СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

ДУБНА

E2 - 9958

I.G.Bogatskaya, S.M.Eliseev, G.M.Zinovjev

STATISTICAL MECHANISM OF MULTIPARTICLE PRODUCTION IN HADRON-NUCLEI COLLISIONS

1976

E2 - 9958

I.G.Bogatskaya, S.M.Eliseev, G.M.Zinovjev

STATISTICAL MECHANISM OF MULTIPARTICLE PRODUCTION IN HADRON-NUCLEI COLLISIONS

An increasing interest in high energy hadron-nuclei collisions could be observed recently. It was caused mainly by the fact that theorists have realized the importance of these processes as an essential source of information about the hadron interaction dynamics [1,2]. In particular, one may hope that the study of hadron-nuclei collisions will help us to find out what longitudinal distances and time scales are essential in strong interactions at high energies. Some arguments about the increase of longitudinal distances with energy within the parton model have generated an interesting mechanism of hadron-nuclei interaction and have induced predictions which at present are in quite a good agreement with experimental data [3,4]. As is known, the parton model implies that a fast moving hadron dissociates according to a certain multiperipheral mechanism into a system of point-like components. Further only the "wee" partons interact with the target. A rather strong interaction can occur between "wee" partons also, and the parton lifetime proves to be proportional to its momentum. Being applied to the hadron-nucleus scattering this conception gives rise to a certain natural space scale, the radius of a nucleus r, distinguishing the partons. Those partons which have lifetime $\tau \geq r$ or momentum $p \geq rm^2$ (where m is some characteristic mass) do not interact with the nucleus because they have no time to dissociate up to the "wee" partons. The rest of them in principle, can interact with the nucleus and assuming that their density does not change with increasing energy of initial hadron one can rather easy understand, e.g., the limiting fragmentation of the nucleus [3,4].

Quantitative estimations of the secondary spectra within the parton model, as is known, are based on the assumption that their shapes are similar to those of the parton distributions. However, as is noted in [4] and shown in [5], this similarity can be slightly violated in hadron-nuclei processes due to the rise of multiladder exchanges which effect manifests in difference between inclusive spectra of hadron-nucleus and hadron-hadron collisions.

A detailed study of the behaviour of inclusive spectra is evidently related to the investigation of the peculiarities of intranuclear interactions and parton-hadron transition. In the present paper we consider the possibility to describe the data on hadron-nucleus interaction within a statistical model allowing also in some sense an interpretation of parton-hadron transition as developing on the basis of a certain statistical mechanism. Note that effectiveness of the statistical description of a parton system has been advocated earlier [6] and in our opinion this description is valid in our problem for partons having $\tau \geq r$, i.e., those being able to interact with the nucleus.

Turning to the analysis of the experimental data [2] one can infer that the models of hadron-nucleus scattering taking into account a possible formation of nucleon systems allowing the collective description seem to be obviously favoured over the models treating the nucleus as a set of independent nucleons. This observation supports our assumption that an incident high-energy hadron interacts with the tube of nucleons lying on its path. We assume also that this collision results in the formation of a certain parton system moving to the thermodynamical equilibrium due to the parton interactions and generating secondaries after leaving the nucleus. If these interactions proceed via parton exchanges between each two neighbouring partons, the Hamiltonian of such a system allows the decomposition into normal modes [6].

The idea of normal modes brings to the following expression for the energy of the state with k_i quanta of the corresponding normal mode in accordance with the phonon theory

$$E\{k_i\} = \sum_{i=1}^{k_c} k_i E_i \qquad (1)$$

(neglecting the contribution of the ground state which does not influence the forthcoming calculations). Hence the total number of states is defined as

$$Q = \sum_{\{k_i\}} \exp(-\beta E\{k_i\}). \qquad (2)$$

Considering the events when partons behave collectively we use, for convenience, the approximation of continuum, i.e., we pass to the string /6/. In this case the cut present in (1) and (2) can, in principle, be introduced by analogy with the Debye cut in the theory of phonons. However, we treat k_c as a certain parameter the physical meaning of which will be clarified later.

If all hadrons are assumed to be produced from the parton phase only, one can choose the condition of parton-hadron transition as follows

$$\sum_{i=1}^{k_c} k_i = n. \qquad (3)$$

It means that the probability P_n of particle generation by a system formed in a hadron-nucleus collision is given by expression (2) which can be rewritten in the form

$$P_n = N \sum_{k_1} \sum_{k_2} \dots \sum_{k_c} \exp(-\beta \sum_{i=1}^{k_c} k_i E_i), \qquad (4)$$

where N is an appropriate normalizing constant. The direct calculation of (4) with constraint (3) is extremely difficult so in statistical mechanism one usually passes to considering of the grand partition function and obtains P_n performing path integration by means of the steepest-descent

method. In the present case we prefer to change summation by integration in (4). This transition facilitates taking into account condition (3) and makes possible the obtaining of accurate analytic expression. We rewrite (4) as the product of integrals

$$P_n = N' \int dk_1 \int dk_2 \dots \int dk_c \delta(n - \sum_{i=1}^{k_c} k_i) \exp(-\beta \sum_{i=1}^{k_c} k_i E_i), (5)$$

where normalization is to be performed anew and generally speaking differs from N in (4). The validity of this transition from (3) and (4) to definition (5) naturally needs to be proved specially. We omit here the discussion of this question noting only that in the approximation $\beta E_i \ll 1$ taken in the present model for the reasons sketched below, the path integration and calculation of (5) bring to the same result.

Performing the multiple integration in (5) one gets the multiplicity distribution

$$P_n = N' \sum_{i=1}^{k_c} \frac{\exp(-\beta n E_i)}{\beta^{k_c - 1} \prod_{i \neq j} (E_i - E_j)}. \qquad (6)$$

As a consequence, we obtain the following normalization constant

$$N' = \{ \sum_{i=1}^{k_c} [\beta^{k_c - 1} \prod_{i \neq j} (E_i - E_j)(1 - e^{-\beta E_i})] \}^{-1} . \qquad (7)$$

The energy spectrum of the string, as is known, is given by $E_j = jE_0$. In fact, in our model the values of j are bounded, i.e., $j = 1,2,\dots k_c$. Inserting this spectrum into (6) after not very complicated but rather cumbersome calculations we arrive at

$$P_n = k_c E_0 \beta \exp(-\beta n E_0)[1-\exp(-\beta n E_0)]^{k_c - 1}. \qquad (8)$$

This expression for distribution implies the following behaviour of the average multiplicity of secondaries from hadron-nucleus collisions

$$\bar{n} = \frac{1}{\beta E_0}\sum_{j=1}^{k_c}\frac{1}{j} = \frac{1}{\beta E_0}[C+\ln k_c + \frac{1}{2k_c} - 0(\frac{1}{k_c^2})] \equiv \frac{f(k_c)}{\beta E_0}, \qquad (9)$$

C - is the Euler constant.

To make meaningful the comparison of (8) and (9) with experimental data, the above expressions should naturally be averaged over all possible positions of the tube in the nucleus and over the number of nucleons in it. In our model that results in averaging over the parameter k_c. It is clear that using the appropriate phenomenological weight functions one can achieve sufficiently good agreement of the obtained results with experimental data. However, here we are mainly concerned with qualitative estimates and general agreement with the data therefore for simplicity we consider below the distributions obtained for certain average value of $\bar{k}_c$.

Before starting the discussions of the results derived above, consider the validity of the proposed description. Evidently, the initial energy s dependence enters into expressions (8) and (9) through $\beta = 1/T$. Nevertheless, establishing of the detalied relation between s and temperature of the system T in the framework of statistical description appears to be a question far from being trivial /7/. Therefore, to avoid complication of the problem, we can, in principle, choose this relation on the basis of experimental behaviour of $\bar{n}$. It means that if we assume the logarithmic dependence of the average multiplicity on initial energy in accordance with the parton model (the pure statistical description seems to favour fractional power of s), the sense of the condition $\beta E_i \ll 1$ becomes clear. Due to the limitedness of E_i it proves to be the same as the standard assumption of the parton model which validity, as is known, requires not only large initial energy but large ln s too /6/.

Considering (9) from such a phenomenological point of view one can obtain, using the present model, the ratio between the mean number $\bar{n}$ of the secondaries produced in the proton-nucleus collision and the mean number of secondaries from the proton-proton collision at the same energy $\bar{n}_H$

$$R = \bar{n}/\bar{n}_H = \sum_{j=1}^{k_c}\frac{1}{j} = f(k_c). \qquad (10)$$

As is seen from (10), this ratio does not depend on the initial energy that agrees with recent experimental results. Now in accordance with (10) we plot R as a function of k_c taking $\bar{n}_H$ for proton-proton interactions from experiment, then compare this curve to the experimental data on the dependence of R[8] on the average number of collisions $\bar{\nu}$ with the individual nucleons inside the nucleus and find the remarkable coincidence of both the plots. This fact appears to be rather essential for general scheme of our considerations for it helps to attribute a physical meaning to the last unclear parameter in our model, viz., k_c. It seems to be coincident with the number of collisions or with that of nucleons in the tube, that is the same, and herewith $\bar{\nu} \approx \bar{k}_c \sim A^{1/3}$. In (10) it brings to the dependence of R on the atomic number of the nucleus being in an excellent agreement with experiment. Also the fact seems rather important that multiplicity distribution (8) possesses also the property of KNO-scaling /9/. In fact, it follows from (8) and (9) that the product

$$\bar{n}P_n = \Psi(z) \tag{11}$$

does not depend on initial energy because the function $\Psi(z)$ is determined in the form

$$\Psi(z) = k_c f(k_c) \exp[-f(k_c)z]\{1-\exp[-f(k_c)z]\} \tag{12}$$

and $z = n/\bar{n}$.

The dispersion ratio $D=\sqrt{\overline{n^2}-\bar{n}^2}$ also displays the energy independence. These results are in good agreement with recent experimental data. Moreover, the data reveal only slight difference between KNO-functions for hadron-nucleus and hadron-hadron processes. In our case, however, tracing this result is not a simple task because, as we noted above, $\bar{n}$ and P_n should be integrated along with certain weight function of k_c. Nevertheless, being confined to the approximation of average k_c one can prove that the shape of the function KNO is similar to that observed in experiment though weak ($\sim \ln A$) dependence on the atomic number is present.

In conclusion we note that the above considerations can serve as a basis for concrete realization of mechanism of the multiple generation in the framework of the quark-gluon model /10/. The valence quarks of the initial hadron, as before, will bring to explanation of the existence of leading particles in hadron-nucleus collisions, and the glue of the incident hadron interacting strongly with the glue of nucleons composing the nucleus will bring to the formation of a statistical system similar to the one considered in the present model. This system will be the source of secondaries (because dynamics of the glue field is unknown).

One can see that simple considerations developed in the paper make it possible to understand a number of experimental facts and to interpret rather naturally cetrain theoretical statements. In spite of that we think our results should mainly be considered not as arguments in favour of uniqueness of the proposed model but as an evidence for the validity of statistical approach to the description of hadron-nucleus interaction at high energy.

The authors are grateful to M.I.Gorenshtein, V.S.Stavinsky and V.D.Toneev for stimulating discussions. One of us (G.M.Z.) has profited much from conversation with L.Susskind who turned his attention to the fruitful idea of the Debye cut in studying the nature of scaling behaviour.

REFERENCES

1. E.L.Feinberg. Problems of Theoretical Physics, a Memorial volume to Igor Tamm. Nauka, Moscow, 248 (1972).
2. K.Gottfried. CERN preprint TH 1735(1972).
3. O.V.Kancheli. JETP Letters, 18, 465 (1973).
4. N.N.Nikolaev, V.I.Zakharov. L.D.Landau ITP preprint (1975); N.N.Nikolaev. Preprint L.D.Landau ITP-18 (1975).
5. J.Koplik and A.N.Mueller. University of Columbia. Preprint CO-2271-59(1975).
6. J.Kogut and L.Susskind. Physics Reports, C8, 78 (1973); H.B.Nielsen, L.Susskind and A.B.Kraemmer. Nucl.Phys., B28, 34 (1971).
7. M.I.Gorenstein, V.A.Miransky, V.P.Shelest, G.M.Zinovjev, and H.Satz. Nucl. Phys., B76, 473 (1974).
8. B.Andersson, J.Otterlund. Lund University, Preprint LUIP-CP-74-15 (1974).

9. Z.Koba, H.B.Nielsen, and P.Olesen. Nucl. Phys., B40, 317 (1972).
10. L. Van Hove. CERN preprint TH 2121 (1976).

Received by Publishing Department
on July 26, 1976.