ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

ДУБНА

E2 - 9985

V.Ogievetsky, E.Sokatchev

ON VECTOR SUPERFIELD GENERATED BY SUPERCURRENT

1976

E2 - 9985

V.Ogievetsky, E.Sokatchev

ON VECTOR SUPERFIELD GENERATED BY SUPERCURRENT

Submitted to "Nuclear Physics" B

It is very tempting to find a supersymmetric version of the gravitational theory [1]. The main feature of such theories is that the graviton being combined with a fermion is treated as an irreducible zero-mass representation of supersymmetry. The simplest proper supermultiplet contains helicities ±2 and ±3/2. The simplest superfield involving the corresponding tensor and spin-vector fields is the vector one, $h_\mu(x, \theta)$. Its role as a basis for a possible supergravitational theory will be discussed in the present paper.

The search for supergravity proceeds in different ways. One of them is the direct generalization of the Riemannian geometry to the superspace [2-5]. Every "superspace point" $z^M(x^\mu, \theta^\alpha)$ is parametrized by four anticommuting spinor coordinates θ^α in addition to the standard coordinates x^μ and a certain metric tensor $g_{MN}(z)$ is introduced. In fact, $g_{MN}(z)$ consists of higher superfields, namely tensor $g_{\mu\nu}(x, \theta)$, spin-vector $g_{\alpha\mu}(x, \theta)$ and spin-spinor $g_{\alpha\beta}(x, \theta)$ ones. These superfields involve higher spins (e.g., $g_{\mu\nu}(x,\theta)$ contains spins 3 and 5/2). A careful analysis of the field equations of motion is required. Spins 3, 5/2 and superfluous spins 2 have to be excluded from these equations and the equations of motion for gravitational and accompanying fields should posses an acceptable form. We have not yet seen such a precise analysis anywhere. As an alternative in several remarkable papers /6-8/ a supergravity theory is constructed directly in terms of the gravitational and spin-vector fields without using superfields. An elegant and reasonable scheme is proposed for the interaction of the spin-vector field with the gravitational one. This scheme possesses invariance both under the general covariance group and under some spinor gauge transformations generalizing the gauge transformations of the Rarita-Schwinger equations (local supersymmetry).

However, a superfield formulation of every supersymmetric theory remains more preferable in our opinion because the explicit invariance is ensured at all stages.

In the present paper an attempt is made to construct a supergravity theory using the simplest possible superfield, just the vector one*. This approach is based on the following observation. The Einstein equation can be represented in the form of and can be derived /9/ as the equation of motion for a symmetric tensor field $h_{\mu\nu}(x)$. The source of this field is the energy-momentum tensor

---

*The main results of this attempt were reported at the IV International Conference on Nonlocal and Nonlinear Field Theory, D2-9788, Alushta, April 1976.

$\theta_{\mu\nu}(x)$ for all fields including the gravitational one

$$\Box h_{\mu\nu} - \partial_\mu \partial^\lambda h_{\lambda\nu} - \partial_\nu \partial^\lambda h_{\lambda\mu} + \eta_{\mu\nu} \partial^\lambda \partial^\rho h_{\lambda\rho} - \frac{1}{3}(\Box \eta_{\mu\nu} - \partial_\mu \partial_\nu) h^\lambda_\lambda = -a\theta_{\mu\nu}. \tag{1}$$

The coupling constant a has dimensionality cm (in units $\hbar = c = 1$) and it is the square root of the Einstein constant, $a^2 = \kappa$.

Such a field treatment of gravity can be supersymmetrized. The energy-momentum tensor together with the supersymmetry spin-vector current enter into a real vector superfield, just supercurrent $V_\mu(x,\theta)$. This remarkable fact was established by Ferrara and Zumino /10/ . In direct analogy with the gravitational theory one can suppose that the supergravitational theory is that of a real vector superfield $h_\mu(x,\theta)$ the source of which is the supercurrent.

In other words, we suggest that the equations of motion for the gravitational superfield have the form

$$\pi_\mu^{\ \nu} h_\nu(x,\theta) = -aV_\mu(x,\theta), \tag{2}$$

where a is the same gravitational coupling constant as in Eq. (1) and $\pi_{\mu\nu}$ is the operator of the free equation for h_μ.

In the general case the supercurrent obeys the "conservation" law /10/ of the third order in the spinor derivatives D_α (in real four-component notations; see Appendix A)

$$D_\alpha(\bar{D}D)V_\mu - 2i\partial_\mu(\gamma^\nu D)_\alpha V_\nu = 0 \tag{3}$$

which contains, in particular, the conservation laws for the energy-momentum tensor and for the supersymmetry spin-vector current

$$\partial^{\mu}\theta_{\mu\nu}(x) = 0, \quad \partial^{\mu}J_{\alpha\mu}(x) = 0. \qquad (4)$$

We wish the conservation law (3) to follow from the equation of motion (2) (as well as the conservation law $\partial^{\mu}\theta_{\mu\nu}=0$ follows from the equation (1)). Therefore the equation operator $\pi_{\mu\nu}$ is required to obey identically the same condition (3) as the supercurrent does

$$D_{\alpha}(\bar{D}D)\pi_{\mu\nu} - 2i\partial_{\mu}(\gamma^{\lambda}D)_{\alpha}\pi_{\lambda\nu} \equiv 0. \qquad (5)$$

Further, it is not hard to establish the dimensionality of the operator $\pi_{\mu\nu}$. The gravitational field and the energy-momentum tensor have dimensionalities cm^{-1} and cm^{-4}, respectively. They are included into the coefficients for $\bar{\theta}i\gamma_{\mu}\gamma_5\theta$ in the decompositions (A.1) and (A.2) of the superfields $h_{\mu}(x,\theta)$ and $V_{\mu}(x,\theta)$. . Taking into account the dimensionality of θ_{α} ($cm^{-1/2}$) we find that $[h_{\mu}] = cm^{0}$ and $[V_{\mu}] = cm^{-3}$. Finally, from Eq. (2) it follows that

$$[\pi_{\mu\nu}] = cm^{-2}. \qquad (6)$$

In order to find an operator with such properties we are going to use the "root" method /11/ based on the projection superspin operators/12/ and their roots. To begin with, we have to examine what superspins are described by Eq. (2). The general vector superfield $\Phi_{\mu}(x,\theta)$ contains irreducible representations of supersymmetry with superspins 3/2, 1, 1/2 and 0 /12/ .

The highest superspin 3/2 is singled out by the differential condition

$$(\gamma^{\mu} D)_{\alpha} \Phi_{\mu} = 0. \tag{7}$$

The superspin content of the supercurrent is easily elucidated when Eq. (3) is written in the equivalent form [10]

$$(\gamma^{\mu} D)_{\alpha} V_{\mu} = D_{\alpha} S, \tag{8}$$

where S is some model-dependent scalar superfield with superspin 0 (a chiral one). Comparing Eq. (8) with Eq. (7) one concludes that the supercurrent contains a mixture of superspins 3/2 and 0.

Following the root method the next step is to write down the corresponding mixture of projection operators [12]

$$\Pi^{3/2}_{\mu\nu} + \alpha \Pi^{0}_{\mu\nu}, \tag{9}$$

where

$$\Pi^{3/2}_{\mu\nu} = \frac{2}{3}\left[\left(1 + \frac{(\bar{D}D)^2}{4\Box}\right)\left(\eta_{\mu\nu} - \frac{\partial_{\mu}\partial_{\nu}}{\Box}\right) + \frac{1}{4\Box}\epsilon_{\mu\nu\lambda\rho}\, \partial^{\lambda}\bar{D} i \gamma^{\rho}\gamma_5 D\right],$$

$$\Pi^{0}_{\mu\nu} = -\frac{(\bar{D}D)^2}{4\Box}\,\frac{\partial_{\mu}\partial_{\nu}}{\Box}.$$

Then the operator $\pi_{\mu\nu}$ is defined as the square root of the localized operator (9):

$$\pi_{\mu}{}^{\lambda}\pi_{\lambda\nu} = \Box^2 (\Pi^{3/2} + \alpha\Pi^{0})_{\mu\nu} \; .$$

It turns out that there exists only one root of this kind (at $\alpha = 4/9$) and it has the form

$$\pi_{\mu\nu} = \frac{1}{6}[4(\eta_{\mu\nu}\Box - \partial_\mu\partial_\nu) + \eta_{\mu\nu}(\bar{D}D)^2 + \epsilon_{\mu\nu\lambda\rho}\partial^\lambda \bar{D} i\gamma^\rho \gamma_5 D]. \quad (10)$$

This operator has the right dimensionality cm^{-2} and satisfies identity (5). Finally, Eq. (2) with operator (10) contains correct equations for the component fields (see Appendix B). In particular, the symmetric tensor field $h_{\mu\nu}(x)$ included in the superfield $h_\mu(x,\theta)$ obeys Eq.(1) and the spin-vector field $\phi_{a\mu}(x)$ obeys the Rarita-Schwinger equation. In the free case the equation obtained describes only two chiralities: ±2 and ±3/2.

Note that Eq. (2) with operator (10) is invariant up to the lowest order in the coupling constant a under the gauge transformations

$$\delta_0 h_\mu = \bar{D}\gamma_\mu \Psi, \quad (11a)$$

where

$$\Psi_a(x,\theta) = (\gamma^\nu D)_a \bar{D}\Psi_\nu(x,\theta) - 2i\partial^\nu \Psi_{a\nu}(x,\theta) \quad (11b)$$

and $\Psi_{a\nu}(x,\theta)$ is arbitrary gauge spin-vector superfunction. In particular, this transformation contains the corresponding invariance transformations of Eq. (1)

$$\delta_0 h_{\mu\nu} = \partial_\mu w_\nu + \partial_\nu w_\mu - 2\eta_{\mu\nu}\partial^\lambda w_\lambda \quad (12)$$

and of the Rarita-Schwinger equation.

The most direct way to find the supersymmetric version of the Einstein equation would be as follows. One has to find out the supercurrent for the left-hand side of Eq. (2), then to put it into the R.H.S. (together with the supercurrent for matter fields), afterwards to compute the supercurrent for the system obtained and to continue these iterations further. Such a way seems to be too straightforward and difficult. There is also another difficulty: nobody has succeeded in deriving an algorithm to deduce the supercurrent. Ferrara and Zumino [10] guessed fortunately the supercurrent for the simplest models of chiral and general scalar superfields.

However, there exists another way for the realization of the programme proposed. That is the search for a superfield transformation group (a generalization of the general covariance group) and we have made the first attempts.

Consider the real scalar superfield $\Phi(x, \theta)$. Its free Lagrangian has the form [13,14] $(S = \int d^4x d^4\theta\, \mathfrak{L}(x,\theta))$

$$\mathfrak{L}_0^{\Phi} = -\frac{1}{4}\bar{D}^{\alpha}\bar{D}^{\beta}\Phi \cdot D_{\alpha} D_{\beta}\Phi + m^2\Phi^2 . \tag{13}$$

The corresponding supercurrent is [10]

$$\begin{aligned} V_{\mu}(x,\theta) = \bar{D}D \cdot \bar{D}^{\alpha}\Phi \cdot \bar{D}D(i\gamma_{\mu}\gamma_5 D)_{\alpha}\Phi - \\ - \frac{4}{3}m^2[\bar{D}^{\alpha}\Phi(i\gamma_{\mu}\gamma_5 D)_{\alpha}\Phi - 2\Phi\bar{D}i\gamma_{\mu}\gamma_5 D\Phi] . \end{aligned} \tag{14}$$

According to our basic hypothesis (2) we suppose that in the lowest order in the coupling constant a the interaction of $\Phi(x,\theta)$ with the gravitational superfield $h_\mu(x,\theta)$ is going through the supercurrent (14), i.e.,

$$\mathfrak{L} = \mathfrak{L}_0^\Phi + aV^\mu h_\mu + \mathfrak{L}_0^h + a\mathfrak{L}_{int}^h, \qquad (15)$$

where

$$\mathfrak{L}_0^h = \frac{1}{2} h^\mu \pi_{\mu\nu} h^\nu \qquad (16)$$

and $a\mathfrak{L}_{int}^h$ denotes the self-interaction Lagrangian for $h_\mu(x,\theta)$. Unlike $\mathfrak{L}_0^h$ the terms $a\mathfrak{L}_{int}^h$ and $aV^\mu h_\mu$ are not invariant under the gauge transformation (11). To achieve the invariance of the total Lagrangian (15) one has to accompany (11) by some additional transformations. The first of them is a multiplicative transformation of the vector superfield $h_\mu(x,\theta)$ and it would be arranged to compensate for the remainder due to the noninvariant term $a\mathfrak{L}_{int}^h$. In this paper we restrict ourselves to a multiplicative transformation of the scalar superfield $\Phi(x,\theta)$ which is needed to ensure the invariance of the first three terms in $\mathfrak{L}$. This transformation is uniquely defined as

$$\begin{aligned}\delta\Phi(x,\theta) = a[&4\lambda_\mu(x,\theta)\partial^\mu\Phi(x,\theta) - \\ &-\bar{D}^\alpha\lambda_\mu(x,\theta)(i\gamma^\mu D)_\alpha\Phi(x,\theta) - 2z_\mu(x,\theta)\bar{D}i\gamma^\mu\gamma_5 D\Phi(x,\theta) - \\ &-\bar{D}^\alpha z_\mu(x,\theta)(i\gamma^\mu\gamma_5 D)_\alpha\Phi(x,\theta)],\end{aligned} \qquad (17a)$$

where

$$\lambda_\mu(x,\theta) = \bar{D}\gamma_\mu\gamma_5\Psi(x,\theta), \quad z_\mu(x,\theta) = \bar{D}\gamma_\mu\Psi(x,\theta) \qquad (17b)$$

and $\Psi(x,\theta)$ is defined in (11b).

Note the following important analogy. Let the scalar field $\phi(x)$ interact with the gravitational one $h_{\mu\nu}(x)$ in the framework of the usual gravity. Then the transformation (12) of $h_{\mu\nu}$ · is accompanied by a transformation of the scalar field itself, just $\delta\phi(x) = a w_\mu(x)\partial^\mu\phi(x)$. By a suitable choice of the gauge function $w_\mu(x)$ this transformation reduces to translations (w_μ = = const), to rotations and Lorentz transformations ($w_\mu = \omega_{\mu\nu} x^\nu$), etc. In these cases the additive term (12) vanishes. Exactly in the same way, if $\Psi_\alpha(x,\theta)$ in (11) and (17) is properly choosen, we obtain: at $\Psi_\alpha = \bar{\theta}^\beta(\gamma_\mu\gamma_5)_{\beta\alpha} c^\mu$ -translations with constant parameters c_μ; at $\Psi_\alpha = \bar{\theta}\theta(\gamma_5\epsilon)_\alpha + \bar{\theta}\gamma_5\theta\epsilon_\alpha$ - supertranslations with parameters ϵ_α; at $\Psi_\alpha = \bar{\theta}^\beta(\gamma^\mu\gamma_5)_{\beta\alpha}\omega_{\mu\nu}x^\nu$ - rotations and Lorentz transformations with parameters $\omega_{\mu\nu}$, etc. In all these cases $z_\mu = \bar{D}\gamma_\mu\Psi = 0$, i.e., the superfield $h_\mu(x,\theta)$ does not receive the additive term (11).

The most remarkable feature of the transformation (17) is the presence of the term with two spinor derivatives ($\bar{D} i\gamma_\mu\gamma_5 D$) on the superfield. In contrast to all the remaining terms it does not reduce to transformations of the coordinates x_μ and θ_α . This fact indicates that the supersymmetric generalization of the general covariance group under consideration cannot be reduced to some group in the superspace (x_μ, θ_α) but it involves essentially transformations over the superfields themselves. This situation seems to be natural as, e.g., even in the linear subgroup SL(4, R) of the general covariance group, spinors do not exist (following to Cartan) whereas a spinor θ_α enters into the pair (x_μ, θ_α) . It is just the reason why in the gravitational theory one has to introduce the vierbein degrees of freedom or to use a nonlinear realization to describe spinors [9]. .

Above the lowest-order transformations of the general scalar superfield were derived. The same procedure can be carried out for the chiral superfield model [14,15]

$$S = \int d^4x d^4\theta [-\frac{1}{2}\Phi\Phi^+ + \frac{m}{2}(\delta(\bar\theta)\Phi^2 - \delta(\theta)\Phi^{+2}) + \frac{2}{3}g(\delta(\bar\theta)\Phi^3 - \delta(\theta)\Phi^{+3})]. \quad (18)$$

Here $\delta(\theta)$ and $\delta(\bar\theta)$ are δ-functions on the Grassmann algebra [14] (now we use the two-component notations; see Appendix A). The superfield $\Phi(x,\theta,\bar\theta)$ and its conjugate satisfy the chirality conditions

$$\bar D_{\dot\alpha}\Phi = 0, \; D_\alpha\Phi^+ = 0. \quad (19)$$

The supercurrent for this model is [10] $(V_\mu = \tilde\sigma_\mu^{\dot\alpha\alpha} V_{\alpha\dot\alpha})$

$$V_{\alpha\dot\alpha} = \frac{2}{3}(i\Phi\sigma^\mu_{\alpha\dot\alpha}\overleftrightarrow{\partial}_\mu\Phi^+ + \frac{1}{2}D_\alpha\Phi\bar D_{\dot\alpha}\Phi^+). \quad (20)$$

According to our assumption (15) the interaction with the gravitational superfield $h_{\alpha\dot{\alpha}}$ $(h_{\mu} = \tilde{\sigma}_{\mu}^{\dot{\alpha}\alpha} h_{\alpha\dot{\alpha}})$ is written down as

$$\mathfrak{L}_{int} = 2aV_{\alpha\dot{\alpha}} h^{\alpha\dot{\alpha}} .$$

Then we find that transformation (11) $(\delta h_{\alpha\dot{\alpha}} = \frac{1}{2}(D_{\alpha}\bar{\psi}_{\dot{\alpha}} + \bar{D}_{\dot{\alpha}}\psi_{\alpha})$ with $\frac{1}{\sqrt{2}}\psi_{\alpha}$ and $\frac{1}{\sqrt{2}}\bar{\psi}_{\dot{\alpha}}$ forming the Majorana spinor $\Psi_{\alpha})$ has to be accompained by the transformation

$$\delta\Phi = a[-4i\bar{D}\tilde{\sigma}_{\mu}\psi.\partial^{\mu}\Phi + \bar{D}\bar{D}.\psi^{\alpha}D_{\alpha}\Phi + \frac{1}{3}\bar{D}\bar{D}.D^{\alpha}\psi_{\alpha}.\Phi]. \qquad (21)$$

and by its conjugate for Φ^{+}. It is not hard to verify that Eq. (21) differs from Eq.(17) only by the last term. This term without derivatives on the superfield serves as a weight factor and it is connected with the nonzero dimensionality (cm^{-1}) of the chiral superfield (the general scalar superfield in (13) has dimensionality 0). Note that the terms with two spinor derivatives on the superfield vanish due to the specific properties (19) of the chiral superfields.

In conclusion we should stress that the true Lagrangian and superfield equations will contain more terms of higher order in the gravitational constant a. They are expected to be very nonlinear in the gravitational superfield $h_{\mu}(x,\theta)$ and some indications for this fact can be found analysing the results of Ref. /8/. Trying to close the algebra of the local supersymmetry Freedman and van Nieuwenhuizen come to transformations with parameters depending on the fields. This situation is typical of the nonlinear realizations [16].

We hope to find answers to these and other related questions in future.

Appendix A

In the present paper we mainly use the four-component Majorana formalism accepted, e.g., in Ref. [17] (the only difference: we define $\epsilon^{0123} = 1$). However, in the case of the chiral scalar superfields we use the more adequate two-component Van der Waerden formalism (see, e.g., Refs. [10, 14]).

The vector superfield $h_\mu(x,\theta)$ has the decomposition

$$
\begin{aligned}
h_\mu(x,\theta) &= A_\mu(x) + \bar\theta^\alpha \psi_{\alpha\mu}(x) + \frac{1}{4}\bar\theta\theta F_\mu(x) + \\
&+ \bar\theta\gamma_5\theta G_\mu(x) + \frac{1}{4}\bar\theta i\gamma^\nu\gamma_5\theta A_{\nu\mu}(x) + \frac{1}{4}\bar\theta\theta.\bar\theta^\alpha \chi_{\alpha\mu}(x) + \\
&+ \frac{1}{32}(\bar\theta\theta)^2 D_\mu(x).
\end{aligned} \tag{A.1}
$$

The supercurrent $V_\mu(x,\theta)$ obeying condition (3) has components

$$
\begin{aligned}
V_\mu(x,\theta) &= v_\mu(x) + \bar\theta^\alpha \lambda_{\alpha\mu}(x) + \frac{1}{4}\bar\theta\theta\partial_\mu f(x) + \\
&+ \frac{1}{4}\bar\theta\gamma_5\theta\partial_\mu g(x) + \frac{1}{4}\bar\theta i\gamma^\nu\gamma_5\theta(t_{\nu\mu}(x) + \frac{1}{2}\epsilon_{\nu\mu\lambda\rho}\partial^\lambda v^\rho(x)) + \\
&+ \frac{1}{4}\bar\theta\theta.\bar\theta^\alpha(i\partial\!\!\!/ \lambda_\mu(x) - 2i\partial_\mu\gamma^\nu\lambda_\nu(x))_\alpha +
\end{aligned} \tag{A.2}
$$

$$+\frac{1}{32}(\bar{\theta}\theta)^2(\Box v_\mu(x) - 2\partial_\mu\partial^\nu v_\nu(x)),$$

where

$$\sigma^{\mu\nu}\partial_\mu\lambda_\nu = 0, \quad t_{\mu\nu} = t_{\nu\mu}, \partial^\mu t_{\mu\nu} = \partial_\nu t^\lambda_\lambda. \qquad (A.3)$$

Appendix B

Equation (2) in terms of the component fields (see decompositions (A.1) and (A.2)) reduces to a set of field equations:

$$C_\mu \equiv \frac{1}{3}(D_\mu + \Box A_\mu - 2\partial_\mu\partial^\nu A_\nu + \epsilon_{\mu\nu\lambda\rho}\partial^\lambda A^{\rho\nu}) = -av_\mu, \qquad (B.1)$$

$$\frac{2}{3}\partial_\mu\partial^\nu F_\nu = a\partial_\mu f, \quad \frac{2}{3}\partial_\mu\partial^\nu G_\nu = a\partial_\mu g, \qquad (B.2)$$

$$\frac{1}{6}[4\Box A_{\mu\nu} - 4\partial_\mu\partial^\lambda A_{\nu\lambda} - 4\partial_\mu\partial^\lambda A_{\lambda\nu} + \qquad (B.3)$$

$$+\epsilon_{\nu\lambda\kappa\rho}\partial^\kappa(\delta^\rho_\mu(D^\lambda + \Box A^\lambda) + 2\epsilon^\rho_{\mu\omega\tau}\partial^\omega A^{\tau\lambda})] =$$

$$= -at_{\mu\nu} - \frac{a}{2}\epsilon_{\mu\nu\lambda\rho}\partial^\lambda v^\rho,$$

$$\frac{1}{3}(\Box D_{\mu} - 2\partial_{\mu}\partial^{\nu} D_{\nu} + \Box^{2} A_{\mu} + \epsilon_{\mu\nu\lambda\rho}\ \Box \partial^{\lambda} A^{\rho\nu}) = \quad (B.4)$$

$$= - a(\Box v_{\mu} - 2\partial_{\mu}\partial^{\nu} v_{\nu}),$$

$$\frac{1}{6}[2\Box \psi_{\mu} - 4\partial_{\mu}\partial^{\nu}\psi_{\nu} - 2i\not\partial \chi_{\mu} + \quad (B.5)$$

$$+ i\epsilon_{\mu\nu\lambda\rho}\partial^{\lambda}(\gamma^{\rho}\gamma_{5}\chi^{\nu} + i\not\partial\gamma^{\rho}\gamma_{5}\psi^{\nu})] = -a\lambda_{\mu},$$

$$\frac{1}{6}[2\Box \chi_{\mu} - 2\partial_{\mu}\partial^{\nu}\chi_{\nu} + 2\Box i\not\partial\psi_{\mu} - \quad (B.6)$$

$$- i\epsilon_{\mu\nu\lambda\rho}\partial^{\lambda}(i\gamma^{\rho}\not\partial\gamma_{5}\chi^{\nu} + \Box\gamma^{\rho}\gamma_{5}\psi^{\nu})] =$$

$$= -a(i\not\partial\lambda_{\mu} - 2i\partial_{\mu}\gamma^{\nu}\lambda_{\nu}).$$

Equation (B.4) follows from Eq. (B.1). Inserting Eq. (B.1) into Eq. (B.3) we obtain

$$\Box h_{\mu\nu} - \partial_{\mu}\partial^{\lambda}h_{\lambda\nu} - \partial_{\nu}\partial^{\lambda}h_{\lambda\mu} + \frac{1}{3}\eta_{\mu\nu}\partial^{\lambda}\partial^{\rho}h_{\lambda\rho} -$$

$$- \frac{1}{3}(\Box\eta_{\mu\nu} - \partial_{\mu}\partial_{\nu})h^{\lambda}_{\ \lambda} = -at_{\mu\nu}, \quad (B.7)$$

where $h_{\mu\nu} = 1/2(A_{\mu\nu} + A_{\nu\mu})$. Taking the trace of Eq. (B.7) and putting the result back into Eq. (B.7) we get Eq. (1) with the energy-momentum tensor (see Eq. (A.3))

$$\theta_{\mu\nu} = t_{\mu\nu} - \eta_{\mu\nu} t^{\lambda}{}_{\lambda} .$$

Consider now the fermion equations (B.5) and (B.6). The second is a corollary of the first one. We make a change of the field variables in Eq. (B.5):

$$\phi_\mu = \chi_\mu + i\partial\!\!\!/ \psi_\mu - 2i\gamma_\mu \partial^\nu \psi_\nu$$

and obtain

$$\partial\!\!\!/ \phi_\mu - \frac{1}{3}\partial_\mu \gamma^\nu \phi_\nu - \frac{1}{3}\gamma_\mu \partial^\nu \phi_\nu + \frac{1}{3}\gamma_\mu \partial\!\!\!/ \gamma^\nu \phi_\nu = 2ia\lambda_\mu, \quad (B.8)$$

which is a Rarita-Schwinger equation. It can be rewritten in the form (after multiplying by γ^μ and inserting the result back into Eq. (B.8))

$$\partial\!\!\!/ \phi_\mu - \frac{1}{3}\partial_\mu \gamma^\nu \phi_\nu - \gamma_\mu \partial^\nu \phi_\nu + \frac{1}{3}\gamma_\mu \partial\!\!\!/ \gamma^\nu \phi_\nu = 2iaJ_\mu, \quad (B.9)$$

where

$$J_\mu = \lambda_\mu - \gamma_\mu \gamma^\nu \lambda_\nu$$

is the supersymmetry current (see Eq. (A.3)).

Now it becomes clear that Eq. (2) describes in fact a mixture of superspins 3/2 and 0. The superspin 3/2 multiplet consists of: spin 2 (field $h_{\mu\nu}$, equation (1)); spin 3/2 (field $\phi_{\alpha\mu}$, equation (B.9)); spin 1 is carried by the field C_μ which has no derivatives in Eq. (B.1) and is expressed directly in terms of the source field v_μ. The superspin 0 is represented by the scalar fields $\partial^\mu C_\mu$, $\partial^\mu F_\mu$, $\partial^\mu G_\mu$, $\partial^\mu\partial^\nu h_{\mu\nu}$ and by the spinor field $\partial^\mu\phi_{\alpha\mu}$. All these fields are expressed also in terms of their sources $\partial^\mu v_\mu$, f, g, $\theta^\mu{}_\mu$ and $(\gamma^\mu J_\mu)_\alpha$.

In the free case $(a = 0)$ a number of degrees of freedom drop out. Some of them (the fields without derivatives mentioned above) vanish together with their sources. The remaining become arbitrary due to the gauge invariance (11) (in particular, this invariance provides the necessary invariances of Eq. (1) and (B.9)). Thus at $a = 0$ Eq. (2) describes chiralities ±2 and ±3/2 only.

Reference

1. D.Volkov, V.Soroka. Pisma JETP, 18, 529 (1973).
2. P.Nath, R.Arnowitt. Phys.Lett., 56B, 177 (1975).
3. R.Arnowitt, P.Nath, B.Zumino. Phys.Lett., 56B, 81 (1975).
4. V.Akulov, D.Volkov, V.Soroka. Pisma JETP, 22, 396 (1975).
5. B.Zumino. CERN preprint TH-2120 (1975).
6. D.Z.Freedman, P. van Nieuwenhuizen. S.Ferrara. Phys. Rev., D13, 3214 (1976).
7. S.Deser, B.Zumino.Phys.Lett.,B62,335 (1976).
8. D.Z.Freedman, P. van Nieuwenhuizen, Phys.Rev., D15, 912 (1976).
9. V.Ogievetsky, I. Polubarinov. Ann. Phys., 35, 167 (1965); JETP, 48, 1625 (1965).
10. S.Ferrara, B.Zumino. Nucl.Phys., B87, 207 (1975).
11. V.Ogievetsky, E.Sokatchev. JINR preprint E2-9984, Dubna (1976).

12. E.Sokatchev. Nucl.Phys., B99, 96 (1975).
13. J.Wess. Fermi-Bose Supersymmetry, Karlsruhe preprint (1974).
14. V.Ogievetsky, L.Mezincescu. Usp.Fiz.Nauk, 117, 637 (1975).
15. J.Wess, B.Zumino. Phys.Lett., 49B, 52 (1974).
16. A.Borisov, V.Ogievetsky. TMP, 21, 329 (1974); JINR preprint E2-7684, Dubna (1974).
17. A. Salam, J.Strathdee. Phys.Rev., D11, 1521 (1975).

Received by Publishing Department
on July 22, 1976.