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A NEW POLARIZED NEUTRONS METHOD
FOR STUDYING DEPTH INHOMOGENEOUSLY
MAGNETIZED MAGNETIC FILMS

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To uncerstand the nature of ferromagnetic states of materials they should be studied on a small scale. There are two typical scales in the picture of space behaviour of a local magnetization vector. The first scale is a crystal lattice scale, the second one is the domain scale. The thermal neutron diffraction method on the lattice of ordered atomic spins makes it possible to investigate an atomic magnetic structure. Śspensions or magneto-optical methods are usually used to examine the surface domain structure. The intermediate scale determined by the limits of interdomain boundaries ( $\sim 1000 \mathrm{~A}$ ) is in existence. The phenomenologic theory describing magnetic structures and their dynamics on this scale (micromagnetism $/ 1 /$ ) has been propounded in $/ 2 /$. Theoretical studies of equilibrated magnetic structures of thin magnetic films have revealed that there exists a new type of films magnetized distribution of which (in the basic state) in the direction $z$ perpendicular to a surface is inhomogeneous $/ 3 /$. The imitation of such inhomogeneous state of magnetization is said to be connected with the distinctions between the values and the types of anisotropy constants on a film surface and inside it. These physical causes are based not only on the distinctions between the inner and surface symmetries of the magnetic ion surrounding, but also on the existence of light element impurities in the skin. The influence of a basic state of the film, being magnetized inhomogeneously along $z$, on the peculiarities of spin-wave resonance absorption spectra has been theoretically studied $/ 4$ /.

Yet, the methods, determining experimentally the details of onedimensional inhomogeneous magnetic structure on a scale of interdomain behaviour ( -1000 A ), are unknown, i.e. we are still not able to define experimentally the one-dimensional function $\vec{M}(z)$ of a local magnetization vector. The point is that well-developed methods make it possible either to state the existence and type of an inhomogeneous basic state of a film (spin-wave resonance), or to estimate the thickness of the ferromagnet inhomogeneously magnetized skin and to define anisotropy inside this skin (the Kerr magneto-optical effect $/ 5,6 \%$ ). As to the latter methodiits space resolution mounts, as a rule, to $0.1+0.2 \mu$.

Let us point out that well-developed methods of the Lorentz microscopy are inaplicable in the study of inhomogeneous along the depth distribution of magnetization $\vec{M}$. These methods make it possible to observe magnetic inhomogeneities of a sample plane only. Yet, there are no experimental methods determining the $\vec{M}(z)$ function with high resolution.

Below we are going to analyze the possibility of getting the detailed information on $\vec{M}(z)$ in a magnetic film with the help of polarized neutron reflection from its surface.
2. It is known that the problem of detecting the reflection coefficient of neutrons from a flat boundary of non-magnetic medium is reduced to the solution of a one-dimensional quantum mechanical problem in reflection from a potential jump $U$, where $U$ is the value of an effective energy of neutron-nuclear interaction being related to the mean value of the coherent nuclear scattering length $b$ and numbers of nuclei in the unit volume N by the following relation
$\mathrm{U}=4 \pi \frac{\hbar^{2}}{2 \mathrm{~m}} \mathrm{~N} \cdot \mathrm{~b}$.
The $u$ values are in the region of $\sim 10^{-7} \mathrm{eV}$. Film composition inhomogeneities arising, for example, on its surface due to impurities, lead to the dependence of $U$ on a coordinate directed along the normal to the film (z). In case of ferromagnetics the potential of interaction with a medium takes new additive:
$\mathrm{v}_{\mathrm{m}}=-\vec{\mu} \cdot\left(\overrightarrow{\mathrm{B}}-\overrightarrow{\mathrm{H}}_{\mathrm{o}}\right)$,
where $\vec{\mu}$ is the neutron magnetic moment, $\vec{H}_{0}$ is the vector of an external magnetic field, $\vec{B}$ is the vector of magnetic induction of medium $(\vec{B}=4 \pi \vec{M}+\vec{H}), \vec{H}$ is the sum of all magnetic fields in the medium: an external and demagnetization fields of the sample poles. $U_{m}$ is, as a rule, of the same order of magnitude as $U$. For inhomogeneous magnetized structures $\vec{B}=\vec{B}(z)$, where $z$ is the coordinate directed along the normal to the surface and it leads to the dependence $U_{m}=$ $=U_{m}(z)$. Let's deal in detail with the behaviour of vector $\vec{B}(z)$ for main types of inhomogeneous equilibrated magnetic structures in magnetic films. Hereafter, the coordinate $z$ will be taken to be directed along the inner normal to a plane; $x$ and $y$ will be lying in the film plane. The film itself. is to be flat and limitless along $x$ and $y$. Let us denote a continuous vector of local magnetization by $\vec{M}$ and consider that $\vec{M}$ depends only on $z$. For brevity sake, such structures will be
termed $z$-structures. Let us formulate the assertion important for the subsequent discussion: for the limitless parallel plates at any noncollinear z-structures, i.e. under any dependences of the vector $\vec{M}$ on 2
$\vec{M}(z)=M_{x}(z) \vec{n}_{x}+M_{y}(z) \vec{n}_{y}+M_{z}(z) \vec{n}_{z}$
the vector $\vec{B}(z)$ equals
$\vec{B}(z)=4 \pi \vec{M}_{S}(z)+\vec{H}_{0}$
where $\vec{M}_{s}(z)=M_{x}(z) \vec{n}_{x}+M_{y}(z) \vec{n}_{y}$ is a component of vector $\vec{M}(z)$ coincident with the film plane. This assertion is easily proved (e.g., see $/ 7 /$ ) for the plates with $M_{z}(z)=$ const. One may elementary prove the equation (4) for the general case, i.e. when $M_{z}(z) \neq$ const (a plate contains "volumeric magnetic charges"). It is sufficient to brake noncollinear z-structures down to two types. Let us refer to the first type the films where the vector $M_{s}(z)$ conserves its space direction, i.e. when axes are properly selected
$\vec{M}_{S}(z)=M_{X}(z) \vec{n}_{X}$.
The behaviour of $\vec{M}_{s}(z)$ corresponds to such a behaviour of $\vec{M}(z)$ :
$\vec{M}(z)=M_{X}(z) \vec{n}_{X}+M_{z}(z) \vec{n}_{z}$.
Such $\vec{M}(z)$ vector dependence emerges (according to $/ 3,8 /$, in films with volumeric anisotropy of "light plane" or "light axis" types, and with the surface anisotropy of "light axis" of "light plane" types, respectively. Besides, equation (5) is done for one-dimensional anisotropy in the film planes when $|\vec{M}|$ depends on $z$.

Let's refer the films with the spiral $\vec{M}_{S}(z)$ structure to the secona type, i.e.:
$\vec{M}_{s}(z)=M_{x}(z) \vec{n}_{x}+M_{y}(z) \vec{n}_{y} ; \quad M_{x}(z) / M_{y}(z) \neq$ const.
Such dependence of $M_{s}$ on $z$ corresponds to the vector
$\vec{M}(z)=M_{x}(z) \vec{n}_{x}+M_{y}(z) \vec{n}_{y}+M_{z}(z) \vec{n}_{z}$.
领解 structures may emerge, for example, in one-dimensional anisotropy magnetic films (in the external magnetic field) with distinguishing values of plane anisotropy constants and inside the films.
3. Now we turn our attention to the analysis of the neutron reflection from films with a magnetized z-structure. Then, let us write the neutron wave function with regard to the spin
$\psi_{(z)}=\binom{\Psi_{+}(z)}{\Psi_{-}(z)}$.
The neutron wave function (8) in magnetic medium adheres to the pauli one-dimensional equation with potentials (1) and (2)
$\frac{h^{2}}{2 n} \cdot \frac{d^{2}}{d z^{2}} \Psi(z)+\left[E-\left(U-\mu \vec{\sigma}\left(\vec{B}(z)-\vec{H}_{0}^{\prime}\right)\right] \quad \Psi(z)=0\right.$
or with the eq.(4)
$\frac{n^{2}}{2 m} \frac{d^{2}}{d z^{2}} \Psi(z)+\left[E-\left(U-4 \pi \mu\left(\vec{\sigma} \vec{M}_{S}(z)\right)\right] \Psi(z)=0\right.$.
where $E=\frac{\hbar^{2} k_{z}^{2}}{2 m}, k_{z}$ is a wave vector $z$-component, $\vec{\sigma}$ is a vector with components $\sigma_{x}, \sigma_{y^{\prime}} \sigma_{z}$ (the Pauli matrices). The wave functions of falling $\Psi_{i}(z)$ and reflected $\Psi_{f}(z)(z<0)$ neutrons are written as follows
$\Psi_{i}(z)=e^{i k_{z} z}\binom{\Psi_{+}^{(i)}}{\Psi_{-}^{(i)}} ; \quad \Psi_{f}(z)=e^{-i \dot{k}_{z} z}\binom{\Psi_{+}^{(f)}}{\psi_{-}^{(f)}}$
Spinors $\binom{\Psi_{+}^{(i)}}{\Psi_{-}^{(i)}}$ and $\binom{\Psi_{+}^{(f)}}{\Psi_{-}^{(f)}}$ are bound by the following equation
$\binom{\Psi_{-}^{(i)}}{\Psi_{-}^{(+)}}=R\binom{\psi_{+}^{(i)}}{\psi_{-}^{(i)}}$.
where $R$ is a $2 x 2$ reflection matrix, depending on $K_{z}$ and $\vec{M}_{S}(z)$. Matrix $R$ is to be found from the equation (10) with regard to standard boundaries conditions. The matrix $R_{n, m}$ elements are the essence of the reflection probability amplitude with the neutron spin flipping ( $n \neq m$ ) and without it $(n=m)$. Thus, due to the dependence of $R_{n, m}$ elements on $K_{z}$ we get information on $\vec{B}(z)$; i.e. $R$ is uniquely determined by space dependence of $\vec{M}_{S}(z)$.

Let us associate the values being measured during experiments on polarized neutron reflection with the matrix R. Generally a polari-
zed neutron beam is described by a spin density matrix/9/

$$
\begin{equation*}
\rho=\sum_{n} a_{n}\left|\psi_{n}\right\rangle<\psi_{n} \mid \tag{13}
\end{equation*}
$$

where $a_{n}$ are mixing coefficients of a neutron pure spin state $\left|\Psi_{n}\right\rangle$. The vector of beam polarization is determined by a matrix $\rho$ as follows/9/:
$\overrightarrow{\mathbf{P}}=\operatorname{tr}(\rho \vec{\sigma}) / \operatorname{tr} \rho$.

In order to determine polarization of a reflected beam one has to know the way the matrix $\rho$ is transformed due to the reflection of neutrons. It is easy to show that a falling beam matrix $\rho_{i}$ is transformed to a reflected beam matrix $\rho_{f}$ nearly by the equation

$$
\begin{equation*}
\rho_{\mathrm{f}}=\mathrm{R} \rho_{\mathrm{i}^{\mathrm{R}}}{ }^{+} \tag{15}
\end{equation*}
$$

where $\mathrm{R}^{\boldsymbol{+}}$ is the hermitian conjugated matrix. Let us construct a matrix of a magnetic film polarizing ability
$\varepsilon=\mathrm{RR}^{+}$.

Using the matrix $\varepsilon$ we construct a vector of a film polarizing ability as follows:
$\vec{Q}=\operatorname{tr}(\varepsilon \vec{\sigma}) / \operatorname{tr} \varepsilon$.

It is not difficult to show that the scalar product of vectors of polarisation of a falling beam $\vec{P}_{o}\left(k_{z}\right)$ and that of the vector of a film polarising ability $\vec{Q}\left(K_{z}\right)$ satisfies the equation
$\vec{P}_{o}\left(K_{z}\right) \vec{Q}\left(K_{z}\right)=\left[N_{+}\left(K_{z}\right)-N_{-}\left(K_{z}\right)\right] /\left[N_{+}\left(K_{z}\right)+N_{-}\left(K_{z}\right)\right]$,
where $N_{+}\left(K_{z}\right)$ and $N_{-}\left(K_{z}\right)$ are the intensities of neutrons reflected from a film surface with "on" and "off" spinflipper, respectively. The spinflipper is a polarized beam instrument performing the reverse of the falling beam polarisation vector $\vec{P}_{o}\left(K_{z}\right)$. Thus, the combination (18) of experimentally measured spectra $N_{+}\left(K_{z}\right)$ and $N_{-}\left(K_{z}\right)$ is uniquely related by the reflection matrix $R$. In this case the matrix is uniquely determined by a particular kind of. $z-s t r u c t u r e \vec{M}_{S}(z)$.

In the sense that a definite vector of the film $\vec{Q}\left(K_{z}\right)$ polarising ability corresponds to each concrete z-structure (see (16), (17)),
let us consider specific peculiarities of this vector typical for the above types of structures.

Substituting $\mathrm{M}_{\mathrm{S}}(\mathrm{z})$, appropriating to first type structures; in eq.(10) reduces it to a system of two independent equations with respect to $\Psi_{+}(z)$ and $\Psi_{-}(z)$. With an appropriate choice of a coordinate system (where $\vec{M}_{S}(z)=M_{x}(z) \vec{n}_{x}$ and the matrix $\sigma_{x}$ is diagonal) a reflection matrix becomes diagonal, i.e.
${ }_{R}=\left(\begin{array}{ll}R_{11} & 0 \\ 0 & R_{22}\end{array}\right)$
Clearly in this case $\vec{Q}\left(K_{z}\right)$ II $\vec{n}_{x}$, i.e. it occurs in a film plane and is in direction with the vector of magnetic induction. The latter statements are true for any values of the neutron wave vector $K_{z}$. Using the determination of the vector $Q\left(K_{z}\right)$ it is easy to show that
$Q_{x}\left(K_{z}\right)=\frac{\left|R_{11}\left(K_{z}\right)\right|^{2}-\left|R_{22}\left(K_{z}\right)\right|^{2}}{\left|R_{11}\left(K_{z}\right)\right|^{2}+\left|R_{22}\left(K_{z}\right)\right|^{2}}$
and dependence of $Q_{x}$ on $K_{z}$ is to be defined by $\vec{M}_{S}(z)$. A complete experiment on the determination of the vector $\vec{Q}$ is reduced to the independent measurement of spectra $N_{+}\left(K_{z}\right)$ and $N_{-}\left(K_{z}\right)$ combination (18) at three orthogonal directions of a polarisation vector $\vec{P}_{\mathrm{o}}$ of the falling beam. The fact that $\varphi_{z}=0$ must lead any $K_{z}$ to a zero value of
$\left[N_{+}\left(K_{z}\right)-N_{-}\left(K_{z}\right)\right] /\left[N_{+}\left(K_{z}\right)+N_{-}\left(K_{z}\right)\right]$
if a falling beam vector $\vec{P}_{0}\left(K_{z}\right)$ is perpendicular to a film. Substituting $\vec{M}_{s}(\bar{z})$, appropriating to second type structures, in eq.(10) leads to a system of two interdependent equations with respect to $\Psi_{+}(z)$ and $\Psi_{-}(z)$. From this it follows that the matrix $R$ in the general case is non-diagonal, i.e. all elements of $R_{n, m}$ may be different from zero. These are expressions for the components of the polarising ability vector $Q$ for $z$-structures of the second type:
$Q_{x}\left(K_{z}\right)=\frac{\left[\left|R_{11}\left(K_{z}\right)\right|^{2}+\left|\mathrm{R}_{12}\left(\mathrm{~K}_{z}\right)\right|^{2}\right]-\left[\left|\mathrm{R}_{22}\left(\mathrm{~K}_{z}\right)\right|^{2}+\left|\mathrm{R}_{21}\left(\mathrm{~K}_{z}\right)\right|^{2}\right]}{\sum_{\mathrm{n}, \mathrm{m}}\left|\mathrm{R}_{\mathrm{nm}}\right|^{2}}$
$Q_{Y}\left(K_{z}\right)=\frac{2 J m\left[R_{11}\left(K_{z}\right) R_{21}^{*}\left(K_{z}\right)+R_{12}\left(K_{z}\right) R_{22}^{*}\left(K_{z}\right)\right]}{\sum_{n, m}\left|R_{n m}\right|^{2}}$
$Q_{z}\left(K_{z}\right)=\frac{2 \mathrm{Re}\left[\mathrm{R}_{11}\left(K_{z}\right) \mathrm{R}_{21}^{*}\left(\mathrm{~K}_{\mathrm{z}}\right)+\mathrm{R}_{12}\left(\mathrm{~K}_{\mathrm{z}}\right) \cdot \mathrm{R}_{22}^{*}\left(\mathrm{~K}_{\mathrm{z}}\right)\right]}{\sum_{\mathrm{n}, \mathrm{m}}\left|\mathrm{R}_{\mathrm{nm}}\right|^{2}}$,
where $\mathrm{R}_{\mathrm{nm}}^{*}$ are complex conjugated values. From non-diagonal conditions $R_{12}, R_{21} \neq 0$ it follows that $Q_{2}\left(K_{2}\right)$ being normal to a film, generall differs from zero. This basic pecuilarity of the vector $\vec{Q}$ for a second type structures leads to the difference from zero of a film polarising ability along the direction perpendicular to its surface. The complete experiment, as in the previous case, is reduced to the independent measurement of $\left[N_{+}\left(K_{z}\right)-N_{-}\left(K_{z}\right)\right] /\left[N_{+}\left(K_{z}\right)+N_{-}\left(K_{z}\right)\right]$ for three orthogonal directions of the vector $\vec{P}_{0}\left(K_{z}\right)$. It is clear that the dependence of $Q_{x, y, z}$ on $K_{z}$ is to be determined by the magnetic structure parameters.
.. From what has been said above it follows that there is a possibility to restore a complex $\vec{M}_{s}(z)$ structure of magnetic films by polarised neutron reflection. To perform this it is essential: firstly, to measure spectra dependence of three components of film polarising ability vector $\vec{Q}$, carrying out the above complete experiment; secondly, solving eq. (10) with boundary conditions for a model structure to fit, changing structure parameters, theoretical values of $\vec{Q}\left(K_{z}\right)$ to the experimental ones over a wide range of values $K_{z}$. Thus, the essence of the suggested approach is reduced to the solution of eq. (10) which makes it possible on the base of $\vec{M}(z)$ to calculate the reflection mat rix $R\left(K_{z}\right)$ and the experimentally measured vector $\vec{Q}\left(K_{z}\right)$. It should be noted that in the general case eq. (10) cannot be solved analytically. Hence the solution of eq. (10) with different types of $\vec{M}(z)$ is an independent problem.

The developed approach as well as the method of solution of eq.(10)for the general case, was favoured by the results of experiments on the reflection of polarized neutrons from thin ( 1500 A) FeCo films, investigations of which had been carried out at the pulsed neutron reactor IBR-2, JINR. $\therefore$

## CONCLUSIONS

1. The theoretical approach of explanation of experimental data on polarized neutron reflection from inhomogeneously magnetized thin films is suggested. The very approach makes it possible to determine space behaviour of the local magnetization vector $\vec{M}_{S}(z)$ with space resolution exceeding the resolution of magneto-optical methods approximately by an order.
2. The matrix and vector of polarising ability for a magnetic film are constructed; the peculiarities characteristic for various types of inhomogeneously magnetized films have been considered.

It is shown that for films with the spiral structure magnetization the polarising ability vector must have a non-zero component perpendicular to a film plane?
3. If this approach is realized it will be essential to carry out the complete experiment: to measure three components of the film's polarising ability vector according to the wave vector of incident neutrons.
4. It appears that the complete experiment is reduced to the measurement of the definite spectra of the reflectea neutrons (see eq. (18)) for three orthogonal directions of the incident beam polarisation vector.

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## REFERENCES

1. Brown W.E. - Micromagnetics, Interscience Publishers a Division of John Wiley \& Sons, New York-London, 1963.
2. Landau L.D., Lifshits E.M. - Phys.Zs.Sow., 1935, v.8. P.153.
3. Barjahtar V.G., Klepikov V.F., Sobolev V.L. - Fizika Tverdogo Tela, 1971, v.13, p.1454.
4. Barjahtar V.G., Klepikov V.F., Sobolev V.L. - Fizika Tverdogo Tela, 1971, v.13, p.3517.
5. Krinchik G.S. - Fizika magnitnyh javleniy. - Publ. by Moscow University, Moscow, 1985.
6. Krinchik G.S., Zubov B.E., Nikitin L.V. - Poverhnost, 1982, v.1, p. 22.
7. Tikadzumi S. - Fizika Ferromagnetizma, "Mir", Moscow, 1987.
8. Salanskiy.N.M., Eruhimov M.Sh. - Fizicheskie Svoiystva i Primenenic Magnitnyh Plenok - "Nauka", Novosibirsk, 1975.
9. Blum K. - Density Matrix Theory and Application. Plenum Press, New York and London, 1981.
