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A.I.Frank, D.B.Amandzholova

NEUTRON QUANTUM REFLECTION

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1 Introduction

The problem of non-stationary quantum effects in neutron optics has lately been the subject of extensive discussions in the literature. Moshinsky was, apparently, one of the first persons to approach this problem in 1952 [1]. He considered the evolution of a neutron wave upon instantaneous extraction of a perfect absorber from a beam of monochromatic neutrons. Ya.B.Zel'dovich analyzed the general problem of periodic processes in 1966 and introduced the concept of quasi-energy [2] . The more this issue is dealt with, the more it becomes evident that precisely that publication points to the most effective strategy of experimental investigation of non-stationary processes in neutron optics [3].

In 1976 A.S.Gerasimov and M.V.Kazarnovsky [4] analyzed a series of non-stationary quantum phenomena, which, in principle, can be observed in experiments with ultracold neutrons (UCN). Among the problems considered was the problem of UCN reflection from a weakly oscillating potential. The present article is directly related to this work. Below, we present a simple solution of the problem indicated and discuss a quite realistic experiment for observing a novel quantum phenomenon. Our aim was not to present, here, a detailed review of available theoretical results, especially that, to a certain extent, this has been done in [5]. Nevertheless, we shall draw attention to an experiment [6], performed not long ago, in which quantum effects occurring, when neutrons are reflected from a vibrating surface, were observed. This problem was also dealt with in [4]. Fig. 1 illustrates the difference in formulation between the problems of neutron scattering from a vibrating surface and from an oscillating potential.

2 Exertion of a periodic influence on a neutron wave

In [2] a new physical characteristic, quasi-energy, is shown to arise and permit quite a clear formulation, when the Hamiltonian exhibits an ex-

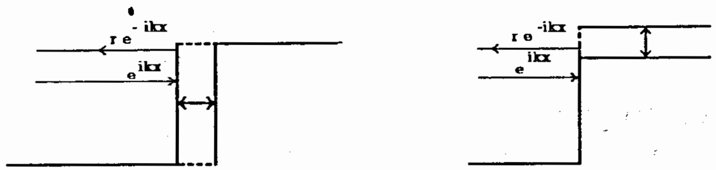

Figure 1: Reflection from a vibrating mirror and a mirror with an oscillating potential

plicit periodic time dependence.

By definition quasi-energy is ambiguous. It may be supplemented with any integer multiple of the quantity

$$\Delta E = \frac{2\pi\hbar}{T}, \tag{1}$$

where T is the period. A concrete case of such a phenomenon has been considered in [3],[7]. The problem considered therein is related to the periodic influence of a perfect chopper, which after a time of $T/2$ instantaneously cuts off and then opens, an initially monochromatic neutron beam. On the basis of the Moshinsky solution [1] and the principle of superposition, the following wave function is found in [3] for a periodic chopper:

$$\psi(x,t) = \frac{1}{2}e^{i(kx-\omega t)} + \frac{i}{\pi}\sum_{n=-\infty}^{\infty}\frac{e^{i(k_n x-\omega_n t)}}{2n-1}, \tag{2}$$

$$\omega_n = \omega + \frac{2\pi(2n-1)}{T}; \qquad k_n = \left(\frac{2m\omega_n}{\hbar}\right)^{1/2}. \tag{3}$$

Thus, the state in the semi-space to the right represents a non-stationary superposition of waves, each of which has an energy $\hbar\omega_n$ and a corresponding wave number k_n. These equidistant satellites are related correspond, together with the non-shifted line ω, to a sole particle quasi-energy. It is to be noted that the amplitudes of the satellites are Fourier coefficients of a rectangular function characterizing the influence of the chopper.

The relationship with the Fourier transform becomes even more evident, when the problem is being solved of the diffraction of monochromatic neutrons on a moving grating [8],[5]. Solution of the problem of neutron diffraction in a moving reference system connected with the grating reduces to Fourier transformation of the coordinate part of the wave function. The wave function in the laboratory reference system is found by subsequent application of the Galilean transformation. In the limit, when the grating has a high velocity V and its period is $2a$, the value of $T = 2a/V$ remaining constant, one readily arrives at the formula (2),(3), given above. In the same way was a solution found, also, for the phase grating.

It has been shown in [9],[5] that a solution can be found without invoking the precise solution of the Schrödinger equation with the corresponding time-dependent potential, in the case of a monochromatic neutron beam under periodic influence. Consider the action of some device located at the origin of the reference system being a periodic variation of the amplitude or phase of the initial plane wave. Then, at small distances from this device the wave function will have the form:

$$\psi(x,t) \cong f(t)e^{i(kx-\omega t)}, \qquad k^{-1} < x \ll vT \tag{4}$$

where $f(t)$, generally, is a complex function of period T, and v is the neutron velocity. Representing $f(t)$ in the form of a Fourier expansion over the frequencies $n\Omega$,

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{-in\Omega t}, \qquad \Omega = \frac{2\pi}{T}, \tag{5}$$

we obtain for $x > 0$:

$$\psi(x,t) = \sum_{n=-\infty}^{\infty} C_n e^{i(kx-\omega t)} e^{i(k_n-k)x} e^{-in\Omega t} = \sum_{n=-\infty}^{\infty} C_n e^{i(k_n x-\omega_n t)}, \tag{6}$$

where

$$\omega_n = \omega + n\Omega, \qquad k_n = k(1+n\gamma)^{1/2}, \qquad \gamma = \Omega/\omega \ll 1. \tag{7}$$

We shall term the function $f(t)$ the modulation function. It is readily seen that in the case of an absorbing chopper and a rectangular modulation function we arrive at expression (2).

3 Neutron reflection from an oscillating potential

The above arguments make it easy to obtain a solution of the problem for neutrons reflected from an oscillating potential barrier. In the stationary case, the amplitude r of the wave reflected from a potential U is given by the usual solution of the stationary Schrödinger equation. In the case of a time-dependent potential the amplitude $r(t)$ is also readily found by formal substitution of the quantity $U(t)$ into the corresponding expression for the amplitude. The state characterizing the reflected wave is a superposition of coherent waves, the amplitudes of which are the Fourier coefficients of the function $r(t)$.

Now consider the most simple, although important from a practical point of view, example of the reflection of neutrons from a mirror characterized by a variable potential. Its constant part is just the optical potential

$$U_{opt} = \frac{2\pi\hbar^2}{m} Nb, \tag{8}$$

where N is the nuclear density, b is the coherent scattering length, and m is the neutron mass. It is natural to assume the time-dependent part of the potential $u(t)$ to be determined by the interaction related to the spin orientation. In the simplest case this is just the magnetic interaction [4]:

$$u(t) = [\gamma(\vec{\sigma}\vec{B})], \tag{9}$$

where γ is the neutron gyromagnetic ratio, $\vec{\sigma}$ is the spin operator, $\vec{B}$ is the magnetic induction of the material of which the mirror is made. The time dependence may be either related to rapid changes of spin orientation or, in the case of a ferro-magnetic mirror, to remagnetization of the material of the mirror. We have:

$$r(t) = \frac{\sqrt{E} - \sqrt{E - U_{opt} - u(t)}}{\sqrt{E} + \sqrt{E - U_{opt} - u(t)}}, \tag{10}$$

where E is the neutron energy. Hence, the wave function of the state corresponding to reflected waves is

$$\psi_r(x,t) = \sum_{n=-\infty}^{\infty} C_n e^{i(k_n x - \omega_n t)}, \tag{11}$$

$$C_n = \frac{1}{T}\int_0^T r(t) e^{in\Omega t} dt. \tag{12}$$

Hitherto we implicitly assumed the neutrons to be normally incident upon the medium. In this case, noticeable neutron reflection occurs, if the neutron energy E is at least not too high, as compared with the optical potential, i.e. in the case of UCN. The above calculations and arguments are, naturally, also valid in the case of thermal or cold neutrons exhibiting grazing incidence on the mirror. In this case the normal component $k_\perp$ and the quantity

$$E_p = \frac{\hbar^2}{2m} k_\perp^2.$$

must be substituted for the wave number k and the energy E, respectively, in all the above expressions. Thus, in accordance with (6),(7) the reflection results in a set of reflected waves with differing normal wave number components

$$k_{\perp n} = k_\perp \left[1 + n\left(\frac{2m\Omega}{\hbar k_\perp^2}\right)\right]^{1/2}, \tag{13}$$

where $k_\perp$ corresponds to the wave number of the incident wave.

Invariance with respect to translation along the surface of the mirror obviously results in the longitudinal component of the wave vector not changing, when reflection occurs. Consequently, reflected waves corresponding to various satellites will exhibit different reflection angles and energies. The picture that arises can be illustrated by Fig.2.

Figure 2: Quantum reflection

It seems extremely similar to diffraction on a plane grating. This similarity, however, is purely superficial. Ordinary spatial diffraction is determined by mutual Fourier transformations of coordinates and wave numbers, which is reflected in the terminology, where the concepts of quasi-momentum and inverse lattice vector are usual. Correspondingly, the difference between the wave vectors of any adjacent diffractive arrangements represents the inverse lattice vector. In the case considered, the adjoined variables of the Fourier transformation are time and frequency (and, consequently, energy, also). All the components of the wave pattern of a state resulting from reflection pertain to a sole quasi-energy. The energy of adjacent satellites differs by the quantity $\hbar\Omega$, while the difference between their wave numbers is essentially dependent upon the order n. We shall term this phenomenon "quantum reflection".

4 The possibility of experimental observation

The concrete form of the dependence $u(t)$ and, consequently, of $r(t)$ is determined by the experimental conditions. For definiteness and for simplifying the calculations we shall deal with the case of a ferromagnetic mirror magnetized up to saturation and assume the orientation of magnetization to change after a time $T/2$ and the remagnetization time to be significantly shorter, than T. Then the quantity $r(t)$ is constant during a semi-period and is determined by the expression:

$$r_{\pm}(t) = \frac{\sqrt{E_p} - \sqrt{E_p - U_{opt} \pm \mu B}}{\sqrt{E_p} + \sqrt{E_p - U_{opt} \pm \mu B}}, \tag{14}$$

where μ is the magnetic moment of the neutron. In this case, the calculation of the Fourier coefficients in (12) becomes trivial, and the intensities of the respective partial waves are given by the relations:

$$|C_0|^2 = \frac{|r_+ + r_-|^2}{4}, \quad |C_n|^2 = \frac{|r_+ - r_-|^2}{\pi^2(2n-1)^2}. \tag{15}$$

The intensities of reflected waves of the zeroth (specular reflection) and first orders are plotted in Fig.3 versus the normal component of the incident neutron velocity. The latter is represented in arbitrary units $v_\perp/v_b$, where

$$v_b = \sqrt{2U_{opt}/m}.$$

A typical value of U_{opt}=150neV is assumed for the optical potential.

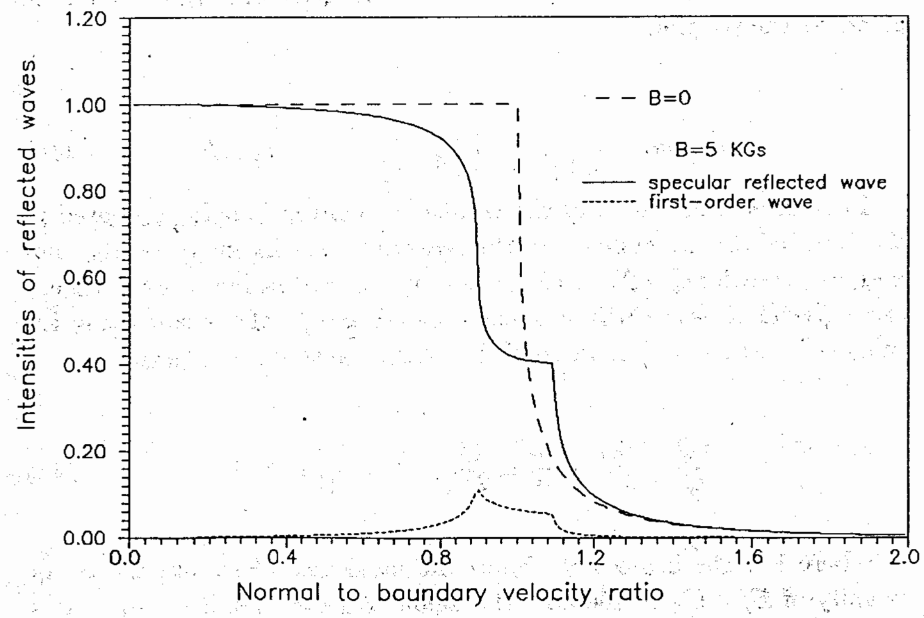

Figure 3: The intensities of reflected waves

As one can see from (15), the intensities of waves of higher orders just decrease as the squares of odd numbers, so they are not presented in the plot. We note that the region, where the intensity of first-order waves rises, corresponds to neutron energies lower than the potential barrier. In this case, total external reflection occurs, $|r_+|^2 = |r_-|^2 = 1$, and the quantum effects are only related to phase modulation of the reflected wave.

As it was to be expected, the intensity of satellites depends on the magnetic induction, i.e. on the depth of modulation. The intensity of the closest satellites amounts to ten-twenty percent in the case of a relatively low (2-3Kgauss) magnetic induction of the medium, so observation of the phenomenon will present no difficulty, if the usual technique of neutron reflectometry is applied. Of course, it is, then, necessary to provide the conditions for separation in space of the specular component (zeroth order) from the satellites. The angular distribution of reflected waves is given by the relation

$$\alpha \approx tg\alpha = \frac{k_\perp}{k\parallel}\left[1+n\left(\frac{2m\Omega}{\hbar k_\perp^2}\right)\right]^{1/2}, \qquad k_\parallel \approx k, \tag{16}$$

In neutron reflectometry the results are conventionally presented in the form of the dependence of the reflected wave intensity on the momentum transfer $Q \simeq 2k_\perp$. Consequently, for estimating to what extent the experiment is realistic, one can take advantage of the resolution expressed in terms of Q, instead of the angular resolution. Then,

$$\frac{\delta Q}{Q} \simeq \frac{\delta k_\perp}{k_\perp} \simeq \frac{\delta\alpha}{\alpha} \simeq \frac{\hbar\Omega}{2E_p}, \quad \Omega = \frac{2\pi}{T} = 2\pi f, \tag{17}$$

where f is the frequency. Since the maximum effect occurs in the vicinity of $E_p \approx U_{opt} \approx 150neV$, the following resolution is required at a remagnetization (or polarization reversal) frequency 5MHz, as one can see from (17):

$$\delta Q/Q \simeq 0.07,$$

which is quite within the range of resolutions exhibited by ordinary reflectormeters.

5 Discussion

Experimental observation of a new quantum effect, such as, for instance, the quantum reflection of neutrons considered above, is evidently of great interest in its own virtue. One may, however, try to foresee the possible applications of the said effect. First of all, we note that quantum reflection results in coherent separation of the beams. In essence, this gives rise to the possibility of creating a neutron interferometer based on a new principle. The arrangement of this device may be quite similar to that of an interferometer based on diffraction gratings [10]. Such a proposal has already been discussed in connection with the observed division of a wave undergoing reflection from a vibrating surface [11]. We note that the authors of the latter reference claim that such interferometers, being based on non-stationary quantum phenomena, may, in principle, provide an experimental answer to the question, whether a neutron wave packet is a superposition of states or only their non-coherent mixture. It has been shown, previously, that conventional interferometers are essentially not capable of distinguishing between these two possibilities [12].

Quantum reflection may turn out to be useful for the development of new techniques for magnetic studies in neutron reflectometry. We note that in such studies it has recently become traditional to measure the spin dependence of the reflection curve, or of the so-called polarization ratio. However, quite often the difference between the reflection coefficients of two spin components is insignificant, so a comparatively small effect has to be measured. Dealing only with satellites, the origin of which is itself related to the difference between the reflection amplitudes of spin components, may lead to an essential improvement of the situation.

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Франк А.И., Аманджолова Д.Б. E3-93-418
Квантовое отражение нейтронов

Рассмотрена задача отражения нейтронов от вещества с осциллирующим потенциалом. Состояние отраженной волны представляет собой суперпозицию когерентных волн с различающимися нормальными компонентами волновых чисел и энергиями. При отражении тепловых и холодных нейтронов эти волны разделяются в пространстве. Обсуждается возможность эксперимента по наблюдению этого явления и возможные применения в нейтронной рефлектометрии и интерферометрии.

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Frank A.I., Amandzholova D.B. E3-93-418
Neutron Quantum Reflection

The problem of neutron reflection from a medium with an oscillating potential is considered. The reflected wave state represents a superposition of coherent waves of various normal wave number components and energies. When thermal and cold neutrons are reflected, these waves are separated in space. Possible experimental observation of this phenomenon and, also, possible applications in reflectometry and interferometry are discussed.

The investigation has been performed at the Frank Laboratory of Neutron Physics, JINR.

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