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THE EFFECTIVE GRADIENT METHOD IN THE FUEL BOWING PROBLEM

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This paper is dedicated to the memory of the EBR-1 reactor accident which happened 40 years ago because of fuel rod bowing. The incident made the necessity of subassembly construction of reactor cores clear.

INTRODUCTION

The importance and complexity of the bowing phenomena in nuclear reactors are well known [1-4], apparently since the accident at the EBR-1 reactor in 1955 [4]. Fuel bowing is most likely to result from a temperature gradient transverse to the axial direction of the subassemblies. Despite the fact that a subassembly is a rather tight cluster of rods, the motion of the fuel due to bowing of the fuel elements within subassemblies or the subassemblies themselves may be noticeable; especially in fast reactors. In a large (pancaked) reactor the bowing of fuel elements within subassemblies, that are themselves constrained not to bow, has a negligible effect on reactivity, but in EBR-size reactors the effect attains a value of the order of 1$ per mm of effective decrease in the core radius [1].

The periodically pulsed reactor is much more sensitive to reactivity insertions in comparison to steady-state reactors. For example, even a very small (say 0.01mm) radial displacement of the periphery row of subassemblies in the IBR-2 reactor (2-MW, plutonium-dioxide sodium-cooled periodically pulsed reactor in Dubna, Russia [5]) causes a noticeable effect - a power rise with a period of about 80 sec, whereas in a steady-state operation mode, the period would be equal to 25 min. The example indicates that bowing effects not only can, but must be noticeable in the IBR-2 reactor. The time-dependence of the IBR-2 response function [6] has some typical features of a bowing nature, namely: abrupt threshold-type behaviour, parametric instability from measurement to measurement, feedback nonlinearity [7], high sensitivity to average power level.

From the calculational point of view, determination of the overall bowing effects, such as reactivity feedback, is a huge problem that becomes unsolvable for large cores containing hundreds of close-packed subassemblies which represent a complicative statically undetermined system [1]. In the cases like this, only estimations are available, and hence, analytical or semianalytical models describing the fundamental bowing mechanisms by simple equations would be useful.

The method proposed here permits one to bypass the solution of partial differential equations, practically without a loss of accuracy, provided traditional cylindrical geometry of the fuel elements and linearity of the equations. The idea of the method is to extract the first order Fourier harmonic from the multi-dimentional temperature distribution, and hence, to reduce the number of variables by converting the $T(r,\varphi,t)$ function into some vector with two variables (r and t).

This procedure of "vectorization" of functions, named here the "effective gradient method", as well as its accuracy, are demonstrated by comparison of two ways (direct and "effective") of solving the special steady-state boundary-value problem, that was set for the single eccentrically bonded fuel element, placed in the radially distorted neutron flux and coolant temperature [8].

The "vectorized" model can be generalized to account for the thermo-mechanical and thermo-hydraulic interactions of fuel elements. That will give a possibility for qualitative description of the behaviour of fuel elements within the subassembly as well as the subassemblies themselves, by means of ordinary equations.

IDEA OF THE METHOD

The Omega Operator. The bowing of core components, such as fuel rods, casings, etc., caused by temperature T depends on the integral over the rod cross section

$$\int \vec{r}\,T(\vec{r},t)\,dxdy\,.$$

The linear coefficient of thermal expansion (β) must be included in the integral, but here it is assumed to be a constant. The plane radius-vector $\vec{r} = ix+jy$, originating from the neutral axis z of the rod, can be represented as $\vec{r} = r\vec{\Omega}$, where $\vec{\Omega} = i\cos\varphi + j\cos\varphi$ is a unit vector.

The angular part of the integral can be defined as the operation

$$\hat{\Omega}T(r,\varphi) \equiv \frac{1}{\pi}\int_0^{2\pi} T(r,\varphi)\vec{\Omega}\,d\varphi = \vec{\Theta}(r) \qquad (1)$$

Here we have introduced the operator $\hat{\Omega}$ which turns an angular dependent T function into some vector $\vec{\Theta}(r)$. The limit of the ($\vec{\Theta}/r$) ratio at zero is equal to the gradient of the T function at $r = 0$ transverse to the z direction. The ratio $\vec{\Theta}/R$ gives the effective gradient that characterizes the transverse distortion of the T function on a circle of radius R.

The x,y components of the $\vec{\Theta}$ vector are equal to the corresponding components (a_1, b_1) of the first harmonic of the Fourier series of T function. In general, for the time-dependent case

$$T(r,\varphi,t) = a_0 + \sum_{n=1}^{\infty}(a_n\cos n\varphi + b_n\sin n\varphi)\,. \qquad (2)$$

Therefore, the omega operator extracts the first-order Fourier harmonic from an arbitrary periodical angular dependence. If some boundary-value problem can be formulated in terms of "vector of function" the problem becomes much easier because of a reduction in the number of variables. For example, the action of the omega operator on the linear equation of thermal conductivity

$$\frac{1}{a}\frac{\partial T}{\partial t} = \Delta T(r,\varphi,t) + \frac{1}{\lambda}q_v(r,\varphi,t)\,, \qquad (3)$$

where a and λ are constants, and

$$\Delta = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}\,,$$

gives a couple of φ independent equations of the form

$$\frac{1}{a}\frac{\partial\vec{\Theta}}{\partial t} = \Delta\vec{\Theta}(r,t) - \frac{1}{r^2}\vec{\Theta}(r,t) + \frac{1}{\lambda}\vec{Q}(r,t) \qquad (4)$$

where Δ is now a radial part of the Laplasian. If the same operation is available for the boundary conditions the "vectorization" of the problem will be an exact procedure.

Some Useful Properties of the Omega Operator. It is clear that the vector of a constant is a zero vector. Some other results of the action of the omega operator are listed below for use later on:

(a) $\hat{\Omega}\,\varepsilon\cos(\varphi - \varphi_0) = \vec{\varepsilon}\,.$

The vector of the cosine function of constant amplitude ε and phase shift φ_0 is the φ_0 directed vector:

$$\vec{\varepsilon} = i\varepsilon \cos\varphi_0 + j\varepsilon \sin\varphi_0 \equiv (\varepsilon \cos\varphi_0, \varepsilon \sin\varphi_0).$$

(b) $\hat{\Omega}\ T(\varphi) = \vec{T}_1 = (a_1, b_1)$.

The vector of an arbitrary function is equal to the vector of its first Fourier harmonic of a series like Eq.(2). The vector $\vec{T}_1$ of function T can be also named the "first" vector. So, the "second" vector will be $\vec{T}_2 = (a_2, b_2)$.

(c) $\hat{\Omega}\ \varepsilon\cos(\varphi - \varphi_0)T(\varphi) = a_0\vec{\varepsilon} + \frac{1}{2}\varepsilon\vec{T}_2(-\varphi_0)$.

The action of the omega operator on the product of the cosine and arbitrary functions gives two vectors. One of them, denoted by $\vec{T}_2(-\varphi_0)$, is the "second" vector $\vec{T}_2 = (a_2, b_2)$ of T function turned on an angle φ_0 in a negative direction relative to its natural orientation, i.e.

$$\vec{T}_2(-\varphi_0) =$$
$$= (a_2\cos\varphi_0 + b_2\sin\varphi_0, b_2\cos\varphi_0 - a_2\sin\varphi_0).$$

The cosine function ejects the "first" vector of the T function.

(d) $\hat{\Omega}\ d^2T / d\varphi^2 = -\vec{T}_1$.

The vector of the second derivative of the arbitrary function is equal to its inverse vector. This property was used in Eq.(4).

(e) $\hat{\Omega}\ T(\varphi)F(\varphi) = f_0\vec{T}_1 + a_0\vec{F}_1 + \frac{1}{2}\sum_{n=1}^{\infty}\vec{R}_n$.

The action of the omega operator on the product of two functions gives the combination of their vectors and the infinite sequence of the additional components.

The Effective Bowing Gradients. The temperature integral divided by the momentum of inertia I of the rod cross section gives the definition of the bowing temperature gradient

$$\vec{\nabla}T(z) = I^{-1}\int T(r,\varphi)\vec{r}rdrd\varphi . \quad (5)$$

This vector is used directly in bowing calculations. For example, the free deflection $\vec{u}$ of the rod would be described by the equation

$$\frac{d^2\vec{u}}{dz^2} = -\beta\vec{\nabla}T . \quad (6)$$

The bowing gradients for the fuel rod and cladding are the aim of the calculations of this paper.

THE STEADY-STATE TEMPERATURE DISTRIBUTION . EXACT SOLUTION.

The Setting of the Problem. The small fuel fragment of unit length and radius R_1 is placed eccentrically inside the cladding of inner and outer radii, R_2 and R (Fig.1). The gap between fuel and cladding is filled by a movable heat-conducting medium. The cladding is cooled by a liquid of known angular dependent temperature $T(\varphi)$. The axial location z of the fragment is consided as a parameter.

The termal conductivity coefficients of the fuel (λ_1), cladding (λ_2) and gap medium (λ'), as well as the coolant film heat transfer coefficient (α), are constants. The axial heat flux is set to zero. The tangential component of the heat flux in the gap, which may be caused by heat conduction or natural convection, is also negligeble.

Our aim is to find the steady-state temperature distributions in the fuel $T_1(r,\varphi)$ and cladding $T_2(r,\varphi)$ by direct solution of the corresponding boundary-value problem which will be set under the following assumptions.

1. The heat generation per unit volume of fuel has a distribution which is typical for fast reactors:

$$q_v(r,\varphi) = q + \nabla q\ r\cos\varphi, \quad 0 \le r \le R_1 . \quad (7)$$

Here q is the average heat generation at a distance z along the fuel rod, ∇q is the absolute value of the heat generation gradient $\vec{\nabla}q$ across the fuel rods, which is proportional to the global neutron flux gradient transverse to the z direction at the fuel fragment location in the core. Variables r and φ are the distance from the neutral axis and the angle measured relative to the x direction, respectively. The latter is chosen to coincide with the direction of vector $\vec{\nabla} q$ (see Fig.1).

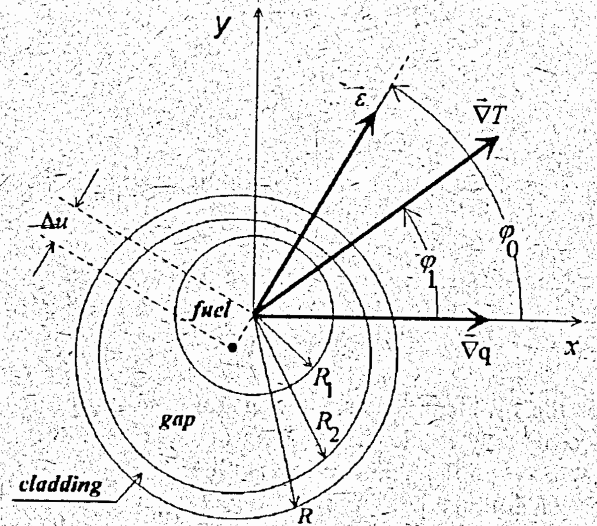

Figure 1. The initial vectors of asymmetry of a cylindrical fuel element.

2. The gap is small ($R_2 - R_1 \ll R_1$), and hence its heat transfer coefficient per unit length can be approximated by a cosine dependence of the form [9]

$$\frac{\alpha'}{1-\varepsilon\cos(\varphi-\varphi_0)}. \tag{8}$$

Here $\alpha' = 2\pi\lambda' / \ln(R_2/R_1)$ is the symmetrical heat transfer coefficient, $\varepsilon = \Delta u/(R_2 - R_1)$ is the value of eccentricity, where Δu is a fuel rod displacement.

Thus, the vector $\vec{\varepsilon}$ $(0 \le \varepsilon \le 1)$ aimed in the φ_0 - direction, characterizes the eccentric position of the fuel fragment inside the cladding (Fig.1).

The main assumption in setting the boundary-value problem is that the geometrically asymmetrical gap can be replaced by a symmetrical one but having asymmetrical heat transfer properties, according to expression (8). This assumption permits us to use a common coordinate system for both fuel and cladding.

3. Beyond the boundary layer, the coolant temperature can be represented by the cosine function

$$T(\varphi) = \Theta\cos(\varphi-\varphi_1) \tag{9}$$

i.e., we use only the first Fourier harmonic of the angular distribution, and set the average fluid temperature to zero. Parameter φ_1 characterizes the direction in which the temperature difference of the liquid in the channel reaches its maximum value, 2Θ , so $\vec{\Theta}$ is the vector of coolant temperature aimed in direction φ_1 (see Fig.1).

For the peripheral fuel elements in the subassembly, the cosine dependence of the coolant temperature has been confirmed experimentally [10-12]. For the aims of the bowing problem, it can be generalized to central fuel elements too, despite the fact that for central cells the main harmonic is, for example, the sixth harmonic for a triangular lattice or the fourth one in case of a quadratic lattice [11]. Higher-order harmonics do not contribute to bowing when α is constant.

The vector of coolant temperature per unit length across the channel gives the effective temperature gradient $\vec{\nabla} T$. Further, it will be useful to define it to include the coolant film thickness:

$$\vec{\nabla} T = \frac{\vec{\Theta}}{R+\lambda_2/\alpha}. \tag{10}$$

Thus, three vectors, namely $\vec{\nabla} q$ $\vec{\varepsilon}$ and $\vec{\nabla} T$, are the asymmetry factors of the problem characterising, respectively, the distortion of fission density across the fuel pin, eccentric arrangement of the fuel inside the cladding, and angular asymmetry of the coolant flow temperature. They depend on the axial distance z as on a parameter, and all of them are considered here as known vectors.

The equations for temperature distributions in the fuel pin and cladding are well known:

$$\Delta T_1(r,\varphi) = -\frac{1}{\lambda_1} q_v(r,\varphi); \tag{11}$$

$$\Delta T_2(r,\varphi)=0. \qquad (12)$$

The boundary conditions on the gap sides and on the outer surface of the cladding are as follows.

$$T_1(\varphi,R_1)-T_2(\varphi,R_2)=-2\pi R_1\lambda_1\frac{\partial T_1(\varphi,R_1)}{\partial r}\frac{1-\varepsilon\cos(\varphi-\varphi_0)}{\alpha'}, \qquad (13)$$

$$\lambda_1 R_1\frac{\partial T_2(\varphi,R_1)}{\partial r}=\lambda_2 R_2\frac{\partial T_2(\varphi,R_2)}{\partial r}, \qquad (14)$$

$$-\lambda_2\frac{\partial T_2(\varphi,R)}{\partial r}=\alpha\left[T_2(\varphi,R)-\Theta\cos(\varphi-\varphi_1)\right]. \qquad (15)$$

The Solution of the Eqs.(11-12) is found in the form [8]:

$$T_1(r,\varphi)=\Phi_1(r)+\sum_{n=1}^{\infty}(a_n\cos n\varphi+b_n\sin n\varphi), \qquad (16)$$

$$T_2(r,\varphi)=\Phi_2(r)+\sum_{n=1}^{\infty}(A_n\cos n\varphi+B_n\sin n\varphi), \qquad (17)$$

where

$$\Phi_1(r)=T_0-\frac{qr^2}{4\lambda_1}$$

and

$$\Phi_2(r)=\frac{qR_1^2}{2\lambda_2}(\ln\frac{R}{r}+\frac{\lambda_2}{\alpha R})$$

are the solutions of the corresponding symmetrical problem. The Fourier coefficients are the following functions:

$$a_n(r)=\begin{cases}c_{21}r-\nabla qr^3/8\lambda_1, & n=1,\\ c_{2n}r^n, & n>1;\end{cases} \qquad (18a)$$

$$b_n(r)=c_{1n}r^n; \qquad (18b)$$

$$A_n(r)=c_{5n}r^n+c_{6n}r^{-n}; \qquad (19a)$$

$$B_n(r)=c_{3n}r^n+c_{4n}r^{-n}. \qquad (19b)$$

The temperature at the fuel pin center T_0 and constants c_{in} ($i = 1, 2,\ldots,6$; $n \ge 1$) are determined from boundary conditions (13-15).

The presence of the cosine function on the right side of Eq.(13) does not permit one to calculate the six constants c_{in} separately for every fixed n. When $\varepsilon\neq 0$ all the constants are defined by the infinite system of equations. The task was solved by means of the following procedure. Because the series (16-17) are certainly convergent, at any rate for small ε, both of them can be interrupted at some number $n=N$ for obtaining the N-th approximation for the unknown constants T_0 and c_{in}. The exact solution is obtained as a limit at $N\to\infty$. The result of this procedure is listed below.

The coefficients c_{in} can be written in the form ($n \ge 1$):

$$c_{1n}=\frac{\nabla b_n}{nR_1^{n-1}}; \qquad (20a)$$

$$c_{2n}=\frac{\nabla a_n}{nR_1^{n-1}}+\frac{3\nabla q}{8\lambda_1}R_1^2\cdot\delta_{n1}; \qquad (20b)$$

$$c_{3n}=\frac{k_n}{1+k_n}\frac{\mu\nabla a_n}{nR_2^{n-1}}+\frac{\nabla T}{1+k_1}\sin\varphi_1\cdot\delta_{n1}; \qquad (20c)$$

$$c_{4n}=-\frac{R_2^{n+1}}{1+k_n}\left(\frac{\mu\nabla b_n}{n}-\nabla T\sin\varphi_i\cdot\delta_{n1}\right); \qquad (20d)$$

$$c_{5n}=\frac{k_n}{1+k_n}\frac{\mu\nabla a_n}{nR_2^{n-1}}+\frac{\nabla T}{1+k_1}\cos\varphi_1\cdot\delta_{n1}; \qquad (20e)$$

$$c_{6n}=-\frac{R_2^{n+1}}{1+k_n}\left(\frac{\mu\nabla a_n}{n}-\nabla T\cos\varphi_i\cdot\delta_{n1}\right). \qquad (20f)$$

In these equations ∇a_n and ∇b_n are the first derivatives of the a_n and b_n functions at the fuel rod boundary, $r = R_1$, $\mu=\lambda_1 R_1/\lambda_2 R_2$, δ_{ni} is a Kronecker delta and

$$k_n = \left(\frac{R_2}{R}\right)^{2n} \frac{\alpha R - n\lambda_2}{\alpha R + n\lambda_2} \tag{21}$$

The coefficients ∇a_n and ∇b_n are calculated with the help of the recurrent relations:

$$\nabla a_{n+1} = g_{n+1}(\nabla a_n \cos\varphi_0 - \nabla b_n \sin\varphi_0), \tag{22}$$

$$\nabla b_{n+1} = g_{n+1}(\nabla b_n \cos\varphi_0 + \nabla a_n \sin\varphi_0). \tag{23}$$

The coefficients g_n are defined by the continued fraction

$$g_n = \cfrac{\Gamma_n}{1 - \cfrac{\Gamma_n \Gamma_{n+1}}{1 - \cfrac{\Gamma_{n+1}\Gamma_{n+2}}{1 - \cdots}}}; \tag{24}$$

where

$$\Gamma_n = \frac{\pi n \lambda_1 \varepsilon}{(1+\gamma_n)\alpha' + 2\pi n \lambda_1}, \tag{25}$$

$$\gamma_n = \frac{\lambda_1}{\lambda_2} \frac{1-k_n}{1+k_n}. \tag{26}$$

It should be noted that in the limiting case

$$g_\infty = \frac{\varepsilon}{1+\sqrt{1-\varepsilon^2}}; \quad \Gamma_\infty = \varepsilon/2; \quad \gamma_\infty = \frac{\lambda_1}{\lambda_2}.$$

The first-step values in Eqs.(22-23) are calculated directly from the equations

$$(1+\gamma)\cdot\nabla b_1 = \\ = -\frac{q_z \varepsilon}{\alpha' R_1}\sin\varphi_0 + 2\frac{R_2/R_1}{1+k_1}\nabla T \sin\varphi_1$$

and

$$(1+\gamma)\cdot\nabla a_1 = -\frac{\nabla q R_1^2}{4\lambda_1} - \\ -\frac{q_z}{\alpha' R_1}\varepsilon\cos\varphi_0 + 2\frac{R_2/R_1}{1+k_1}\nabla T \cos\varphi_1$$

where k_1 is defined by Eq.(21), $q_z = q\pi R_1^2$ is the heat generation per unit length of the fuel rod, and

$$\gamma = \gamma_1 + \frac{2\pi\lambda_1}{\alpha_1}\left(1 - g_2(\varepsilon)\frac{\varepsilon}{2}\right) \tag{27}$$

is the parameter characterising the thermal "stiffness" of the fuel rod relative to asymmetrical thermal loads; its clearer meaning will be given later. Here we must note that γ depends on parameter g_2 (see Eq.(24)) which is a function of ε and is produced by the second Fourier harmonic. The value of γ_1 is determined by Eq.(26).

Thus, as the first parameters ∇a_1 and ∇b_1 are known, it is easy to calculate ∇a_n and ∇b_n by Eqs.(22) and (23), and therefore, all the coefficients c_{in} defined by Eqs.(20a-20f).

The last parameter, the temperature T_0 at the center of the fuel fragment, can be expressed relative to the average temperature of the fuel fragment $\langle T_1 \rangle$:

$$T_0 = \langle T_1 \rangle + q_z / 8\pi\lambda_1;$$

$$\langle T_1 \rangle = \frac{q_z}{\alpha_0} + \Delta T_0.$$

Here

$$\frac{1}{\alpha_0} = \frac{1}{8\pi\lambda_1} + \frac{1}{\alpha'} + \frac{1}{2\pi\lambda_2}\ln\frac{R}{R_2} + \frac{1}{2\pi R\alpha}$$

is the usual expression for the thermal resistance of the cylindrical fuel element; α_0 is the overall heat-transfer coefficient per unit length.

The additional term ΔT_0 is the "overheating" of the fuel rod caused by the factors of asymmetry. It is determined by the scalar product of the vectors of asymmetry:

$$\Delta T_0 = \frac{\pi\lambda_1}{\alpha'}\frac{\bar{\varepsilon}}{1+\gamma}\cdot\left(-\frac{q_z}{\alpha'}\bar{\varepsilon}-\frac{R^3}{4\lambda_1}\vec{\nabla}q+\frac{2R_2}{1+k_1}\vec{\nabla}T\right). \tag{28}$$

According to Eq.(28) the "overheating" effect is absent for symmetrical fuel elements, and is always negative when $\nabla q = \nabla T = 0$.

The problem has been solved. Sequences (16-17) converge very fast even at the point of fuel-cladding contact (when ε =1). The convergence is faster for smaller g_n. In particular, for a symmetrical arrangement of the fuel rod (ε=0, g_n=0) all higher-order harmonics (n>1) are absent, as well as the "overheating" of the fuel. It is clear from Eq.(28) that the fuel rod displacement by itself does not lead to overheating. On the contrary, it decreases the average fuel temperature because of a reduction of the overall thermal resistance due to the eccentricity ε.

The Approximate Estimation of the fuel peak temperature is given by [8]:

$$T_m \cong T_0 + qr_m^2/4\lambda_1,$$

where

$$r_m \cong \frac{2\lambda_1}{q}\sqrt{c_{11}^2+c_{21}^2}$$

is the radial peak location, the azimuth of which can be estimated by

$$\cos\varphi_m \cong c_{21}/\sqrt{c_{11}^2+c_{21}^2}.$$

For example, in the case of the IBR-2 central fuel element, provided full contact (ε = 1) at $\nabla q = \nabla T = 0$ and pin power of q_z = 135 W/cm, we should have

$$r_m/R_1 \cong \frac{2\pi\lambda_1}{\alpha'}\cdot\frac{\varepsilon}{1+\gamma} = 0.29;$$

$qr_m^2/4\lambda_1 = 36K;$
$\Delta T_0 = -50K;$
$T_0 = 525K$ (at zero sodium temperature).

The thermal stiffness $\gamma(\varepsilon)$ for IBR-2 fuel elements varies from $\gamma(1) = 0.36$ to $\gamma(0) = 0.43$.

The special case of the solved problem, namely symmetrical rods and uniform heat generation (ε=∇q=0), were considered in [10,11].

The Bowing Gradients. The exact solution of the boundary-value problem considered above can be used now for direct calculation of the effective bowing gradients for the fuel rod ($\vec{\nabla}T_1$) and cladding ($\vec{\nabla}T_2$) in accordance with definition (5).

If we introduce, by analogy to γ (see Eq.(27)), the thermal stiffness of the cladding Γ, defined by equation

$$\frac{1}{1+\Gamma} = \frac{\gamma_1}{1+\gamma}\left(1-\frac{\delta}{1-k_1}\right), \tag{29}$$

where

$$\delta \equiv \frac{R^2-R_2^2}{R^2+R_2^2}$$

is the definition of the dimensionless cladding thickness, the result of integration can be written in the form:

$$\vec{\nabla}T_1 = \frac{2}{1+\gamma}\cdot\frac{R_2}{R_1}\frac{\vec{\nabla}T}{1+k_1}+\frac{q_z}{1+\gamma}\left(-\frac{\bar{\varepsilon}}{\alpha' R_1}+\frac{1+7\gamma}{24\pi\lambda_1}\cdot\frac{\vec{\nabla}q}{q}\right); \tag{30}$$

$$\vec{\nabla}T_2(z) = \left(\frac{2\Gamma}{1+\Gamma}-\delta\right)\frac{\vec{\nabla}T}{1+k_1}+\frac{q_z}{1+\Gamma}\left(\frac{\bar{\varepsilon}}{\alpha' R_2}+\frac{R_1/R_2}{4\pi\lambda_1}\cdot\frac{\vec{\nabla}q}{q}\right). \tag{31}$$

Eqs.(30) and (31) show that the vector of eccentricity $\bar{\varepsilon}$, which determines the displacement of the fuel relative to the cladding, affects the fuel and cladding in opposite directions.

The thermal stiffness is a barrier which prevents the penetration of the temperature gradient (not temperature) into the body from the outside. A small-sized body with a good thermal conductivity must have a high thermal stiffness, and vice versa. For example, the IBR-2 clad-to-pin stiffness ratio is

$$\Gamma / \gamma = 81 / 0.43 \cong 200.$$

In the case of thin cladding, when $\delta \ll 1$ and $\delta \ll \alpha R / \lambda_2$, the stiffnesses γ and Γ can be determined by equations:

$$\frac{\gamma}{2\pi\lambda_1} \cong \frac{1}{\alpha'}\left(1 - g_2\frac{\varepsilon}{2}\right) + \frac{\delta}{2\pi\lambda_2} + \frac{1}{2\pi R\alpha}, \tag{32}$$

$$\frac{1}{1+\Gamma} = \frac{\lambda_1}{1+\gamma}\left(\frac{1}{\alpha R} + \frac{\delta}{2\lambda_2}\right) \tag{33}$$

Now the sense of the stiffness parameter becomes clear. Eq.(32) shows us that the rod stiffness γ is equal to a quarter of the ratio of the summary thermal resistance of the rod jackets (i.e. gap, cladding and boundary layer listed in the right-hand side of Eq.(32)) to the thermal resistance $1/8\pi\lambda_1$ of the rod itself.

THE EFFECTIVE GRADIENT METHOD

Now we shall calculate the bowing gradients with the help of omega operator defined by Eq.(1), i.e. evading the solution of the boundary-value problem (11-15).

Since vectorisation is an exact procedure for the linear equation of thermal conductivity (see Eqs.(3-4)) the accuracy of the method should be determined by the adequacy of the vectorised boundary conditions (13-15). The eccentricity ε of the fuel makes Eq.(13) the single source of such inaccuracy, provided α is constant.

With the help of the properties of the omega operator listed above the result of the action of the omega operator on Eqs.(11-15) can be obtained quite easily. The result for Eqs.(11-12) is almost immediate:

$$\Delta\vec{\Theta}_1 - \frac{1}{r^2}\vec{\Theta}_1(r) = -\frac{1}{\lambda_1}\vec{Q}(r); \tag{34}$$

$$\Delta\vec{\Theta}_2 - \frac{1}{r^2}\vec{\Theta}_2(r) = 0. \tag{35}$$

Here $\vec{\Theta}_1$, $\vec{\Theta}_2$ and $\vec{Q}$ are the vectors (or "first" vectors) of the functions T_1, T_2, and q_v, respectively.

The boundary conditions can be vectorised in an exact form if we may use the g_2 factor (Eq. (24)) and the relations (22-23) of the direct theory. Then, the action of omega operator on the Eqs.(13-15) will give the vectorised conditions in the form

$$\vec{\Theta}_1(R_1) - \vec{\Theta}_2(R_2) = -\frac{2\pi R_1\lambda_1}{\alpha'} \times \left\{\left(1 - g_2\frac{\varepsilon}{2}\right)\nabla\vec{\Theta}_1(R_1) - \vec{\varepsilon}\nabla\Phi_1(R_1)\right\}; \tag{36}$$

$$\lambda_1 R_1 \nabla\vec{\Theta}_1(R_1) = \lambda_2 R_2 \nabla\vec{\Theta}_2(R_2); \tag{37}$$

$$-\lambda_2\nabla\vec{\Theta}_2(R) = \alpha\left[\vec{\Theta}_2(R) - \vec{\Theta}\right]; \tag{38}$$

where ∇ is a denotion for the first derivative at a boundary in Eqs.(13-15). The film heat transfer coefficient α is a constant as formerly, but q_v and T represent arbitrary functions. Note that the vectorised boundary-value problem (Eqs.(34-38)) can be easily generalised for the time-dependent case.

In the case of Eq.(7) for the q_v function, we obtain a simple vector of power generation

$$\vec{Q} = r\vec{\nabla}q,$$

and a simple solution to Eqs.(34-35):

$$\vec{\Theta}_1(r) = \vec{C}_1 r - \frac{\vec{\nabla}q}{8\lambda_1}r^3$$

$$\vec{\Theta}_2(r) = \vec{C}_2 r + \vec{C}_3 / r,$$

where C vectors can be easily found from Eqs.(36-38). The substitution of these equations into the bowing gradients

$$\vec{\nabla}T_i = \pi I_i^{-1}\int \vec{\Theta}_i(r)\; r^2 dr$$

for the fuel rod (i = 1) and cladding (i = 2) gives expressions

$$\vec{\nabla}T_1 = \vec{C}_1 - \frac{\vec{\nabla}qR_1^2}{12\lambda_1},$$

$$\vec{\nabla}T_2 = \vec{C}_2 - \frac{2\vec{C}_3}{R^2 + R_2^2},$$

that become exactly the same as Eqs.(30-31) after calculation of the vector constants.

Estimation of the Accuracy of the Method. The vectorized problem (34-38) is an exact one owing to the presence of the g_2 factor in Eq.(36). That happened because Eqs.(22-23) make the "second" vector of the T_1 function (turned to the angle $-\varphi_0$) to be proportional to its (first) vector $\vec{\Theta}_1$ with g_2 as the coefficient of proportionality.

Being the product of exact theory, the g_2 parameter may be considered here as an unknown value that makes the fuel rod thermal stiffness $\gamma(\varepsilon)$ (see Eq.(27) or (32)) uncertain.

The upper limit of this uncertainty is

$$\frac{\gamma(0)-\gamma(\varepsilon)}{\gamma(0)} < \frac{g_2(\varepsilon)\varepsilon/2}{1+\alpha'\left(\dfrac{\delta}{2\pi\lambda_2}+\dfrac{1}{2\pi\alpha R}\right)}.$$

The uncertainty may be appreciable for gas bonded fuel elements (small α'). For the helium bonded fuel elements of the IBR-2 reactor, $1 - \gamma(1)/\gamma(0) = 0.16$.

Angular Dependence of Alpha. By analogy to Eq.(8) for α', one may suppose a similar dependence for the film transfer coefficient:

$$\frac{\alpha}{1-e\cos(\varphi-\varphi_2)}.$$

Here α is the former constant, e is the eccentricity of the coolant cell. Then, boundary condition (38) must be replaced by

$$-\lambda_2\left\{\left(1-\frac{h_2}{2}\right)\nabla\vec{\Theta}_2(R) - \vec{e}\,\nabla\Phi_2(R)\right\} = \alpha\left[\vec{\Theta}_2(R) - \vec{\Theta}\right]$$

where $\vec{e}$ is now the forth factor of asymmetry, and $h_2(e)$ is an unknown parameter, the second source of uncertainty. For a high heat conduction of the coolant ($\lambda_2/\alpha R << 1$) $h_2 \approx e\lambda_2/\alpha R$, and hence this parameter can be omitted ($\lambda_2/\alpha R = 0.03$ for the sodium cooled IBR-2 reactor).

In order to take into account the α variation for the calculation of bowing gradients, the replacements

$$\vec{\Theta} \to \vec{\Theta} - \frac{q_z}{2\pi\alpha R}\vec{e}, \qquad \text{and} \qquad \alpha \to \frac{\alpha}{1-h_2 e/2}$$

must be made in Eqs.(30-31) and in the definitions of k_1 (Eq.(21)), γ and Γ (Eq.(27) or (32), and Eq.(29) or (33)).

THERMOHYDRAULICS

The Thermo-Hydraulic Vectors of Asymmetry. Up to now we considered the effective temperature gradient $\vec{\nabla}T$ of the coolant as a known vector. In reality, it depends on the distortion of the cladding temperature, which is dependent on all the factors of asymmetry, including $\vec{\nabla}T$ itself, and, on the other hand, upon the hydrodynamic asymmetry of the coolant flow in the channel. Both factors can be represented by two vectors [8]: the vector of asymmetry of the heat flux at the cladding surface ($\vec{J}$), and the vector of asymmetry of the coolant flow across the coolant channel ($\vec{G}$). The values of the vectors are normalised relative to the overall linear heat flux (q_z) and coolant mass flow (g) per fuel element.

The heat propagates from the cladding surface to the coolant cell at $r = R$. Let us divide the cell into two parts, 1 and 2, by a separating line along the cladding diameter and find the flux and flow differences. There are two orientations of the line at which the differences between average fluxes $\langle J\rangle = (J_1 - J_2)$ and flows $\langle G\rangle = (G_1 - G_2)$ in the subcells attain maximum values. Then, the vectors of heat flux and coolant flow functions can be defined (according to [8] and [13]) as:

$$J = \frac{\pi}{2}\langle J \rangle = -\frac{2\pi R}{q_z}\lambda_2 \nabla\bar{\Theta}_2(R), \tag{39}$$

$$G = \frac{\pi}{2}\langle G \rangle = \frac{\pi}{2}\left[\frac{p_1}{p}\left(\frac{d_1}{d}\right)^{\frac{3}{2-m}} - \frac{p_2}{p}\left(\frac{d_2}{d}\right)^{\frac{3}{2-m}}\right].$$

Here p_i and d_i are the hydraulic perimeter and equivalent diameter of the subcell, i; p and $d = 4s/p$ are the same values for cell. Parameter m depends on the Reynolds criterium for the subcell: $m = 1$ for laminar flow, $m = 0.25$ for turbulent flow ($10^4 < Re < 10^5$), and $m = 0$ when $Re > 10^5$ [12].

The vector $\vec{G}$ is aimed from the "small" subcell (1) to the "big" one (2) (In ref.[8] the opposite direction for $\vec{G}$ was chosen, and the $\vec{G}$ vector itself was treated as the average vector). The cell and subcell parameters depend on the cladding position relative to the positions of adjacent fuel elements. The approximate dependence of $\vec{G}$ on the cladding displacement $\vec{u}_2$ can be represented by [8]:

$$\vec{G} = \vec{G}_0 - \frac{3\pi}{2-m}\cdot\frac{h}{2s}(\vec{u}_2 - \vec{u}_c). \tag{40}$$

Here $\vec{G}_0$ is the initial hydraulic asymmetry of the cell, h is the pitch of the lattice, s is the coolant cell cross section and $\vec{u}_c$ is the displacement of the cell itself due to the bowing of adjacent fuel elements. Vector $\vec{u}_c$ characterises the new position of the hydraulic center of the cell (where $\vec{G} = 0$) and is determined by the summary displacement of the adjacent fuel elements.

Correlation with the Vector of Coolant Temperature. The action of the omega operator on the elementary angular dependent equation of the heat-balance in the coolant cell gives, with the help of Eq.(39), the approximate equation for the vector of coolant temperature:

$$\frac{d\vec{\Theta}}{dz} + \lambda\vec{\Theta} = \frac{q_z}{c_p g}\frac{1}{1-G^2} \times \left[-\vec{G} + \frac{\pi\lambda_1}{1+\gamma}\left(\frac{\vec{\varepsilon}}{\alpha'} + \frac{R_1\vec{\nabla}q}{4\pi\lambda_1 q}\right)\right] \tag{41}$$

in which $g = \rho v s$ is the coolant mass flow through the cell, c_p is the special heat capacity of the coolant, and

$$\lambda = \frac{1}{c_p g}\cdot\frac{1}{1-G^2}\cdot\frac{2\pi}{1+\gamma} \times \left\{\lambda_1 + \lambda_2\delta\left[1 + 2\pi\frac{\lambda_1}{\alpha_2}\left(1 - g_2\frac{\varepsilon}{2}\right)\right]\right\} \tag{42}$$

is the reciprocal relaxation length due to heat transfer across the fuel element.

Because the relaxation length is of the order of $2dPe$ [8], where Pe is the Peclet criterium (the ratio of $\rho c_p v d$ and thermal conductivity) the relaxation mechanism has to be taken into account in the constant-heat-flux measurements [12,13] at low flow rates (practically, at $Pe < 100$).

In the special case (eccentric annular cell, $\lambda = 0$, $\vec{\nabla}q = \vec{\varepsilon} = 0$, $m = 0.25$) Eq.(41) coincides with the solution of [13], which also has a singularity at $G = 1$. However, the accuracy analysis of Eq.(41) and its subcell representation indicate that at high hydraulic asymmetries (practically at $G^2 > 0.5$) the use of $\langle G \rangle^2$ instead of G^2 in the denominators of Eqs.(41-42) becomes more preferable.

Thus, the equations of thermoelasticity (Eq.(6) is the simplest case) for the displacements of the fuel rod ($\vec{u}_1$) and cladding ($\vec{u}_2$), the obvious relation for the fuel eccentricity

$$(R_2 - R_1)\,\vec{\varepsilon} = \vec{u}_1 - \vec{u}_2,$$

Eqs.(30-31) for the bowing gradients, Eq.(10) and (41) for $\vec{\nabla}T$ and $\vec{\Theta}$, as well as Eq.(40) for $\vec{G}$, represent the closed set of coupled equations for estimation of single fuel element bowing. The displacement $\vec{u}_c$ of the hydraulic coolant-cell center couples the behaviour of the fuel element with the behaviour of other fuel elements within the subassembly.

The system of equations becomes linear when G^2 and $g_2\varepsilon/2$ are negligible terms. The additional equation (and an additional source of nonlinearity) appears when the α variation should be taken into account.

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Ломидзе В.Л. E3-95-509

Метод эффективных градиентов в задаче об искривлении тепловыделяющих элементов ядерного реактора

Предложена аналитическая модель искривления тепловыделяющих элементов ядерного реактора, учитывающая три основных фактора изгиба: (1) неравномерность плотности делений по азимуту топливного сердечника, (2) эксцентричное расположение сердечника внутри оболочки твэла и (3) азимутальную зависимость температуры теплоносителя, омывающего твэл. Суммарное воздействие трех указанных факторов есть некий вектор (изгибающий температурный градиент), непосредственно определяющий характер искривления твэлов. Разработанный метод позволяет свести стационарную задачу к системе обыкновенных дифференциальных уравнений и сравнительно просто рассчитать характер изгиба любого твэла в тепловыделяющей сборке.

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Lomidze V.L. E3-95-509

The Effective Gradient Method in the Fuel Bowing Problem

An analytical model of reactor fuel element bowing, caused by three main factors: (1) nonuniform fission density across the fuel pin, (2) eccentric arrangement of the pin inside the cladding and (3) nonsymmetrical angular dependence of the cooling temperature, is suggested. The summary action of the factors mentioned gives some vector (bowing temperature gradient) which directly determines the character of the bowing of the fuel elements. The system of ordinary equations obtained allows one to calculate, comparatively easily, the bowing of any element contained in a fuel assembly.

The investigation has been performed at the Frank Laboratory of Neutron Physics, JINR.

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