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V.I.Ogievetsky, E.Sokatchev

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1. Introduction

The derivation of the field equations of motion was considered in many papers which used different approaches. One can find in [1,2] references to the literature on this subject.

We should turn to this problem in connection with the superfield theory. The superfields (SF below) are rather complicated objects, each of them contains many fields of integer and half-integer spins. Therefore the derivation of the adequate equations for them is an urgent and nontrivial task. Notice that only the simplest scalar SF-general and chiral - have been considered in detail up to now. The equations of motion for them were conjectured by some apt unification of the equations of motion for fields entering into their composition [3,4]. However, at present some higher SF are also of interest, in particular, the spinor and vector ones. The first one is connected with an attempt to find the general supersymmetric version [5] of the Yang-Mills theory. In such a theory the spinor SF is the gauge SF. The vector SF generated by the supercurrent of Ferrara and Zumino [6] is needed in a possible supersymmetric generalization of the gravitational theory.

The starting point in these models is the derivation of the free equations of motion. Now it becomes impossible to seek for these equations by some sorting due to the higher complexity and due to increasing number of the operator structures. A certain clear algorithm is needed to obtain them. In the present paper such a procedure is proposed. It is based on the properties of the projection operators selecting the irreducible representations. The new feature consists in establishing the role and in using the roots of the projection operators (i.e., the squares of these root operators are the projection operators). Besides we clear up that the Rarita-Schwinger equations for spin-vector field and the Pauli-Fierz equations for symmetric tensor field in fact contain square roots of the projection operators. Therefore the approach under consideration has some pedagogical value in the ordinary field theory also. The use of the projection properties permits one to define easily the Green functions.

The paper is planned as follows. We begin with some necessary information concerning the SF theory and in particular we recall the composition of irreducible supermultiplets in the SF with arbitrary spin. Further we formulate a general idea of the derivation of the equations. It is illustrated then by the examples of the standard equations for the spin 3/2 and 2 fields. Afterwards the equation for the spinor SF is discussed in detail.

II. Preliminaries

We use the following notations: θ_a denotes four-component Majorana spinor coordinates; $\frac{1}{2}\{\gamma_\mu,\gamma_\nu\}=\eta_{\mu\nu}=\mathrm{diag}(+---)$; $\gamma_5=\gamma_0\gamma_1\gamma_2\gamma_3$;

$\sigma_{\mu\nu}=\frac{i}{2}[\gamma_\mu,\gamma_\nu]$; $\epsilon^{0123}=1$; $\bar\theta^\beta=(C^{-1})^{\beta a}\theta_a$,

where $C=i\gamma^0\gamma^2$ is the charge conjugation matrix; $\Box=\partial_\mu\partial^\mu$.

The supersymmetry algebra

$$[J_{\mu\nu},S_a]=-\frac{1}{2}(\sigma_{\mu\nu})_a^{\ \beta}S_\beta,\quad [P_\mu,S_a]=0,$$
$$\{S_a,\bar S^\beta\}=(\gamma_\mu)_a^{\ \beta}P_\mu \tag{1}$$

is realized on the SF

$$\Phi_i(x,\theta)=A_i(x)+\bar\theta^a\psi_{ai}(x)+\frac{1}{4}\bar\theta\theta F_i(x)+\frac{1}{4}\bar\theta\gamma_5\theta G_i(x)$$
$$+\frac{1}{4}\bar\theta i\gamma_\mu\gamma_5\theta A_i^\mu(x)+\frac{1}{4}\bar\theta\theta.\bar\theta^a\psi_{ai}(x)+ \tag{2}$$
$$+\frac{1}{32}(\bar\theta\theta)^2 D_i(x).$$

Here i is some external Lorentz index (e.g., the scalar SF $\Phi(x,\theta)$, the spinor SF $\Phi_a(x,\theta)$, the vector SF $\Phi_\mu(x,\theta)$, etc.). We say that the SF $\Phi_i(x,\theta)$ has <u>external spin</u> j if it obeys the irreducibility conditions for Poincare spin j with respect to the index i. For example, the SF $\Phi_\mu(x,\theta)$ has external spin 1 if $\partial_\mu\Phi^\mu=0$ and spin 0 if $\partial_\mu\Phi_\nu=\partial_\nu\Phi_\mu$.

The irreducible representations of algebra (1) (with nonzero mass) are labelled by the eigenvalues of the second Casimir operator (a generalization of the square of the Pauli-Lubanski vector)/7/

$W^2=-m^2Y(Y+1)$,

where Y is an integer or half-integer called superspin. A representation with superspin Y contains four ordinary (Poincare) spins J :

$$J = Y - \frac{1}{2}, Y, Y, Y + \frac{1}{2}. \qquad (3)$$

In Ref. /8/ it was shown that the SF (2) realize reducible representations of the supersymmetry. Note the remarkable duality: an SF with external spin j contains four irreducible multiplets with superspins Y :

$$Y = j - \frac{1}{2}, j, j, j + \frac{1}{2}. \qquad (4)$$

The projection operators extracting these irreducible representations out of the SF with arbitrary spin j are also calculated and the corresponding supplementary conditions are derived /8/ .

Finally, let us remind an important operator - the spinor derivative D_a /9/ . It obeys the same commutation relations (1) as S_a and anticommutes with S_a

$$\{S_a, D_\beta\} = 0.$$

For this reason all the operators invariant under the supersymmetry transformations are constructed out of D_a . In particular, the projection operators mentioned above and the equation of motion operators we are interested in are polynomials in D_a .

III. Equations of Motion

It is instructive to start with an analysis of some features of the standard equations for the ordinary fields. In the field theory the elementary particles are described by fields which are functions

of the coordinates $\phi_i(x)$ transforming according to some representations of the Lorentz group (i stands for a set of Lorentz indices). At the same time these fields give also representations of the Poincare group (with P_μ realized as $i\partial_\mu$). The latter are reducible (at least because the value of P^2 is not fixed). On the other hand, it is natural to associate such characteristics of the particles as mass and spin with the irreducible representations of the Poincare group. So we have to impose certain conditions on the functions $\phi_i(x)$ that single out the corresponding irreducible part. First of all, we require that the particle momentum P_μ lies on the mass-shell

$$P^2 \phi_i = m^2 \phi_i . \qquad (5)$$

Further, depending on the Lorentz index i , the field can describe one or more spins. It is conventionally assumed that one field describes one spin (as a rule, the highest it contains). In this connection the supplementary conditions are imposed:

$$R_{ij} \phi_j = 0, \qquad (6)$$

where R_{ij} means a set of differential operators. Equation (6) excludes all the spins except the highest one.

However, certain troubles arise when the Klein-Gordon equation (5) and the supplementary conditions (6) are written down separately. In this case the introduction of the interaction can lead to contradictions. Therefore it is strongly preferable to write Eqs.(5) and (6) in the form of a single differential equation

$$\pi_{ij}\phi_j = 0. \tag{7}$$

Now Eqs. (5) and (6) are obtained as corollaries of Eq. (7). For instance, when spin 1 is described by a vector field $a_\mu(x)$ Eqs. (5) and (6) read

$$\Box a_\mu(x) + m^2 a_\mu(x) = 0, \quad \partial^\mu a_\mu(x) = 0.$$

This couple of equations is equivalent to the Proca equation

$$\Box a_\mu(x) - \partial_\mu \partial^\nu a_\nu(x) + m^2 a_\mu(x) = 0. \tag{8}$$

There is also one more requirement which concerns the order of the operator π_{ij} in Eq. (7). One assumes that π_{ij} is of the first order for the Fermi fields and of the second - for the Bose fields.

How to deduce equations of the type (7) satisfying all the requirements formulated above? The answer is prompted by the Proca equation (8). Let us rewrite it in the form

$$-\Box(\Pi^1)_\mu{}^\nu a_\nu = m^2 a_\mu, \tag{9}$$

where

$$(\Pi^1)_{\mu\nu} = \eta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\Box}$$

is the projection operator for spin 1. Now it is clear that the field $a_\mu(x) = -\Box/m^2.(\Pi^1)_\mu{}^\nu a_\nu(x)$ obeys the supplementary condition (of the type (6)) $\partial^\mu a_\mu = 0$. Then $(\Pi^1)_\mu{}^\nu a_\nu = a_\mu$ and Eq. (9) reduces to Eq. (5).

This example suggests a general idea. Let Π_{ij} be the projection operator extracting the representation we are dealing

with out of the field $\phi_i(x)$. Multiply it by the $-\Box$ in power q sufficient to cancel the nonlocality and write the equation

$$(-\Box)^q \Pi_{ij}\phi_j = (m^2)^q \phi_i. \tag{10}$$

Thus the irreducible representation is singled out. However, the order of Eq. (10) may be too high. Suppose that, e.g., $q = 2$ and a second order equation is required. Then one can find (in general, not uniquelly) an operator $\pi = \sqrt{(-\Box)^2\Pi}$ defined by

$$\pi_{ij}\pi_{jk} = (-\Box)^2 \Pi_{ik} \tag{11}$$

and write an equation of the right order

$$\pi_{ij}\phi_j - m^2\phi_i = 0. \tag{12}$$

Equation (10) follows from Eq. (12)

$$(-\Box)^2\Pi_{ij}\phi_j = \pi_{ik}\pi_{kj}\phi_j = \pi_{ik}(m^2\phi_k) = m^4\phi_i.$$

This means that Eq. (12) selects the same representation.

A classical illustration for this "root" trick is the Dirac equation. The bispinor field $\psi_\alpha(x)$ describes spin 1/2 only. So here the projection operator is simply 1 and Eq. (10) takes the form of Eq. (5)

$$-\Box\psi_\alpha(x) = m^2\psi_\alpha(x).$$

We need a first order equation so we find the $\pi_\alpha{}^\beta$

$$\pi = \sqrt{-\Box} = i\,\not\partial$$

and arrive to the common Dirac equation

$$i\,\not\partial\psi - m\psi = 0.$$

The derivation of some other known field equations gives nontrivial examples how to handle the "root method". So, spin 3/2 is usually described by a spin-vector field $\psi_{a\mu}(x)$. At the same time this field contains two spins 1/2. The supplementary conditions excluding these superfluous spins are

$$\partial_\mu \psi_a^{\ \mu} = 0, \qquad (\gamma_\mu \psi^\mu)_a = 0.$$

We want to find an equation of motion containing these supplementary conditions. Consider the projection operator extracting spin 3/2 out of the field $\psi_{a\mu}$

$$\Pi_{\mu\nu,\alpha\beta} = \eta_{\mu\nu} 1_{\alpha\beta} - \frac{2}{3}\frac{\partial_\mu \partial_\nu}{\square} 1_{\alpha\beta} - \frac{1}{3}(\gamma_\mu \gamma_\nu)_{\alpha\beta} + \frac{1}{3\square}[\not\partial(\partial_\mu \gamma_\nu - \partial_\nu \gamma_\mu)]_{\alpha\beta}. \tag{13}$$

Then we write down the localized operator $(-\square)\Pi_{\mu\nu,\alpha\beta}$. It includes second order derivatives and we need a first order equation ($\psi_{a\mu}$ is a fermion field). So we have to extract the square root of $(-\square)\,\Pi$. There exists a one-parameter set of such roots. In the equations obtained a one-parameter change of field variables $\psi_\mu \to \psi_\mu + \beta \gamma_\mu \gamma^\nu \psi_\nu$ can be made. Finally, the restriction that the equations must correspond to hermitian Lagrangians leads to the Rarita-Schwinger set of equations

$$(\not\partial - m)\psi_\mu - a(\partial_\mu \gamma^\nu \psi_\nu + \gamma_\mu \partial^\nu \psi_\nu) + \frac{1}{2}(3a^2 - 2a + 1)\gamma_\mu \not\partial \gamma^\nu \psi_\nu + (3a^2 - 3a + 1) m \gamma_\mu \gamma^\nu \psi_\nu = 0. \tag{14}$$

Here a is an arbitrary real parameter.

The next example is the symmetric tensor field $h_{\mu\nu}$ describing spin 2 and superfluous spins 0 and 1 too. The corresponding supplementary conditions

$$\partial^\mu h_{\mu\nu} = 0, \quad h^\mu_{\ \mu} = 0$$

should follow from the equations of motion. The projection operator for spin 2 is ($\bar\eta_{\mu\nu} = \eta_{\mu\nu} - \partial_\mu \partial_\nu / \square$)

$$\Pi_{\mu\nu,\lambda\rho} = \frac{1}{2}\bar\eta_{\mu\lambda}\bar\eta_{\nu\rho} + \frac{1}{2}\bar\eta_{\mu\rho}\bar\eta_{\nu\lambda} - \frac{1}{3}\bar\eta_{\mu\nu}\bar\eta_{\lambda\rho} \tag{15}$$

and it has terms with $\square^{-2}$. The operator $(-\square)^2\Pi$ is local but has too high order derivatives. So a square root is required again. Just as in the previous case we obtain a one-parameter set, then introduce the changes $h_{\mu\nu} \to h_{\mu\nu} + \beta \eta_{\mu\nu} h^\lambda_{\ \lambda}$, restrict ourselves to the Lagrangian-type equations and come to the common Fierz-Pauli set of equations

$$\square h_{\mu\nu} - \partial_\mu \partial^\lambda h_{\lambda\nu} - \partial_\nu \partial^\lambda h_{\lambda\mu} + \frac{1+a}{1+2a}(\eta_{\mu\nu}\partial^\lambda \partial^\rho h_{\lambda\rho} + \partial_\mu \partial_\nu h^\lambda_{\ \lambda}) - \frac{2+4a+3a^2}{2(1+2a)^2}\eta_{\mu\nu}\square h^\lambda_{\ \lambda} - \frac{1+a+a^2}{(1+2a)^2} m^2 \eta_{\mu\nu} h^\lambda_{\ \lambda} + m^2 h_{\mu\nu} = 0. \tag{16}$$

Finally, a few words about the massless case. The massless equations are obtained from the massive ones simply putting $m = 0$. What about the irreducibility conditions now? Note that the massless equations do not lead to corollaries of type (5) and (6). Moreover, the character of the representations at $m=0$ changes substantially and the conditions (6) lose their sense now. They are replaced by one or more gauge invariances of the equation which make the superfluous degrees of freedom entirely arbitrary, i.e., unessential. For instance, the Proca equation (8) becomes at $m=0$ invariant under the gauge transformation

$$a_\mu(x) \to a_\mu(x) + \partial_\mu \phi(x)$$

where $\phi(x)$ is an arbitrary scalar function. The Rarita-Schwinger equations (14) are invariant at $m=0$ under substitutions

$$\psi_\mu(x) \to \psi_\mu(x) + \partial_\mu \lambda(x) + \frac{a-1}{2a} \not\partial \gamma_\mu \lambda(x)$$

with arbitrary spinor function $\lambda(x)$, etc.

Now we can transfer all these considerations to the SF case. Note one peculiarity only. Each SF contains bosons as well as fermions. To establish the right order of the SF equation operator the following arguments are used. The SF equations must follow from the action principle [10,11]

$$S = \int d^4x\, d^4\theta\, \mathfrak{L}(x, \theta). \qquad (17)$$

Here $\int d^4\theta$ is understood as a Grassmann integral [12], i.e., $\int \theta_a d\theta^\beta = \delta_a^{\ \beta}$. This means that the dimensionality $[d\theta] = -\frac{1}{2}$ (in cm) because $[\theta] = 1/2$. Then $[\mathfrak{L}] = -2$ since $[S] = 0$ (in units $\hbar = c = 1$). Let us write the kinetic term in the Lagrangian in the form

$$\mathfrak{L}_k = \Phi_i \pi_{ij} \Phi_j ,$$

where π_{ij} is the equation of motion operator. Now it is clear that

$$[\pi] = -2 - 2[\Phi]. \qquad (18)$$

Finally, the dimensionality of the SF is determined by the dimensionality of the component field with the leading spin. To make clear this statement consider the general scalar SF $\Phi(x, \theta)$. Suppose we are interested in the highest superspin $Y = 1/2$ which includes the leading spin 1. This spin is carried by the field $A_\mu(x)$ (see the decomposition (2)) and it is natural to ascribe to it the canonical dimensionality cm^{-1}. Therefore the SF dimensionality equals 0 and according to Eq. (18) $[\pi] = -2$. This is just the dimensionality of the localized projection operator for superspin 1/2 [8,9]:

$$\pi = (-\Box)\Pi^{1/2}, \quad \Pi^{1/2} = 1 + \frac{1}{4\Box}(\bar{D}D)^2.$$

Thus we find the equation [4,11]

$$(\Box + \frac{1}{4}(\bar{D}D)^2)\Phi + m^2\Phi = 0. \qquad (19)$$

As was to be expected, the irreducibility condition $\bar{D}D\Phi = 0$ (see Ref. [8,9]) follows from Eq. (19) at $m \neq 0$ and at $m = 0$ there arises a gauge invariance

$$\Phi \to \Phi + \bar{D}D\Lambda ,$$

where $\Lambda(x, \theta)$ is an arbitrary scalar superfunction. If one writes Eq. (19) in terms of the component fields and eliminates the auxiliary fields one obtains a set of the standard equations for a vector, a scalar and two spinors (at $m = 0$ - for a vector and a spinor).

Now we turn to the spinor SF.

IV. The Spinor Superfield

The spinor SF is defined by its decomposition

$$\Psi_a(x,\theta)=\psi^{(1)}_a(x)+\bar\theta^{\beta}\psi^{(2)}_{\beta a}(x)+\frac{1}{4}\bar\theta\theta\psi^{(3)}_a(x)+\frac{1}{4}\bar\theta\gamma_5\theta\psi^{(4)}_a(x)+$$

$$+\frac{1}{4}\bar\theta i\gamma^{\mu}\gamma_5\theta\psi^{(5)}_{a\mu}(x)+\frac{1}{4}\bar\theta\theta.\bar\theta^{\beta}\psi^{(6)}_{\beta a}(x)+\frac{1}{32}(\bar\theta\theta)^2\psi^{(7)}_a(x); \qquad (20)$$

$$\psi^{(2)}_{\beta a}=(U_1 1+U_2\gamma_5+iU_3^{\mu}\gamma_{\mu}+iU_4^{\mu}\gamma_{\mu}\gamma_5+iU_5^{\mu\nu}\sigma_{\mu\nu})_{\beta a}$$

$$\psi^{(6)}_{\beta a}=(u_1 1+u_2\gamma_5+iu_3^{\mu}\gamma_{\mu}+iu_4^{\mu}\gamma_{\mu}\gamma_5+iu_5^{\mu\nu}\sigma_{\mu\nu})_{\beta a}.$$

If Ψ_a is a Majorana SF then all the Fermi fields - $\psi^{(1)}_a(x)$, $\psi^{(3)}_a(x)$, $\psi^{(4)}_a(x)$, $\psi^{(5)}_a(x)$, $\psi^{(7)}_a(x)$ are also Majorana fields and all the Bose-fields $U_{1\ldots5}$, $u_{1\ldots5}$ are real.

These fields involve a considerable number of spins among which the leading spin 3/2 is the most interesting (it is connected with the field $\psi^{(5)}_{a\mu}$). The leading spin enters into the supermultiplet with the highest superspin 1, which is singled out by the projection operator /8/

$$(\Pi^1)_a^{\beta}=\frac{3}{4}\left(1+\frac{(\bar D D)^2}{4\Box}\right)1_a^{\beta}+\frac{1}{8\Box}i\partial_{\mu}\bar D i\gamma_{\nu}\gamma_5 D(\sigma^{\mu\nu}\gamma_5)_a^{\beta} \qquad (21)$$

or equivalently by the supplementary conditions

$$\bar D D\Psi_a=0,\quad \bar D^{a}\Psi_a=0. \qquad (22)$$

We wish to find an equation which describes only this superspin 1 but not superspins 1/2 and 0.

Let the spinvector field $\psi^{(5)}_{a\mu}$ have the canonical dimensionality $cm^{-3/2}$. Then the dimensionality of the SF $\Psi_a(x,\theta)$ is $cm^{-1/2}$ and according to Eq.(18) the equation operator must have dimensionality cm^{-1}. The localized projection operator (21) has dimensionality cm^{-2} and therefore we have to extract its square root. There exists a family of such roots with arbitrary parameters ξ, η:

$$\pi(\xi,\eta)=\frac{1}{8}[(\cos\xi+\gamma_5\sin\xi)(6i\partial\!\!\!/-i\gamma_{\mu}\gamma_5\bar D i\gamma^{\mu}\gamma_5 D)+$$

$$+\cos\eta.(\bar D D+3\gamma_5\bar D\gamma_5 D)+\sin\eta.\gamma_5(3\bar D D+\gamma_5\bar D\gamma_5 D)].$$

However, all these roots are in fact equivalent because the equations following from them are connected which each other by γ_5 -transformations:

$$\Psi_a\to(e^{\xi\gamma_5}\Psi)_a,\quad \theta_a\to(e^{\eta\gamma_5}\theta)_a,$$

$$\Psi'(x,\theta')=e^{\xi\gamma_5}\Psi(x,\theta).$$

Therefore we choose one of these roots ($\xi=\eta=0$) and write down the equation

$$\frac{1}{8}(6i\partial\!\!\!/+\bar D D+3\gamma_5\bar D\gamma_5 D-i\gamma_{\mu}\gamma_5\bar D i\gamma^{\mu}\gamma_5 D)\Psi-m\Psi=0. \qquad (23)$$

Using the fact that Eq.(23) contains $\pi=\sqrt{(-\Box)\Pi}$ one can easily find the inverse operator

$$\frac{1}{\pi - m} = -\frac{\pi + m}{\Box + m^2}\left[1 + \frac{\Box}{m^2}(1 - \Pi)\right]$$

which defines the Green function and is needed for the perturbation calculations.

Equation (23) can be obtained from the action principle

$$S = \int d^4x d^4\theta \, \mathcal{L}(x,\theta) = \frac{1}{2}\int d^4x d^4\theta \, \bar{\Psi}(\pi - m)\Psi .$$

One can represent the Lagrange density in a more convenient form

$$\mathcal{L} = \frac{1}{32}\left[\bar{\Psi} i \partial\!\!\!/ \Psi - \frac{1}{2}(\bar{D}\gamma_\mu \Psi)^2 + \frac{1}{12}(\bar{D}\sigma_{\mu\nu}\Psi)^2\right] - \frac{1}{2} m \bar{\Psi}\Psi \qquad (24)$$

using the algebraic properties of the spinor derivatives D_a /9/ and integrating by parts.

To be convinced once more that this Lagrangian describes the superspin 1 multiplet it is useful to write it down in terms of component fields (see decomposition (20)). The final result will be maximally compact and illustrative if one excluded the superfluous degrees of freedom. This is usually done by means of the equations of motion. We prefer another procedure which is more legitimate. This procedure consists in suitable changes of the field variables (which are a propos also suggested by the equations of motion). After these changes the superfluous fields remain still in the Lagrangian but it becomes evident that they are unessential (as the equations of motion for them are trivial).

So the fermionic component fields are replaced by

$$\psi^{(1)} \equiv \psi$$

$$\psi^{(3)} = -\frac{1}{4}\phi + \frac{1}{4}\gamma_5 \chi + i\gamma_5\gamma_\mu \psi^\mu - \frac{1}{3}(i\partial\!\!\!/ - m)\psi$$

$$\psi^{(4)} = \frac{3}{4}\chi - \frac{1}{4}\gamma_5 \phi + \gamma_5 (i\partial\!\!\!/ - m)\psi \qquad (25a)$$

$$\psi^{(5)}_\mu = \psi_\mu + \frac{1}{3} i\gamma_5 (i\partial_\mu - m\gamma_\mu)\psi + \frac{1}{4} i\gamma_5\gamma_\mu \phi - \frac{1}{4} i\gamma_\mu \chi$$

$$\psi^{(7)} = \lambda - \frac{i}{2}\partial\!\!\!/ (\phi + \gamma_5 \chi) + \frac{2}{3}\gamma_5 (\partial\!\!\!/ \gamma_\mu - \partial_\mu)\psi^\mu - \Box\psi + \frac{2}{3} m (i\partial\!\!\!/ - 4m)\psi + m\gamma_5 \chi$$

and the bosonic ones by

$$U_1 = A, \quad U_2 = B, \quad U_{3\mu} = V_\mu$$

$$U_{4\mu} = A_\mu - \frac{1}{m}\epsilon_{\mu\lambda\sigma\kappa}\partial^\lambda E^{\sigma\kappa}$$

$$U_{5\mu\nu} = E_{\mu\nu} + \frac{1}{2m}(\partial_\mu V_\nu - \partial_\nu V_\mu)$$

$$u_1 = a + \partial_\mu V^\mu$$

$$u_2 = b - \partial_\mu A^\mu - mB$$

$$u_{3\mu} = v_\mu + \partial_\mu A + 2\partial^\nu E_{\nu\mu} + 2m V_\mu + \frac{1}{m}(\Box V_\mu - \partial_\mu \partial^\nu V_\nu)$$

$$u_{4\mu} = a_\mu - \partial_\mu B + \epsilon_{\mu\lambda\sigma\kappa}\, \partial^\lambda E^{\sigma\kappa}$$

$$u_{5\mu\nu} = e_{\mu\nu} + \frac{1}{2}(\partial_\mu V_\nu - \partial_\nu V_\mu) + \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}\, \partial^\lambda A^\rho -$$

$$- 2mE_{\mu\nu} - \frac{1}{m}(\Box E_{\mu\nu} + \partial_\mu \partial^\lambda E_{\nu\lambda} - \partial_\nu \partial^\lambda E_{\mu\lambda}). \qquad (25b)$$

Then after the integration over $d^4\theta$ the Lagrangian for the fields takes the form

$$\mathcal{L}(x) = \bar\psi^\mu i\partial\!\!\!/ \psi_\mu - 2\psi^\mu i\partial_\mu \gamma^\nu \psi_\nu + \bar\psi^\mu \gamma_\mu i\partial\!\!\!/ \gamma^\nu \psi_\nu +$$
$$+ m\bar\psi^\mu \gamma_\mu \gamma^\nu \psi_\nu - m\bar\psi^\mu \psi_\mu - \frac{4}{3} m^2 (\bar\psi i\partial\!\!\!/ \psi - m\bar\psi\psi) + \qquad (26)$$
$$+ \frac{3}{4}\bar\chi\gamma_5\chi + \frac{1}{4} m\bar\phi\phi + 8(V^\mu \Box V_\mu - V^\mu \partial_\mu \partial^\nu V_\nu) +$$
$$+ 8m^2 V_\mu^2 - 8E^{\mu\nu}\epsilon_{\mu\nu\lambda\rho}\, \partial^\lambda \partial_\kappa \epsilon^{\rho\kappa\alpha\beta} E_{\alpha\beta} - 16m^2 E_{\mu\nu}^2 +$$
$$+ 4e_{\mu\nu}^2 - 2v_\mu^2 + 8maA - 8ma^\mu A_\mu - 4b^2 + 4m^2 B^2 .$$

The adequacy of the choice of the SF equation is confirmed. Really the irreducible superspin 1 representation contains spin 3/2 (spinvector field ψ_μ having the standard Rarita-Schwinger Lagrangian), two spins 1 (vector V_μ and antisymmetric tensor $E_{\mu\nu}$ fields with correct Lagrangians) and spin 1/2 (the field $m\psi$). All other fields are evidently unessential. Further, it is

easy to verify that fields ψ_μ , V_μ , $E_{\mu\nu}$ and $m\psi$ form an invariant subspace under the supersymmetry transformations.

Now we are going to discuss the zeromass case. The corresponding Lagrangian is obtained by setting in Eq.(24) $m=0$. However we cannot set $m=0$ in Eq. (26) because this form is obtained from Eq.(24) by the singular at $m \to 0$ field changes (25). This can be explained as follows. The original decomposition (20) contains the spinor field ψ with dimensionality $cm^{-1/2}$. Multiplying it by the mass we obtain the canonical dimensionality $cm^{-3/2}$. The corresponding part in the Lagrangian (26) is proportional to m^2 and this ensures its vanishing as $m \to 0$. The situation with the bosonic fields is different. In the decomposition (20) there are no fields having spin 1 and dimensionality less than the canonical one cm^{-1} which would enter into the Lagrangian (26) with a multiplier $\sim m$ and would vanish as $m \to 0$. Therefore at $m=0$ in the field changes (25) one has to omit all the terms which contain m both in the denominators and in numerators. Then the Lagrangian is written down as

$$\mathcal{L}(x) = \bar\psi^\mu i\partial\!\!\!/ \psi_\mu - 2\bar\psi^\mu i\partial_\mu \gamma^\nu \psi_\nu + \bar\psi^\mu \gamma_\mu i\partial\!\!\!/ \gamma^\nu \psi_\nu + \qquad (27)$$
$$+ \frac{3}{4}\bar\lambda\gamma_5\chi + 16(V^\mu \Box V_\mu - V^\mu \partial_\mu \partial^\nu V_\nu) - 8E^{\mu\nu}\epsilon_{\mu\nu\lambda\rho}\, \partial^\lambda a^\rho +$$
$$+ 4e_{\mu\nu}^2 - 2v_\mu^2 - 4b^2 .$$

The essential fields here are ψ_μ (chiralities $\pm 3/2$) and V_μ (chiralities ± 1). (Recall that the zeromass supermultiplets include only two successive chiralities.) The equations for the other fields are trivial. In particular

$$\partial^\lambda \epsilon_{\rho\lambda\mu\nu} E^{\mu\nu} = 0, \quad \epsilon_{\mu\nu\lambda\rho} \partial^\lambda a^\rho = 0$$

and we have

$$E_{\mu\nu} = \partial_\mu w_\nu - \partial_\nu w_\mu , \quad a_\rho = \partial_\rho s , \tag{28}$$

where w_μ and s are arbitrary vector and scalar fields, respectively. In other words the fields $E_{\mu\nu}$ and a_ρ turn out to be unessential due to the invariance of the Lagrangian (27) under the gauge transformations

$$E_{\mu\nu} \to E_{\mu\nu} + \partial_\mu w_\nu - \partial_\nu w_\mu , \quad a_\rho \to a_\rho + \partial_\rho s .$$

The Lagrangian (27) is also invariant under the standard gauge transformations of the vector and spinvector fields

$V_\mu \to V_\mu + \partial_\mu f$, f-arbitrary scalar function (29a),

$\psi_\mu \to \psi_\mu + \partial_\mu \xi$, ξ-arbitrary spinor function (29b).

All these transformations have an SF form. Indeed, the Lagrangian (24) allows the gauge transformations (at $m = 0$)

$$\Psi_a \to \Psi_a + D_a \Lambda , \tag{30a}$$

$$\Psi_a \to \Psi_a + (i \not\partial \gamma_5 D)_a \Sigma , \tag{30b}$$

where $\Lambda(x,\theta)$ and $\Sigma(x,\theta)$ are arbitrary scalar superfunctions. The first one, (30a), is connected with the invariance (29a) of the Proca equation and it enables us to construct the generalization of the Yang-Mills theory [5] mentioned in the Introduction. The second transformation (30b) provides the gauge freedom (29b) of the Rarita-Schwinger equation and causes some troubles when introducing interaction [5].

Ending this section we wish to stress once more the compactness and effectiveness of the SF formalism in comparison with the treatment of the supersymmetric models in terms of the field components.

Unfortunately we are not yet accustomed enough to the SF language. Because of this we have often need in lengthy and tiresome calculations in terms of field components in order to achieve greater confidence and apparent clarity.

In conclusion we note that the equations of motion for other SF can be obtained in a similar way. In connection with the supergravity we are especially interested in the vector SF, the Lagrange theory of which will be discussed in a separate paper.

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