μ shows up to determine not only the interaction of the particle with a magnetic field, but with an electrical field $\mathbf{E}$, as well: the "Schwinger interaction" (next to last term in eq. (2a)) and the "Foldy interaction" [1] (last term in (2a)). Certainly, expression (1a) for an electron ($\mu = \mu'_e$) corresponds completely with the general phenomenological equations (2), (2a).

As far as the parameter ϵ is concerned, no unambiguous conclusion about its physical contents can be drawn immediately from eqs. (2), (2a). The fact that ϵ is incorporated side by side with the terms containing μ in the coefficient prefixed to $div\mathbf{E}(\mathbf{R}) = -\Delta_R \Phi(\mathbf{R}) = -4\pi\rho_e(\mathbf{R})$ in eq. (2a) ($\rho_e(\mathbf{R})$ is the charge density inducing the electric field) does not constrain the equation

$$\epsilon = \frac{1}{6}\int d\mathbf{r}\mathbf{r}^2\rho_n(\mathbf{r}), \quad (3)$$

(here ρ_n- is the density of the charge distribution inside the nucleon), contrary to what was presumed in [1], [8], and [10], as well as in some other publications. In such a phenomenological approach, this quantity ϵ is, as a matter of fact, a fitted-parameter that is allowed to take any value, including zero.

The interaction represented by the last term in eq. (2a) results in the Born amplitude of neutron scattering ($e = 0$)

$$f_{ne}(\mathbf{q}) = -\frac{2M}{\hbar^2}[\epsilon + \mu\frac{\hbar}{2Mc}]\int d\mathbf{R}e^{-i\mathbf{q}\mathbf{R}}\rho_e(\mathbf{R}) \quad (4)$$

in the field, induced by the charge density $\rho_e(\mathbf{R})$, and in particular by the charge distribution of the atomic electrons, the atom being nailed down (bound). With the quantity μ being equal to the experimental value of the neutron anomalous magnetic moment, and the quantity ϵ is assumed to be equal to zero, the ne-scattering length $b_{ne} = -f_{ne}(0)/Z$ proves to be equal to the value $b_{ne} = -1.468\,10^{-16}cm$, coinciding within a small error with up to date experimental results [2] - [7]. This is evidence that the ϵ value is small. However, pursuing this phenomenological approach, no rigorous conclusion about the nucleon structure can be acquired through this result. Certainly, as it was already emphasized long before [1], an approach can not been asserted as expedient if the coefficients assigned to describe the nucleon structure are introduced purely empirically and are never gained through any more-or-less general and profound physical theory or, at least, a model.

As pointed out above, accounting for radiative corrections for the electron itself modifies the coefficients by $\mathbf{H}$ and by $div\mathbf{E}$ in different ways. All the more, when the nucleon, being a much more complicated composite system, interacts with an external electromagnetic field, its structure can not be taken into account merely through modification of the Dirac equation, in particular, through replacement of the point-like-particle magnetic moment by the total one, including the anomalous one as well. At this time, because of the lack of a rigorous, thorough theory, the nucleon structure is investigated in the framework of various approaches and models.

Nucleon properties are known to be successfully described in the Cloudy Bag Model (**CBM**) [11], with the interaction on the surface of the bag between quarks confined inside the bag and the pion field playing an essential role. The energy of the bag-nucleon, including quarks and meson fields, in an electromagnetic field replaces the nucleon interaction with an electromagnetic field in eq. (2). According to **CBM**, this energy is presented as the expectation value

$$< N|V|N >=< N|j^{\mu}A_{\mu}|N >=< N|(j_q^{\mu}+j_{\pi}^{\mu})A_{\mu}|N >, \quad (N=n,p) \tag{5}$$

of the operator V, rendering the interaction of an external electromagnetic field A with the quark and meson fields in the bag-nucleon ground state $|N>$, that is in the three-quark state, having a total spin and isospin 1/2 and the given projections. Here j_q, j_μ are operators of the quark and meson field currents. The energy of the bag–nucleon in its rest frame in a static magnetic field $\mathbf{H}=[\bigtriangledown\times\mathbf{A}]$ has the common form $-(\mathbf{H}\boldsymbol{\sigma})\mu_{\mathbf{n},\mathbf{p}}$. The values of the neutron and proton magnetic moments $\mu_{n,p}$, obtained according (5), prove to be in sufficiently good agreement with experimental data [11]. In the case considered, the energy of a neutron propagating with a constant velocity $|\mathbf{v}|\ll c$ through a constant electric field $\mathbf{E}=-\bigtriangledown\Phi$, is represented, according to **CBM**, as follows:

$$\mathcal{E}=< n|(j_q^0+j_{\pi}^0)\Phi|n > -\mu_n\boldsymbol{\sigma}\mathbf{H}', \tag{6}$$

where $\mathbf{H}'=\frac{1}{c}[\mathbf{E}\times\mathbf{v}]$ is the magnetic field in the neutron rest system. The second term in eq. (6) is the spin-orbit interaction, induces the well-known "Schwinger scattering". The potential Φ varying slowly in the nucleon range, the first term in eq. (6) reduces to

$$\mathcal{E}_{ne}(\mathbf{R})=\triangle_R\Phi(\mathbf{R})\frac{1}{6}\int d\mathbf{r}r^2\rho_n(\mathbf{r})=\triangle_R\Phi(\mathbf{R})\frac{1}{6}< r_n^2 >, \quad \rho_n(\mathbf{r})=< n|j_q^0+j_{\pi}^0|n >, \tag{7}$$

where $\rho_n(\mathbf{r})$ is the neutron charge density. This ne–interaction (7) in the Born-approximation results in the scattering amplitude

$$f_{ne}(\mathbf{q})=-\frac{2M}{\hbar^2}\left(\frac{< r_n^2 >}{6}\right)\int d\mathbf{R}\rho_e(\mathbf{R})e^{-i\mathbf{q}\mathbf{R}}. \tag{8}$$

The straightforward calculation of the expectation value $\rho_n(\mathbf{r})$ according to **CBM** and, consequently, $< r_n^2 >$ in eq. (7) (see for instance [12]), provides an f_{ne} value in (8) that coincides within the accuracy of $\approx 10\%$ with both experimental data and the result obtained according to aforecited phenomenological approach, the quantity ϵ being

suggested to be equal to zero. Thus, we can see that the very physical quantity — the scattering length — is expressed according to two different approaches through different neutron characteristics: through the mean square charge radius in **CBM** and through the anomalous magnetic moment in the phenomenological approach, and yet the values calculated in both cases practically coincide. This has been emphasized as being very significant.

The nucleon mean square charge radius can be acquired without handling the bag-nucleon in its rest system. The nucleon–electron scattering amplitude is determined [9] through the nucleon transition current $J^{\nu}_{N\mathbf{P},\mathbf{P}'}(x)$ between nucleon states with moments $\mathbf{P},\mathbf{P}'$. In the framework of the **CBM** approach, $J^{\nu}=<\Psi_{N\mathbf{P}'}|J^{nu}|\Psi_{N\mathbf{P}}>$ where Ψ are the bag-nucleon wave-functions and the quark and meson fields are accounted for [12]. The Fourier-transform $J^0_{N\mathbf{P},\mathbf{P}'}(x)$, that is the zero-component of the transition current in momentum representation, is presented as follows:

$$J^0_{N\mathbf{P}\mathbf{P}'}(q)=\int d^4xe^{iqx}J^0_{N\mathbf{P}\mathbf{P}'}(x)= = (2\pi)^4\delta(P-P'-q)(\epsilon_N(\mathbf{P})\epsilon_N(\mathbf{P}'))^{-1/2}h_n^0(P,q)/2. \tag{9}$$

The second moment of the charge distribution (the nucleon mean square charge radius), depending now on the nucleon momentum $\mathbf{P}$, is related to (9) via the well-known expression

$$< r_N^2(\mathbf{P}) >=-\frac{\partial^2}{(\partial\mathbf{q})^2}h_N^0(\mathbf{P},q)|_{q=0}=-6\frac{\partial}{\partial|\mathbf{q}|^2}h_N^0(\mathbf{P},q)|_{q=0}, \tag{10}$$

which can be rewritten in the form

$$< r_N^2(\mathbf{P}) >=\int d\mathbf{r}r^2h_N^0(\mathbf{P},\mathbf{r})=\int_0^\infty r^2dr4\pi r^2\tilde{\rho}(\mathbf{P},\mathbf{r}). \tag{11}$$

The quantity involved here, , depending on momentum, strictly speaking, can not be treated as the nucleon charge density, though the charge conservation law for the $\tilde{\rho}(\mathbf{P},\mathbf{r})$ is valid in this treatment [12]. The calculation [12] of the quantities $J^0,\tilde{\rho}$ and, consequently, of $< r_N^2(\mathbf{P}) >$, carried out in the framework of the **CBM**, provides a value which almost coincides at $\mathbf{P}=\mathbf{0}$ with the $< r_N^2 >$ values obtained in the aforecited approaches.

The previous consideration makes us realize that the magnitude of the ne–scattering amplitude, obtained from experimental data, must be immediately compared with the one calculated according to the **CBM** approach [11]. In the framework of the **CBM**, there is nothing proportional to the μ $div\mathbf{E}$ (like (4)) contribution to the ne–scattering amplitude. By the way, it might be instructive here to point out that the question about the "Foldy term" in the electron–nucleon interaction has never arisen, while the Lamb shift is investigated in either the ordinary or in the mesonic atoms (see, for example, [13]). On the other hand, in the framework of the phenomenological Foldy approach [1], the ne–scattering amplitude is expressed through the quantity μ according to eq. (4), yet nothing can be concluded immediately about the hidden physical sense of the quantity ϵ, namely that this approach is only phenomenological. Foldy himself, suggesting eq. (3) as being the plausible physical representation for the quantity ϵ, had, as a matter of fact, only designated such an interpretation, but never made any concise statements. Instead he referred to experiments to disentangle the physical meaning of the quantity ϵ. For years, in successive works, the quantity ϵ has been assumed to be exactly equal to the value of the second momentum of the nucleon charge distribution (3), no new or additional argument having been managed at all.

Obtaining $< r_n^2 >$ with high precision experimentally will promote, in our opinion, the development and improvement of the various models now suggested for describing the nucleon, ignoring for the moment that these approaches are apparently not yet in position to provide an accuracy better than $\sim 10\%$. Beyond question, any theoretical approach must be able to successfully describe not only one characteristic of the nucleon, but the majority of the following, simultaneously: $< r^2 >$, μ, m_N, g_A, and so on. Therefore, it is desirable to obtain the values of all of these quantities from experimental data with equal precision. Specifically, the $< r^2 >$ value needs to be known to the same accuracy as the values of μ, g_A and so forth.

On the other hand, exact determination of the ϵ value will enable us to judge to what degree the anomalous magnetic moment incorporates all the main features and peculiarities of the neutron structure while describing, according to aforecited phenomenological approach, the behavior of a neutron in an external electromagnetic field. If the ϵ value proves to be very small, almost negligible, an amazing and intriguing conclusion will be inescapable: that the spectacular phenomenological approach really does exist in which the anomalous magnetic moment thoroughly renders the structure of the neutron as it interacts with an external electromagnetic field. The goals of further experimental investigations are to obtain the b_{ne} value with better accuracy than before and that these measurements will not require significant corrections in the processing their results.

3. Launching into the new experiment

According to generally accepted notions when describing the neutron scattering length on an atom, the neutron strong interaction with the nucleus and the electromagnetic interaction with the nucleus and atomic electron cloud are taken into account [10]. For nuclei with $I = 0$ and subtracting the Schwinger scattering contribution, neutron scattering on atom is determined by the scattering length

$$b = b_N + b_{ne} Z f(\mathbf{q}) , \qquad (12)$$

where b_N is the nuclear scattering length (including the small contributions from the neutron electric polarizability and the neutron interaction with electric charge of nucleus), b_{ne} is the ne–scattering length, the goal of our investigations, and $f(\mathbf{q})$ is the atomic form–factor. That is valid for diamagnetic atoms where the electron shell spin is equal to zero.

The most precise estimations of b_{ne} have been obtained from the following experiments:

1) the angle distribution of elastically scattered neutrons by noble gases [4];

2) the energy dependence of the total neutron cross section of liquid Bi [2] and Pb [6];

3) the intensities of the diffraction peaks for a single ^{186}W crystal [3];

4) the combination of total cross section and coherent amplitude data for Pb and Bi [5], [7].

Each of these methods has its own drawbacks that can lead to systematic errors in the b_{ne} estimates.

Using the first method, the kinematic nuclear asymmetry and its dependence on the thermal motion of gas atoms have to be taken into account. The corrections for this effect, even for Xe and Kr, appear to be several times higher than the expected ne–interaction effect.

In the second case, the corrections for diffraction effects (collective scattering on neighboring atoms) and for the thermal motion of atoms have to be taken into account. The value of these corrections is close to the ne–scattering contribution.

In the third method, to describe the experimental values of b_{coh} by the same b_{ne} for two single crystals which differ in isotopic composition, the authors had to introduce some scattering of a not quite proved nature that contributes to the Bragg peaks [3].

In the fourth case, the result is determined by the value of the coherent amplitude used in the b_{ne} analysis. Thus, the value of b_{coh} is very critical and is subject to "drift" in time [14].

The mentioned ambiguities about the extraction of b_{ne} from experimental data compel us to search for new methods for setting up experiments and performing analysis.

In the present work, the possibilities of experiments on neutron scattering by noble gases using neutron spectrometry with modern pulsed sources are considered. We also revise the corrections for the thermal motion of atoms and for collective scattering on neighboring atoms.

First of all, we investigated the possibility of obtaining b_{ne} through the energy dependence of the ratios of neutron intensities from elastic neutron scattering by noble gases with the value of the large and small electron numbers, Z. For example, the 4π-detectors could be used for Xe and Ne. Nevertheless, it is necessary to take into account the neutron detector efficiency modification for Ne and Xe because of the different neutron energy losses in scattering events. Estimations of the corrections concerning this effect give a value of $(1-2)\%$, apparently compatible with the effect expected due to ne–scattering. In addition, the cross-section in the domain of slow neutrons ($E_n < 0.1eV$) suffers a kinematic increase (by the factor $\sim V_{rel}/V_0$) because of the influence of the thermal motion of the gas atoms which, certainly, are different for Ne and Xe, essentially distorting the ratio of the intensities:

$$\frac{I_{Xe}}{I_{Ne}} \simeq \frac{4\pi b_{N1}^2 + 8\pi b_{N1} Z_1 b_{ne} f_1(E)}{4\pi b_{N2}^2 + 8\pi b_{N2} Z_2 b_{ne} f_2(E)} .$$

Following [15], we recall the origin of the main formula, taking into account the influence of the motion of atoms in a single-atom gas on the differential and total neutron scattering cross-section.

The scattering probability in the center of mass system is

$$\frac{v_r \sigma_0}{4\pi}\delta(v' - v_0')dv' d\Omega' \tag{13}$$

where v_r –relative velocity, v_0' –initial velocity, v' –final velocity, σ_0 –total scattering cross-section. Then, the differential scattering cross-section in the laboratory system has the form

$$\sigma(v_0 \to v, \mathbf{\Omega}_0 \to \mathbf{\Omega}) dv d\Omega = \frac{\sigma_0}{4\pi}\frac{v_r}{v_0}\delta(v' - v_0')dv' d\Omega' . \tag{14}$$

Putting to use the Jacobian,

$$\frac{dv' d\Omega'}{dv d\Omega} = \frac{v^2}{v'^2} ,$$

we get, suggesting σ_0 to be independent of E,

$$\sigma(v_0 \to v, \mathbf{\Omega}_0 \to \mathbf{\Omega}) = \frac{\sigma_0}{4\pi}\frac{v_r}{v_0}\frac{v^2}{v'^2}\delta(v' - v_0') =$$

$$= \frac{\sigma_0}{4\pi}\left(\frac{A+1}{A}\right)^2 \frac{v^2}{v_0}\delta\left(\frac{v^2 - v_0^2}{2} + \frac{u^2}{2A} - \mathbf{uw}\right), \tag{15}$$

where $\mathbf{u} = \mathbf{v} - \mathbf{v}_0$, $\mathbf{w}$ is the velocity of the atom. Integrating over $d\mathbf{w}$, the Z–axis having been chosen along $\mathbf{u}$, we arrive at

$$d\sigma(v_0 \to v, \mathbf{\Omega}_0 \to \mathbf{\Omega}) = < \sigma(v_0 \to v, \mathbf{\Omega}_0 \to \mathbf{\Omega} >_w =$$
$$= \frac{\sigma_0}{4\pi^{3/2}}\left(\frac{A+1}{A}\right)^2 \frac{\sqrt{A}v^2}{v_0 v_T}\frac{1}{|\mathbf{v} - \mathbf{v}_0|} exp\left\{-\frac{A}{4v_T^2}\left(\frac{v^2 - v_0^2}{|\mathbf{v} - \mathbf{v}_0|} + \frac{|\mathbf{v} - \mathbf{v}_0|}{A}\right)^2\right\}. \tag{16}$$

Here $v_T = \sqrt{\frac{2kT}{m}} = 128.9\sqrt{T}\ m/s$, and thereby, we get the relation

$$d\sigma(v_0 \to v, \mathbf{\Omega}_0 \to \mathbf{\Omega}) = \frac{\sigma_0}{4\pi}\left(\frac{A+1}{A}\right)^2 F(v_0, v, \theta, A), \tag{17}$$

where

$$F(v_0, v, \theta, A) = \frac{1}{\sqrt{\pi} v_0 B_0}\frac{v^2}{\sqrt{v^2 + v_0^2 - 2vv_0\cos\theta}} \times$$
$$exp\left\{\frac{-(v^2 - v_0^2\frac{A-1}{A+1} - \frac{2vv_0\cos\theta}{A+1})^2}{4(\frac{A}{A+1})^2 B_0^2(v^2 + v_0^2 - 2vv_0\cos\theta)}\right\}. \tag{18}$$

Here $B_0 = \sqrt{\frac{2kT}{mA}}$. To obtain the ratio of the intensities of the neutrons scattered at various angles, expression (17) has to be integrated over v from 0 to ∞ at the given angles

$$R = \frac{\int_0^\infty F(v_0, v, \theta_1, A)dv}{\int_0^\infty F(v_0, v, \theta_2, A)dv} = \frac{F_s(v_0, \theta_1, A)}{F_s(v_0, \theta_2, A)}. \tag{19}$$

The ne–interaction being taken into account, the interference term

$$d\sigma = [b_N^2 + 2b_N Z b_{ne} f(E_r, \theta_{cm})] F(v_0, v, \theta, A) , \tag{20}$$

appears, where $f(E_r, \theta_{cm})$ is the atomic form factor corresponding to the relative energy (velocity) of the motion of the atom and the neutron E_r, and to the scattering angle θ_{cm} in the center-of-mass system which leads to the scattering angle θ in the laboratory system. Our calculations of the averaged form factors, using the Monte-Carlo method and considering the thermal motion, show that the average form factor values agree, with good precision, with the form factor values taken directly for E_0 (V_0) and θ in the laboratory system. For this reason, the neutron intensity ratio with ne–scattering and thermal motion contributions may be written as

$$R = \frac{b_N^2 F_s(V_0, \theta_1, A) + 2b_N b_{ne} Z f(V_0, \theta_1)}{b_N^2 F_s(V_0, \theta_2, A) + 2b_N b_{ne} Z f(V_0, \theta_2)} . \tag{21}$$

Integrating (17) over v and θ, we obtain for the total cross section, without accounting for ne–scattering:

$$\sigma_s = \sigma_0 \frac{1}{x_0}[(x_0 + \frac{1}{2Ax_0})erf(\sqrt{A}x_0) + \frac{1}{\sqrt{\pi A}}e^{-Ax_0^2}] \equiv \sigma_0 F_t(V_0, A) , \quad (22)$$

where $x_0 = \frac{V_0}{V_T}$ and $V_T = \sqrt{\frac{2kT}{m}}$. Including the ne–scattering, we get

$$\sigma_s = 4\pi b_N^2 F_t(V_0, A) + 8\pi b_N b_{ne} Z f_t(E), \quad (23)$$

where F_t is the integral of F_s over θ.

In calculations of R and σ_s we used the analytical expressions from [10]:

$$f(E, \theta) = \frac{1}{\sqrt{1 + \nu E \sin^2 \theta/2}}, \quad (24)$$

$$f_t(E) = \frac{2}{\nu E}(\sqrt{1 + \nu E} - 1) , \quad (25)$$

for the form factors, where $\nu = 5797.4/q_0^2 \ eV^{-1}$, and q_0 is the parameter fitted for each kind of gas (see [10]). A comparison of the form-factors calculated according these formulas with the tabulated data from [16] shows a good agreement when using the q_0 values for Kr $q_0 = 6.60$ and for Xe $q_0 = 6.80$ as suggested in [10].

The feasible scheme of the experimental set-up for the measurements is presented in Fig.1.

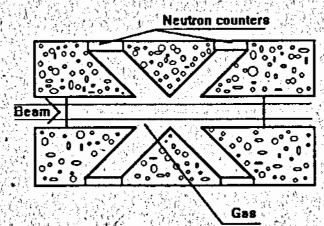

Fig.1. The scheme of the set-up for measuring $R(E)$.

Fig.2 shows the energy dependence of the intensity ratio of neutrons scattered by Xe at 45^0 and 135^0. Calculations were carried out using (21). The steep fall of the R value in the region below 5 meV is determined by the kinematic factors $F_s(V_0, \theta)$ for pure nuclear scattering. In the energy region above 10 eV, the curve R approaches the value for scattering by atoms at rest.

$$W(\theta) = \frac{\left(\frac{1}{A}\cos\theta + \sqrt{1 - \frac{\sin^2\theta}{A^2}}\right)^2}{\sqrt{1 - \frac{\sin^2\theta}{A^2}}} . \quad (26)$$

The position of the $R(E)$ minimum at $E \sim 0.05$ eV is determined by the contribution of ne–scattering with a relative value of $\sim 5.10^{-3}$.

If it is assumed that $R(E)$ can be measured with errors of 2.10^{-4} (in Fig.2, the random points are within such accuracy limits), the converse analysis of pseudo-experimental data using the FUMILI-code (the curve in Fig.2) shows that b_{ne} may be

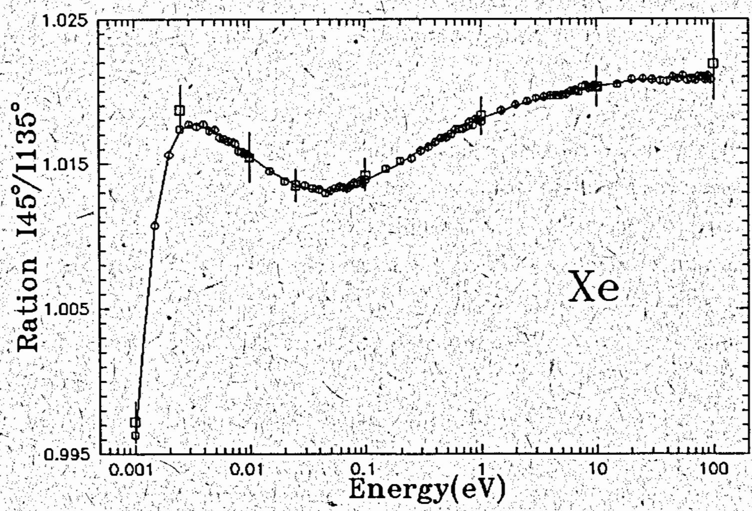

Fig.2. Intensity ratio $R(E)$ for angles of 45^0 and 135^0 as a function on neutron energy E. The circles are the pseudo-experimental results with errors of 0.0002; the curve showes the fitted result: $b_N = 5.85 \pm 0.30 fm$ and $b_{ne} = (-1.32 \pm 0.06)10^{-3} fm$; the squares are data points from Monte Carlo calculations for the same b_N and b_{ne} values.

extracted only with the same accuracy as b_N. This is because of the linear correlation between b_{ne} and b_N that appears in the $R(E)$ analysis. For measurements, it is better to use the heavy isotopes of Xe because they do not have neutron resonances in the eV region. Then, for example, consideration of the -84 and 14.4 eV resonances from

10% ^{131}Xe-dopant retains the true shape of the R curve. The nuclear scattering length can be obtained from total cross section measurements higher than 10 eV where the capture corrections are small. For the existing beams at the IBR-2 reactor and the IBR-30 booster, the b_N estimates can be obtained with an accuracy of better than 2% in several days. Then, from measurement of the angle anisotrophy for 45^0 and 135^0 in the energy interval from 0.002 to 1 eV, with an accuracy of 2.10^{-4} (which is quite possible with the IBR-2 and IBR-30 beams using helium-3 counters at a pressure of 10 atm), b_{ne} can be obtained at better than 2%. In this case, the error of b_{ne} can be determined not by the count statistic, but by the uncertainty Δb_N. For $R(E)$ analysis, it is convenient to use the region below 0.1 eV because here there is no necessity to make corrections for changes in detector efficiency for neutrons scattered at 45^0 and 135^0. To adjust the corrections caused by the thermal motion of the gas atoms, it would be desirable to carry out measurements for noble gases of various A, Z similar to those performed by Kron and Ringo [4].

We obtained for Xe at $E = 0.05$ eV that:

$R = 1.021$ without considering ne–scattering and thermal motion,

$R = 1.020$ considering the thermal motion, only, and

$R = 1.013$ including both ne–scattering and thermal motion.

So, at the maximum of the ne–scattering effect, the anisotropy $R - 1$ due to passing from the center-of-mass system to the laboratory one is equal to 0.021. The correction for the thermal motion reduces this value by 0.001 and the searched for effect by 0.007 more.

Measurement of the total cross section by the neutron transmission for the ^{86}Kr isotope has been more promising. This isotope has a small thermal capture cross section, 3 ± 2 mb [17], so the total cross section is determined, mostly, by scattering. The energy dependence of σ_t calculated for some hypothetical cases are shown in Fig.3. Points show $\sigma_t(E)$; calculated by eq. (23) for the fixed parameters $b_N = 7.73$ fm, $\sigma_\gamma = 2/\sqrt{E(eV)}mb$) (to be more by the factor of 4 than for the pure ^{86}Kr isotope), $b_{ne} = -1.32$ mfm, and is statistically scattered with an error of 2.5 mb. The solid curve displays $\sigma_t(E)$ at $b_{ne} = 0$, that is, the case without ne–scattering. The dashed curve corresponds to $\sigma_t(E)$ for the case without the thermal motion of gas atoms (calculated by eq. (23) at $T \to 0$). One can see that in the 0.1 eV region, the real contribution of ne–scattering is 35 mb, of which 15 mb are compensated for by the thermal motion effect of the atoms. For lower energies, the contribution of ne–scattering increases, but it is considerably overlapped

by the thermal motion effect. So, at $E = 0.001$ eV, $\sigma_{ne} \sim 55$ mb, whereas the thermal motion contribution is more than 100 mb. In Fig.3, the squares show the case

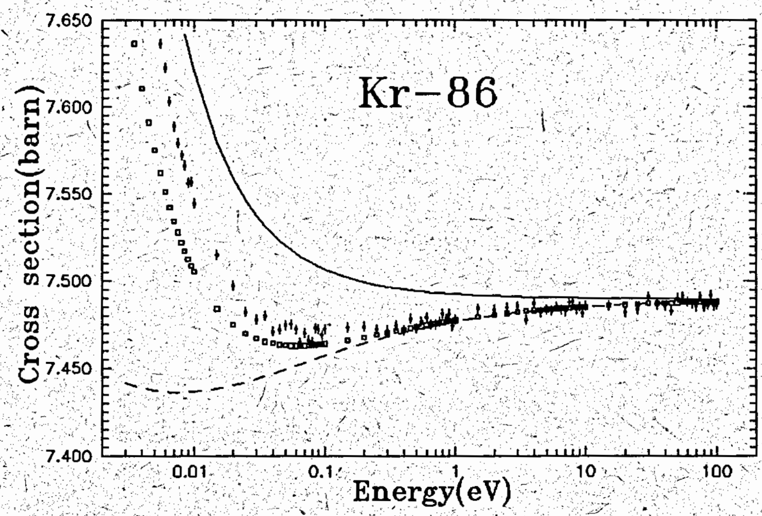

Fig.3. Shape of the total cross sections in relation to a consideration of thermal motion and ne–scattering contributions. The points are pseudo-experimental data at $T = 293^0K$ and $b_{ne} = -1.3210^{-3} fm$; the curve corresponds to $b_{ne} = 0$ and the same T; the dashed line corresponds to $T = 0^0K$ and $b_{ne} = -1.3210^{-3} fm$; squares are the same yet at $T = 184^0K$.

corresponding to a gas temperature of $184K$ (provided that the gas is cooled by liquid xenon). These data essentially relieve the problem of estimating the ne–amplitude.

In Fig.4, the analysis of pseudo-experimental data using the FUMILI-code (with standard errors of 2.5 mb) is presented. If data covered the region of $0.003 - 10$ eV or even $0.003 - 1$ eV, b_{ne} can be obtained with an accuracy of $\sim 5\%$. In this case, the capture cross section can be extracted with an accuracy of 10%. If the data below 0.01 eV are absent, then the errors for b_{ne} and σ_γ increase by ~ 2 times. In this case, it is desirable to know the value of $\sigma_{0\gamma}$ from addition measurements of σ_t in the meV region, which can improve the accuracy of the b_{ne} estimation.

The Russian National Fund for Stable Isotopes has 10 g samples of ^{86}Kr with 99.97%

enrichment. A $S \sim 1.7\ cm^2$ sample can be prepared and its transmission may be $T \sim 0.74$. To attain an accuracy of up to 2.5 mb for such T, the statistic has to be $\sim 3.10^8$ per point. With the mirror neutron guide tube of the IBR-2 reactor in intervals of $3-25\ meV$, such a statistic may be achieved in 24 hours, and in 4 days at the IBR-30 booster for a flight path of 10 m in the region of $0.005-10\ eV$.

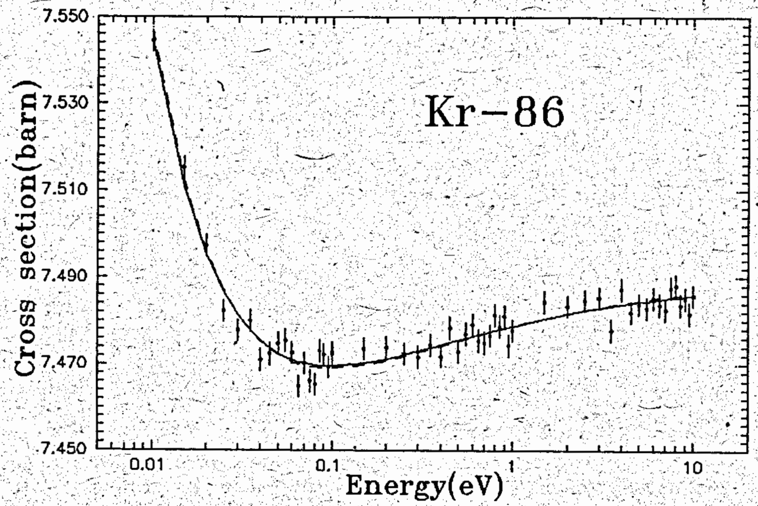

Fig.4. Results of the analysis of the ^{86}Kr total cross section. The points are pseudo-experimental cross sections with errors of $2.5mb$ and $\sigma_\gamma(1eV) = 2mb$. Results of fitting : $b_{ne} = (-1.18 \pm 0.10)10^{-3} fm$, $\sigma_\gamma(1eV) = 1.0 \pm 0.7mb$ or $b_{ne} = (-1.32 \pm 0.02)10^{-3} fm$ by fixed $\sigma_\gamma(1eV) = 2mb$.

The problem of correcting for the diffraction caused by collective scattering of the neutron wave by nearby atoms was considered by Akhieser and Pomeranchuk as far back as 1949 [18]. It has been shown that diffraction effects lead to additional asymmetry in neutron scattering

$$f(\theta) = \frac{\sin\eta}{\eta^3} - \frac{\cos\eta}{\eta^2}, \tag{27}$$

where $\eta = \frac{8\pi R}{\lambda}\sin\frac{\theta}{2}$ and R is the radius of the atom. This effect produces the following

correction in the total cross section

$$\Delta\sigma_s = 2\pi b_N^2 4\pi(2R)^3 n \int_0^\pi f(\theta)\sin\theta d\theta\ , \tag{28}$$

where n is the number of atoms in 1 cm^3.

Our estimations show that for a gas sample with a pressure of 1 atm and 0.5 atm, and at $R = 1.4.10^{-8}\ cm$, the relative corrections $\Delta\sigma_s/\sigma_s$ for the total scattering cross-section result in the values displayed in the following table:

Energy eV	P=1 atm	P=0.5 atm
0.001	1.10^{-3}	6.10^{-4}
0.005	2.10^{-4}	1.10^{-4}
0.01	1.10^{-4}	5.10^{-5}
0.1	1.10^{-5}	5.10^{-6}

Thus, these corrections for the energy above 1 meV prove to be $\sim 1\ mb$, which does not exceed the statistical error. Their influence can be verified by carrying out measurements with various mixtures of the sample.

The research described in this publication was made possible in part by Grant RFS000 from the International Science Foundation and by Grant 94-02-03118-a RFFI.

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Received by Publishing Department
on March 5, 1996.