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SPECIFICATIONS FOR DERIVING NEUTRON ELECTRIC POLARIZABILITY FROM THE TOTAL CROSS SECTIONS OF ^{208}Pb

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1. In spite of numerous and long standing attempts to get an experimental value for the neutron electric polarizability coefficient α_n, the only meaningful result

$$\alpha_n = (1.20 \pm 0.15 \pm 0.20) \cdot 10^{-3}\, fm^3 \tag{1}$$

was communicated in [1]. An analysis and some criticism of [1] was given in [2] about four years ago. Nevertheless, since we regard the measurement of the ^{208}Pb neutron scattering cross section $\sigma(k)$ (k is neutron wave number) performed in [1] as the best one so far, it is very important to understand all the details of the discussed $\sigma(k)$ or $\sigma(E)$ dependence (E is the neutron energy), and not only the $k-$ or $\sqrt{E}-$component of it. In other words, the b and c coefficients in the expression

$$\sigma(k) = \sigma(0) + ak + bk^2 + ck^4 \tag{2}$$

used in [1] must also be physically reasonable. The second motive of the present work is the unexpected results of recent measurements of $\sigma(E)$ for ^{208}Pb performed in Dubna [3]. A very large deficiency in the s-wave resonance strength was discovered in [3] in comparison with that which exists for ^{208}Pb in literature [4,5]. We have found that only by taking an unusually strong resonance into account can α_n be derived from $\sigma(E)$ positive.

2. If $\sigma(k)$ is a "potential" cross section, i.e., it is obtained by subtracting the contributions of all known resonances, it must have the form

$$\sigma(k) = \frac{4\pi}{k^2} sin^2(-ka_{pot}) + \frac{12\pi}{k^2} sin^2\delta_1, \tag{3}$$

where $-ka_{pot}$ and a_{pot} are the s-wave phase shift and scattering length, respectively. For the last one we should write

$$a_{pot} = R'_N + hE + a_pQ, \tag{4}$$

where R'_N is the nuclear part of the so-called scattering radius, hE are the energy dependent "tails" of unknown and distant resonances, and a_pQ is the neutron polarizability contribution; for ^{208}Pb

$$a_p = -0.0392\alpha_n\; fm, \qquad Q = 1 - \frac{5\pi}{18}kR_N + \frac{5}{21}(kR_N)^2 - \frac{2}{243}(kR_N)^4 + \cdots \tag{5}$$

($R_N \simeq 7.1\, fm$, α_n in $10^{-3}\, fm^3$). The h parameter can be expressed to the first power of the E/E_0 approximation via the sum

$$h = -2276\frac{A+1}{A}\sum \frac{g\Gamma_n^{(0)}}{E_0^2}\; fm/eV \tag{6}$$

over all s–wave resonances which were not taken into account in the $\sigma(k)$ calculation. Here $\Gamma_n^{(0)}$ and E_0 are the reduced neutron width and the energy of one resonance. Note, both positive and negative resonances give negative contributions to h. As for the second term in (3), it is the p–wave part of $\sigma(k)$. We use the results of [6] for neutron scattering by natural Pb and the results of following experiment with ^{208}Pb to estimate δ_1 as

$$\delta_1 = -kR + arctg(kR) + arcsin\left[\frac{1}{3}k\frac{(kR)^2}{1+(kR)^2}(R - R_1')\right], \quad R = 8.0\,fm, \quad R_1' = 3.0\,fm, \tag{7}$$

where R is the channel radius and R_1' is the p–wave scattering radius. Finally, it is convenient to join the constant term of a_pQ with R_N' so instead of (4) one gets:

$$a_{pot} = R' + hE - a_p(1 - Q), \tag{8}$$

where $R' = R_N' + a_p$ becomes the value usually seen in experiments.

3. Now, if we present (3) together with (8),(5) and (7) in the form of series (2), its coefficients will be:

$$\sigma(0) = 4\pi R'^2, \tag{9}$$

$$a = 8\pi c_1 R', \tag{10}$$

$$b = 4\pi(c_1^2 + 2c_2R' - \frac{1}{3}R'^4), \tag{11}$$

$$c = 4\pi(c_2^2 - 2c_1^2R'^2 - \frac{4}{3}c_2R'^3 + \frac{2}{45}R'^6 + \frac{1}{3}R^4R_1'^2), \tag{12}$$

where

$$c_1 = -\frac{5\pi}{18}a_pR_N, \; c_2 = 2.09 \cdot 10^5 h + \frac{5}{21}a_pR_N^2.$$

In order to verify the validity of approximation (2) for (3), we found decomposition coefficients of (3) c_3 at k^3, c_5 at k^5 and c_6 at k^6 as well:

$$c_3 = 8\pi c_1\left(c_2 - \frac{2}{3}R'^3\right), \tag{13}$$

$$c_5 = 4\pi c_1 R'\left(\frac{11}{60}R'^4 - 4c_2R' - \frac{4}{3}c_1^2\right), \tag{14}$$

$$c_6 = 4\pi\left[-\frac{1}{3}c_1^4 - 4c_1^2c_2R' - 2c_2^2R'^2 + \frac{5}{12}c_1^2R'^4 + \frac{11}{60}c_2R'^5 - \frac{1}{360}R'^8 + 2R^7R_1'\left(\frac{2}{15} - \frac{R_1'}{3R}\right)\right]. \tag{15}$$

Using the values

$$\sigma(0) = (11.508 \pm 0.005)\,b, \quad a = (0.69 \pm 0.09)\,b \cdot fm, \quad b = (-448 \pm 3)\,b \cdot fm^2, \quad c = (9500 \pm 400)\,b \cdot fm^4 \tag{16}$$

from [1] and above-mentioned quantities for a_p and R_N, it is not difficult to get from equations (9)–(11)

$$R' = (9.5696 \pm 0.0021)\,fm, \quad c_1 = (0.0029 \pm 0.0004)\,b, \quad c_2 = (-0.402 \pm 0.013)\,b \cdot fm, \tag{17}$$

$$a_p = -(0.046 \pm 0.006)\,fm, \quad \alpha_n = (1.18 \pm 0.15) \cdot 10^{-3}\,fm^3, \quad h^{*)} = (-19.0 \pm 0.6) \cdot 10^{-7}\,fm/eV. \tag{18}$$

*) This confirms the result of [3] $h < -11 \cdot 10^{-7}\,fm/eV$.

But substitution of quantities (17) into (12) leads to an unexpected result:

$$\frac{4\pi}{3}R^4R_1'^2 = (-896 \pm 443)\ b\cdot fm^4 \tag{19}$$

instead of

$$\frac{4\pi}{3}R^4R_1'^2 = 1544\ b\cdot fm^4 \tag{20}$$

at R and R_1', according to (7). This means there is an absence of p–wave contribution in $\sigma(k)$ measured in [1]. The deficiency of $\sigma(k)$ at $40\,keV$ is the difference of (20) and (19) times k^4, i.e., $8.9 \pm 1.6\,mb$.

Finally, we estimate the contributions which are proportional to k^3, k^5 and k^6 and which were ignored in [1]. Using (17) for R', c_1, c_2 and (7) for R, R_1' we have:

$$c_3k^3(40\,keV) = (-3.55 \pm 0.49)\,mb, \qquad c_5k^5(40\,keV) = 0.09\,mb, \qquad c_6k^6(40\,keV) = -0.86\,mb. \tag{21}$$

Thus, if the neglect of the two last values of (21) can be regarded as justified, the neglect of c_3k^3, which is $\sim 12\%$ from $ak = 30.18\,mb$ seems rather unjust for a correct α_n determination.

4. The main purpose of this work is to demonstrate an alternative method of $\sigma(E)$ analysis. Having no primary data from [1], we are forced to produce them in an artificial way. One hundred values of $\sigma(E)$ at E from 0.4 to $40\,keV$, calculated according to (2) and (16), were spread randomly with a standard deviation $\Delta\sigma$, and 4 coefficients of (2) were fitted to these quasi–experimental points of $\sigma(E)$ by the least–square method. We stopped at $\Delta\sigma = 2\,mb$ and

$$\sigma(0) = (11.507 \pm 0.001)\,b, \qquad a = (0.68 \pm 0.05)\,b\cdot fm, \qquad b = (-446 \pm 1)\,b\cdot fm^2, \qquad c = (8600 \pm 500)\,b\cdot fm^4, \tag{22}$$

because the coefficient errors increased very abruptly at larger $\Delta\sigma$.

Table

No.	R', fm	$h\cdot 10^7$, fm/eV	$\alpha_n\cdot 10^{42}$, cm^3	E_0, MeV	$\Gamma_n^{(0)}$, eV	χ^2
1	9.5685(6)	-20.4(3)	1.68(19)	–	–	123
2	6.302(58)	0	1.49(26)	-3.9	5770(101)	176
3	6.728(41)	0	1.60(20)	-2.5	3217(46)	122
4	7.048(40)	0	1.66(21)	-1.9	2170(34)	122
5	7.511(37)	0	1.79(22)	-1.3	1211(22)	132
6	8.824(17)	0	3.20(27)	-0.3	101(2)	158
7	7.591(46)	-5	1.63(21)	-1.9	1702(39)	122

So, the obtained set of $\sigma(E)$ points was described first by formulas (3), (8), (5) and (7) with three varied parameters, R', h and α_n. The result of fitting is shown in the first line of the Table (the last column is the χ^2 value for 100 points; parameter errors are in parenthesis). After that, it seemed interesting to see what resonance, instead of h, could fill the deficiency and whether it really exists. With this purpose, we added the resonance and interference terms:

$$\frac{\pi g\Gamma_n^{(0)}\sqrt{E}}{k^2}\cdot\frac{\Gamma_n^{(0)}\sqrt{E} + 2(E-E_0)sin(2ka_{pot}) - 2\Gamma sin^2(ka_{pot})}{(E-E_0)^2 + \Gamma^2/2} \tag{3'}$$

to (3) for one resonance with a given E_0 and $\Gamma_n^{(0)}$, and its interference term:

$$\frac{2\pi}{k^2}\cdot\frac{\Gamma_n^{(0)}\Gamma_{n1}^{(0)}E[(E-E_0)(E-E_{01}) + \Gamma\Gamma_1/4]}{[(E-E_0)^2 + \Gamma^2/4][(E-E_{01})^2 + \Gamma_1^2/4]} \tag{3''}$$

with the strongest resonance having $E_{01} = 507\,keV$, $\Gamma_{n1}^{(0)} = 74\,eV$, where $\Gamma = \Gamma_n^{(0)}\sqrt{E}$, $\Gamma_1 = \Gamma_{n1}^{(0)}\sqrt{E}$.

Fitting $R', \alpha_n, E_0, \Gamma_n^{(0)}$ at the fixed $h = 0$ showed a strong correlation between E_0 and $\Gamma_n^{(0)}$ and no evident minimum of χ^2. So we made several fits with different fixed E_0 displayed in the Table. The lines 2-6 show that any sufficiently distant negative resonance with proper $\Gamma_n^{(0)}$ provides acceptable description of the calculated $\sigma(E)$ points with α_n slightly dependent on E_0. The interval for permissible $E_0 > 0$ is much less and wholly situated at the searched energies [4,5] where the strongest resonance has $\Gamma_n^{(0)} = 74\,eV$. Therefore only a negative resonance can resolve the problem.

Fortunately, the compound–nucleus ^{209}Pb with double-magic ^{208}Pb core has the well–known level scheme up to $\sim 5.5\,MeV$ [7,8] in which there is only one $1/2^+$ level below the neutron binding energy $3.94\,MeV$. It is the single–particle level $4s_{1/2}$ at the energy $2.03\,MeV$ and corresponds to the negative resonance at $E_0 \simeq -1.91\,MeV$. This resonance was used in [5] as "dummy" resonance with $\Gamma_n^{(0)} \simeq 200\,eV$ assigned to it. However, if a resonance is based on the single–particle level with the spectroscopic factor close to 1 (as in our case according to [7]) it may well be that such a resonance has $\Gamma_n^{(0)}$ close to the single–particle Wigner limit which is $\sim 2300\,eV$ for a nuclear radius of $8\,fm$.

Thus we have now solid grounds to declare we know now the searched resonance and consider line 4 of the Table as the most probable among the rest. As for line 7, it shows the possible influence on derived α_n of missed resonances which are equivalent to $h = -5\cdot 10^{-7}\,fm/eV$ (for comparison: the strongest resonance at $E_0 = 507\,keV$ gives $h = -6.6\cdot 10^{-7}\,fm/eV$).

5. Finally, we tried to improve the $\sigma(E)$ analysis used in [1] by adding the cubic term c_3k^3 to (2). Since the fitting of five independent parameters is practically impossible, we got from (10) and (13) the relation

$$c_3 = a(c_2 - \frac{2}{3}R'^3)/R' \simeq -65a, \tag{23}$$

where the approximate equality was obtained by the substitution of values from (17) into the exact equality. This made it possibile to fit four parameters as before, but also taking into account the cubic term $-65ak^3$. The fitted parameters:

$$\sigma(0) = (11.508 \pm 0.002)\,b, \qquad a = (0.54 \pm 0.26)\,b\cdot fm, \qquad b = (-441 \pm 8)\,b\cdot fm^2, \qquad c = (8100 \pm 2100)\,b\cdot fm^4 \tag{24}$$

do not differ much from (16) and (22) and result in

$$R' = (9.5696 \pm 0.0009)\,fm, \qquad h = (-17.7 \pm 1.4)\cdot 10^{-7}\,fm/eV, \qquad \alpha_n = (0.93 \pm 0.40)\cdot 10^{-3}\,fm^3, \tag{25}$$

whose accuracy is a marked degree worse than of (17) and (18).

Conclusion. Detailed analysis of formula (2) and its coefficient values (16) given in [1] has shown the following.

1. Most probably, the cross section $\sigma(k)$ measured in the work [1] is somewhat wrong. This issues from the fact that coefficients b and c describing $\sigma(k)$ are in a wrong relation: the c value at a given b is deficient not only for the p–wave contribution but for the k^4- term from $sin^2(kR')/k^2$, as well (the value of (19) is not zero but negative). The probable reason for this is a distortion due to background difficulties.

2. The systematic error of (1) is obviously underestimated, and for α_n instead of (1) it should be written something like $\alpha_n \sim (0.7 \div 1.9) \cdot 10^{-3}\, fm^3$.

3. In order to get the α_n value from a correctly measured $\sigma(k)$ with errors of about $1 - 2\, mb$, we propose two approaches. The first one (see point 5 above) consists of simultaneous taking into account the linear and cubic in k terms of $\sigma(k)$. The alternative approach is application of formula (3) with the supplements (3′) and (3″) where $E_0 = -1.91\, MeV$ and $\Gamma_n^{(0)}$ is to be fitted. Only the α_n value coincided with these two approaches can be regarded as the reliable one.

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