92-484

сообщения
объединенного
института
ядерных
исследований
дубна

E3-92-484

Yu.A.Alexandrov

DOES THE NEUTRON-ELECTRON SCATTERING LENGTH CHANGE WITH THE INTERRESONANCE INTERFERENCE TAKEN INTO ACCOUNT?

1992

Earlier, the analysis of the experimental data [1] led to the neutron-electron scattering length value in support of that obtained in the diffraction experiment on tungsten [2]. Their average is

$$a_{ne} = (-1.59 \pm 0.04) \times 10^{-3}\ \text{fm}, \qquad (1)$$

which gives the mean square radius of electric charge distribution inside the neutron

$$<r^2_{in}>_N = 3\hbar^2(a_{ne} - a_F) / (Me^2) < 0, \qquad (2)$$

where $a_F = \mu_n e^2 / (2Mc^2) = -1.468\times10^{-3}$ fm is the Foldy's term.

The above value of a_{ne} is in contradiction with the result of [3,4]:

$$a_{ne} = (-1.32\pm0.03) \times 10^{-3}\ \text{fm}, \qquad (3)$$

from which it follows that

$$<r^2_{in}>_N > 0. \qquad (4)$$

Then the question arises: What is the $<r^2_{in}>_N$ still equal to?

First, allow me three remarks:

1) In [1] into σ_{tot} only corrections for Schwinger scattering and solid state effects (making together not more than 0.8%) were introduced. This means that the value of $y = \sigma_{tot}(E) / (4\pi) - a^2_{coh}$ (a_{coh} is the coherent scattering length), being analysed in [1], in the case of bismuth must not turn into zero at $E = 0$ as the authors of [5], criticizing [1], would like to. In the expression for y, at E=0, at least terms due to incoherent scattering on bismuth must remain.

2) In [1] the expression for y contains independent of energy terms p_2 which were varied to achieve better description of the experimental data. These terms make about 15% of the y value.

3) Because in [1] the data are analysed on the basis of the S-matrix of the form

$$S_{nn} = \exp(2i\,\delta_{pot})\left[1 - i\sum_j \Gamma_{nj} / (E - E_j + i\,\Gamma_j / 2)\right], \qquad (5)$$

where δ_{pot} is the potential nuclear scattering phase, Γ_{nj} and Γ_j are the neutron and total widths of the j-th resonance with the energy E_j, the relation

$$\sigma_{tot} = 2\pi g\,(1 - Re S_{nn}) / k^2 \qquad (6)$$

does not contain interresonance interference terms, which is quite natural.

Let us now try to take account of interresonance interference. Terms appear due to it in the expression for σ_{tot}, if one takes S-matrix in the form [6]*

$$S_{nn} = \exp(2i\,\delta_{pot})\left[1 + i\sum_j (\alpha_{nj} + i\beta_{nj})/(\mu_j - E - i\,\nu_j)\right], \qquad (7)$$

where

$$\sum_j (\alpha_{nj} + \beta_{nj}) = \sum_j \Gamma_{nj},\ \sum_j \beta_{nj} = 0,\ \mu_j = \text{Re}\,\tilde{E}_j,\ \nu_j = -\text{Im}\,\tilde{E}_j,\ \delta_{pot} = -kR,$$

$\tilde{E}_j$ is the complex energy of j-th resonance (at $\beta_{nj} = 0$ $\tilde{E}_j = E_j - i\,\Gamma_j/2$). At $\beta_{nj} = 0$ we have σ_{tot} as the sum of Breit—Winger's terms taking into account only interference between the potential and resonance scattering. In general, calculations using the many-level expression for the σ_{tot} on the basis of S-matrix (7) are rather difficult. In [8] from exps. (6) and (7) there was deduced the two-level formula for σ_{tot} in the S-matrix formalism:

$$\sigma_{tot} = \sigma_{pot} + 2\pi g/k^2[(G_1\nu_1 + H_1(\mu_1 - E)) / ((\mu_i - E)^2 + \nu_1^2) +$$
$$+ (G_2\nu_2 + H_2(\mu_2 - E)) / ((\mu_2 - E)^2 + \nu_2^2)], \qquad (8)$$

where

$$G_1 = \alpha_{1n}\cos2\delta_{pot} - \beta_n\sin2\delta_{pot},\ G_2 = \alpha_{2n}\cos2\delta_{pot} + \beta_n\sin2\delta_{pot},$$
$$H_1 = \beta_n\cos2\delta_{pot} + \alpha_{1n}\sin2\delta_{pot},\ H_2 = -\beta_n\cos2\delta_{pot} + \alpha_{2n}\sin2\delta_{pot},$$
$$\alpha_{1n} + i\beta_n = \tilde{\Gamma}_{1n},\ \alpha_{2n} - i\beta_n = \tilde{\Gamma}_{2n},$$
$$2E_{1,2} = E_1 - i\,\Gamma_1/2 + E_2 - i\,\Gamma_2/2 \mp$$
$$\mp[(E_2 - i\,\Gamma_2/2 - E_1 + i\,\Gamma_1/2)^2 - \Gamma^2_{12}]^{1/2},$$
$$\tilde{\Gamma}_{1n} = \Gamma_{1n}\cos^2\varphi + \Gamma_{2n}\sin^2\varphi + \Gamma^{1/2}_{1n}\Gamma^{1/2}_{2n}\sin2\varphi,$$
$$\tilde{\Gamma}_{2n} = \Gamma_{2n}\cos^2\varphi + \Gamma_{1n}\sin^2\varphi - \Gamma^{1/2}_{1n}\Gamma^{1/2}_{2n}\sin2\varphi,$$
$$B = i\,\Gamma_{12}\,[(E_2 - E_1 - i\,\Gamma_2/2 + i\,\Gamma_1/2)^2 - \Gamma^2_{12}]^{-1/2},$$
$$\varphi = \arcsin(B/2),$$

*Attempts made in [5] to choose appropriate S-matrix form are in contradiction with the generally accepted idea of it (see also [7], for example).

E_1 and E_2 are the energies of resonances and Γ_1, Γ_2, Γ_{1n} and Γ_{2n} are the widths of resonances, $\Gamma_{12} = (\Gamma_1 \Gamma_2)^{1/2}$ [8] is the interresonance width, in our case being $\Gamma_{12}^2 << (E_2 - E_1)^2$.

The calculation results of interresonance interference terms by formula (8) for bismuth (resonances $E_1 = 800$ eV and $E_2 = 2310$ eV) and lead-208 ($E_1 = 507$ keV and $E_2 = 1735$ keV) are summarized in the following Table.

Table

	Bismuth (E_1=800 eV, E_2=2310 eV)			Lead-208 (E_1=507 keV, E_2=1735 keV)	
Energy E, eV	1	16	50	1000	25000
$\Delta\sigma_{int}, 10^{-27}$cm^2	±20.7	±20.0	±20.5	±4.9	±4.2

As it follows from the Table $\Delta\sigma_{int}$ is practically independent of energy and its value makes about 0.2% σ_{tot} for bismuth and 0.04% σ_{tot} for lead-208. Its sign depends on the choice of the sign before the square root of Γ_{1n}, Γ_{2n}, B and Γ_{12}.

Another S-matrix form can be used, e.g., from [9,10]:

$$S_{cc'} = \exp(i\,\delta_{pot,c} + i\,\delta_{pot,c'})\left[\delta_{cc'} + i\sum_{\lambda\lambda'} (\Gamma_{\lambda c})^{1/2} (\Gamma_{\lambda' c'})^{1/2} A_{\lambda\lambda'}\right], \qquad (9)$$

where $\delta_{cc'}$ is the delta function,

$$(A^{-1})_{\lambda\lambda'} = (E_\lambda - E)\,\delta_{\lambda\lambda'} - i/2 \sum_c (\Gamma_{\lambda c})^{1/2} (\Gamma_{\lambda' c})^{1/2}.$$

From (9) for the case of two-levels one obtains:

$$S_{nn} = \exp(2i\,\delta_{pot})\,[1 + i\,\Gamma_{1n} / (\Delta E_1 - i/2(\Gamma_1 + \Gamma_2\,\Delta E_1/\Delta E_2)) +$$

$$+ i\,\Gamma_{2n} / (\Delta E_2 - i/2(\Gamma_2 + \Gamma_1 \Delta E_2/\Delta E_1)) +$$

$$+ \Gamma_\gamma(\Gamma_{1n}^{1/2} - \Gamma_{2n}^{1/2})^2 / (2\Delta E_1)/(\Delta E_2 - i/2\,(\Gamma_2 + \Gamma_1 \Delta E_2 / \Delta E_1))\,], \qquad (10)$$

where $\Delta E_\lambda = E_\lambda - E$.

For bismuth at neutron energies below 50 eV the real and imaginary parts of the last term in (10) are considerably smaller than the respective parts of the second and third term and so this last term can be neglected. By expressions (6) and (10) for s-neutrons scattered on bismuth we calculate $\Delta\sigma_{int} = \pm 22\times10^{-27}$ cm^2 at E=1 eV and $\Delta\sigma_{int} = \pm 23\times10^{-27}$ cm^2 at E=50 eV. These are in agreement with the figures in the Table. With the third resonance (E_2=5102 eV) of bismuth accounted for, the value of $\Delta\sigma_{int}$ changes by less than 20% ($\Delta\sigma_{int} = \pm 27\times10^{-27}$ cm^2); and that of σ_{tot}, by no more than 0.5%. From (6) and (10) one may conclude that $\Delta\sigma_{int} \sim \Gamma_{\lambda n}\Gamma_{\lambda+1}/k^2 \cong$ const, i.e., it is practically independent of energy: And the above performed numerical calculation confirms this conclusion. The same conclusion can be made about $\Delta\sigma_{int} = d$ in [5]:

$$d = R^2 \sum_{i\neq j}\sum [\gamma_i^2\gamma_j^2 / ((E - E_{0i})(E - E_{0j}))\,] \cong \text{const} \quad (\text{here } \gamma^2 = \Gamma_n/2kR)).$$

Since in [1] the variable p_2 is also independent of energy, introduction into expression for σ_{tot} of constant in value, small interresonance interference terms obtained in this work or work [5] cannot affect the result a_{ne} of determination in [1], but will of course change p_2 a little.

So from refs. [1,2] is follows that $\langle r_{in}^2\rangle_N < 0$.

In conclusion it is worth noting that the obtained result is an important test of contemporary ideas of the neutron, e.g., of the Cloudy Bag Model [11]. Calculations made in the frame of this model give also $\langle r_{in}^2\rangle_N < 0$ and do not agree, even in sign, with the result of refs. [3,4]. Moreover, the knowledge of the sign and value of the anomalous magnetic moments of the neutron and proton already in the 50's allowed qualitative representation of electric charge distribution $\rho(r)$ inside the nucleon as illustrated in the Figure [12]. Since $\langle r_{in}^2\rangle_N = \int r^2\rho(\vec r)d^3\vec r$, then for the neutron one obtains $\langle r_{in}^2\rangle_N < 0$ due to a negative «tail» of $\rho(r)$ at large r [12—14]. Thus, according to refs. [3,4], once $\langle r_{in}^2\rangle_N > 0$, one should reconsider current ideas of the structure of the neutron.

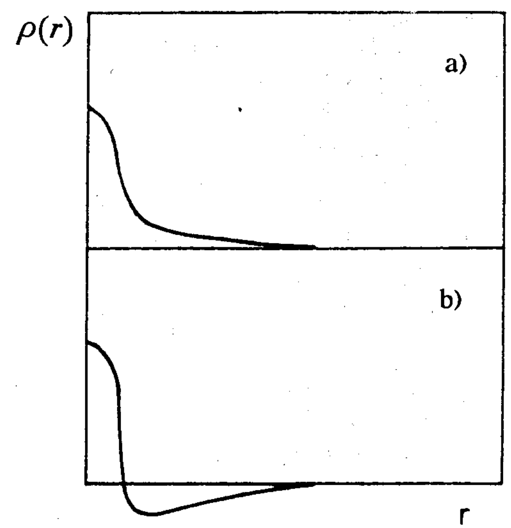

Fig. Expected electric charge distribution inside the nucleon, a) the proton; b) the neutron.

Finally, a few words about already existing in and expected from refs. [3,4] uncertainties: in [4], the resonance scattering and the imaginary part of the scattering amplitude are not fully accounted for. In particular, there is no independent of energy term like p_2 in [1]. Had this term been accounted for, we should immediately have $a_{ne} = -1.59 \times 10^{-3}$ fm. As for ref. [3], one cannot see any incorrectness on the face of it. However, when measuring a 0.5% neutron scattering asymmetry in noble gases with an error less than 3% one must be absolutely sure of the absence of any side effects (p-resonances, light gases admixtures, etc.) leading to false asymmetry. So in the case of xenon the presence at about 0.1 eV neutron energies of a weak ($\Gamma_{n0} \cong 10^{-7} - 10^{-8}$ eV) p-resonance changes essentially the scattering asymmetry observed [14]. Note that a resonance in σ_{tot} with a neutron width of the order of 10^{-7} eV can hardly be detected in a conventional transmission experiment.

The author gratefully acknowledges the help of Mrs. T.F.Drozdova in the preparation of the English version of this paper.

REFERENCES

1. Alexandrov Yu.A., Vrana M., Manrike Garcia J., Machekhina T.A., Sedlakova L.N. — JINR Preprint E3-85-935, Dubna, 1985; Sov. J. Nucl. Phys., 1986, 44, p.900.
2. Alexandrov Yu.A. et al. — Jad. Fiz., 1974, 20, p.1190.
3. Krohn V.E., Ringo G. — Phys. Rev., 1973, 80, p.1305.
4. Koester L., Waschkowski W., Meier J. — Z. Phys. A, 1988, 329, p.229.
5. Nikolenko V.G., Popov A.B. — Z. Phys. A, 1992, 341, p.365.
6. Adler D.B., Adler F.T. — In: Proc. Conf. on Breeding in Fast Reactor, 1965, ANL-6792, p.695.
7. Bethe H.A. — Rev. Mod. Phys., 1937, 9, p.69; Bloch C. — Nucl. Phys., 1957, 4, p.503; Goldberger M.L., Watson K.M. — «Collision Theory». John Wiley and Sons, inc. New York — London — Sydney, 1964; Taylor J.R. — «Scattering Theory», John Wiley and Sons, inc. New York — London — Sydney — Toronto, 1972.
8. Lukyanov A.A. — «The Neutron Crioss Sections Structure», Moscow, Atomizdat, 1978.
9. Wigner E.P. — Phys. Rev., 1946, 70, p.606.
10. Vogt F. — Phys. Rev., 1958, 112, p.203.
11. Thomas A.W. — Adv. Nucl. Phys., 1983, 13, p.1.
12. Yennie D.R., Levy M.M., Ravenhall D.G. — Rev. Mod. Phys., 1957, 29, p.144.
13. Alexandrov Yu.A. — Nucl. Instr. Meth., 1989, A284, p.134.
14. Alexandrov Yu.A. — «Fundamental Properties of the Neutron». Claredon Press, Oxford, 1992.

Received by Publiishing Departmen
on November 23, 1992.