ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
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ДУБНА

E2 - 9936

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1976

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Submitted to "Nuclear Physics"

INTRODUCTION

The problem of understanding the elementary particle dynamics puts forward alongside with the development of the general ideas also the preliminary problem of systematization in quantum numbers [1].

The aim of the present paper is to find the dependence of hadron-hadron total cross-sections on quantum numbers.

The possibility of stating such a problem is based on the representation for the quasipotential nature of hadronic interactions [2], Kadyshevsky's idea for "geometrization" of the relativistic two-body problem [3] and following from here the quantum-mechanical interpretation of the relativistic Fourier analysis [4]. There is also used the experimentally observed geometric scaling of the proton-proton elastic scattering at ISR energies [5].

The arised nonlinear equations are solved with the help of the regularized iteration processes of Gauss-Newton type [6] (the program COMPIL-C 401 in the JINR standard programs library [7] and computer CDC-6400 are used).

The measurements of hadron-hadron total cross-sections of accelerators in Serpukhov [8] - 30-70 GeV, CERN [9] - 300-1500 GeV and Batavia [10] - 50-400 GeV, together with earlier measurements [11] have shown that beginning with the Serpukhov energies the values of total cross-sections grow with increasing energy (see Fig. 1).

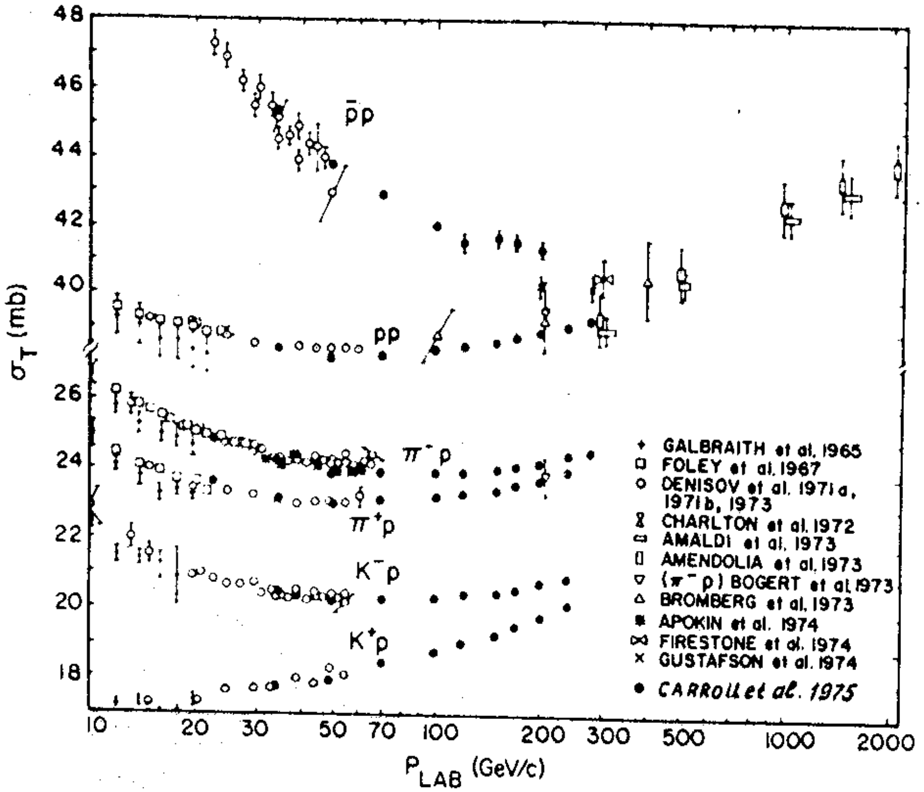

Fig. 1

The discovery of the Serpukhov effect aroused reconsideration of the existing approaches in high energy elementary particle physics and specification of the strict consequences from the basic principles of quantum field theory [12]. There appeared a number of new models of hadrons and new approaches for describing different aspects of their interactions (see, for instance, refs. [13-18]).

In the first section the existence of the interaction effective radius $R(s)$ and usual quantum-mechanical relation between the total cross-section $\sigma_t(s)$ and $R(s)$ is grounded.

In the second section the description of the numerical method which made it possible to specify the assumed general mathematical model of the interaction effective radius is presented.

In the third Section the discussion of the obtained results is given.

I. PROTON-PROTON TOTAL CROSS-SECTION AND THE INTERACTION EFFECTIVE RADIUS

Let us assume the interaction of two hadrons at high energies to be described by a defined in the relativistic relative coordinate space [4] quasipotential [2] $V(r,s)$, where r is the modulus of the relativistic relative coordinate and s, the invariant energy squared of the system.

It was shown [4] that if the function $V(r,s)$ has a simple pole structure, the change of the behaviour of the elastic scattering amplitude from exponential to power with increasing of the momentum transfer squared t has the geometric origin.

Let us choose[19] the quasipotential as

$$V(r,s) = \frac{\lambda_1(s)}{R_1^2(s) + r^2} + \frac{\lambda_2(s)}{R_2^2(s) - r^2}, \qquad (1)$$

where $\lambda_1(s)$, $\lambda_2(s)$, $R_1(s)$, $R_2(s)$ are some unknown functions of the energy.

Using the relativistic Fourier analysis[4] and simple quantum mechanical consideration on the basis of the performed numerical analysis the Born amplitude of the elastic scattering was found in the form[19]

$$T(s,t,R(s)) =$$

$$= \frac{1}{\sqrt{-t(1-t/4)}}\left[\frac{\lambda_1(s)}{F(t,R(s))} + \lambda_2(s)\cos(\rho \ln F(t,R(s)))\right], \qquad (2)$$

where

$$F(t,R(s)) = (1 - t/2 + \sqrt{-t(1-t/4)})^{R(s)},$$

$$\lambda_n(s) = (A_n + i\sqrt{s(s-4)}B_n)R^2(s), \quad n = 1,2$$

and

$$R(s) = R_1 + R_2/s^{R_3} + R_4(\ln s/R_5)^{R_6}, \qquad (3)$$

A_n, B_n and $R_l (l = 1,\ldots,6)$ are unknown parameters (the proton mass is equal to unity).

The obtained amplitude (2) describes[18-19] all specific features of the proton-proton elastic scattering within the whole range in s and $-t$ without $-t \ll 1$.

Because of the kinematic singularity of the amplitude (2) at $t = 0$, it is impossible to use the optical theorem for calculation of the total cross-section.

On the other hand, the experimentally observed property of geometric scaling[5] of the proton-proton elastic scattering means that after some rather a large energy the condition

$$\frac{\partial}{\partial s}\, \frac{1}{\sqrt{\sigma_t(s)}}\, T(s, \sigma_t(s)t) = 0 \qquad (4)$$

is fulfilled. One can numerically prove[19] that the quantity

$$\frac{1}{R(s)}\, T(s, R^2(s)t, R(s)),$$

(where the amplitude is defined by formulae (2) and (3)) satisfies the condition (4).

Therefore, the function $R^2(s)$ (which appeared in the mathematical model of the scattering amplitude (2) and (3)) is proportional to the total cross-section. If the proportionality factor is taken to be equal to $(2\pi)^{-1}$, one obtains usual quantum-mechanical connection between the total cross-section and the interaction effective radius

$$\sigma_t(s) = 2\pi R^2(s). \qquad (5)$$

Thus, the function $R(s)$ may be thought of as the effective radius of hadron-hadron interactions.

II. NUMERICAL DEFINITION OF THE CONCRETE MODEL OF THE EFFECTIVE RADIUS

As is well known, the hadrons can be classified in quantum numbers of mass M, baryon charge B, spin J, electric charge Q, , isotopic spin I and the third projection of isotopic spin $I3$.

Comments:

It is still unknown whether the above quantum numbers are independent or make a complete set.

Let us assume that the hadron-hadron total cross-sections depend on the pointed out quantum numbers also.

We shall use the defined by formula (3) dependence of the effective radius on energy and the hypothesis about the additive character of the dependence on quantum numbers of the functions R_l $(l = 1,\dots,6)$.. More exactly, one assumes that the functions R_l depend on the combinations of quantum numbers of two hadrons

$$\begin{aligned} M &= M_1 + M_2, \\ B &= B_1 + B_2, \\ J(J+1) &= (J_1 + J_2)(J_1 + J_2 + 1), \\ Q &= Q_1 + Q_2, \\ I(I+1) &= (I_1 + I_2)(I_1 + I_2 + 1), \\ I3 &= I3_1 + I3_2 \end{aligned} \tag{6}$$

as follows

$$\begin{aligned} R_l &= A_{l1} + A_{l2}M + A_{l3}M^2 + A_{l4}B + A_{l5}B^2 + \\ &+ A_{l6}J(J+1) + A_{l7}Q + A_{l8}Q^2 + A_{l9}I(I+1) + \\ &+ A_{l10}I3 + A_{l11}I3^2 + A_{l12}I3Q . \end{aligned} \tag{7}$$

Thus, we arrive at the problem of defining unknown quantities $A = \{A_i\}_{i=1,\dots,72}$ (since the index $l = 1,\dots,6$). This problem reduces to the solution of the following overdetermined nonlinear system of equations

$$\sigma_t(s,A,\alpha) = \sigma_t^{EXPT}(s), \tag{8}$$

where $\sigma_t^{EXPT}(s)$ are the experimental values of total cross-sections for the processes in fig. 1, $\sigma_t(s,A,\alpha)$ are defined by formulae (5-7) and α is the set of combinations of quantum numbers (6).

The measurements of the total cross-sections /8-11/ were performed at various accelerators, by different methods, thus they have different errors both of the statistical and nonstatistical nature.

One can write down the overdetermined system (8) in the vector form

$$fx = y, \tag{9}$$

where y denotes the vector of experimental values of the total cross-sections, x is the vector of unknown coefficients A_i

($i = 1,\dots, 72$), n is the number of unknowns and f is the operator determined by relations (5,6,7).

The solution of eq. (9) (in the sense of least squares) reduces to the solution of the following "square" problem

$$f'^{T}(x)P(fx - y) = 0, \qquad (10)$$

where $f'^{T}(x)$ denotes the transposed Jacobi matrix at the point x, and P is the diagonal "matrix of weight".

Bearing in mind the "nonqualitative" character of measurements entering into the vector y, for "solving" eq. (10) we use the following method.

We, first, find the solution x^* of eq. (10) with the matrix P.

On the basis of the obtained solution x^*, we determine the discrepancy vector $fx-y$ and form the matrix

$$P(x^*) = \mathrm{diag}[\frac{1}{(f_1 x^* - y_1)^2}, \dots, \frac{1}{(f_m x^* - y_m)^2}],$$

where m is the number of measurements (the length of the vector y).

Now, we find the solution x^{**} of problem (10) in which the weighted matrix P is replaced by the matrix $P(x^*)$.

Comments:

This process can be repeated and it is specified by fast convergence. The above described method of "solving" the problem (9) was already used in ref. [20].

When determining the concrete dependence of the function $R(s,A,\alpha)$ on quantum numbers, different problems of type (9) have been "solved" (in the sense mentioned above) repeatedly. The regularized iteration processes of the Gauss-Newton type [6] have been used to solve these very problems.

In our case the essential use has been made of the possibilities of the applied method to solve the nonlinear systems with a large number of unknowns as well as its stability in the case of degenerated problems of type (10), when the matrix $f'^{T}(x)f'(x)$ is degenerated at some x, including the solution x^*. In particular, the latter property of R-processes was used to find unknowns A_i of a generalized model with zero values.

One has obtained also that in the combination $A_i Q + A_I I3$ there is valid the numerical equality $A_i = -A_j$. Consequently, from the experimental information on total cross-sections and the generalized mathematical model for the effective interaction radius (7), it follows that the quantum numbers Q and I3 enter into the effective radius parameters through the quantum number of the hypercharge

$$Y = 2(Q - I3),$$

which did not enter into the primary set (6) of quantum numbers.

Analogously, when "solving" the subsequently simplified problems of type /10/, there appeared the quantum number strangeness

$S = Y - B$.

Now, one can write down the obtained approximation for the function $R(s,A,a)$ - concretized mathematical model:

$$R(s,A,a) = R_1 + R_2/(S/SO)^{R_3} + R_4(\ln S/SO)^{R_5},$$

where

$$R_1 = A_1 + A_2 SO + A_3 J(J+1),$$

$$R_2 = A_4 B^2 + A_5 J(J+1) + A_6 I(I+1) + A_7 (Y/2)^2,$$

$$R_3 = A_8 + A_9 B + A_{10} Y + A_{11} S,$$

$$R_4 = A_{12}, \qquad (11)$$

$$R_5 = A_{13} + A_{14} B + A_{15} Y + A_{16} S,$$

$$SO = (M_1 + M_2)^2.$$

Table I represents the obtained (non-zero) values of the parameters A_i $(i = 1, \ldots, 16)$.

Table II represents the values of the functions R_l and the values of the quantity

$$\chi^2 = \sum_{i=1}^{m} (\sigma_l(s_i, A, a) - \sigma_l^{EXPT}(s_i))^2,$$

where m is the number of measurements for any process.

One can note that the quantity

$$\frac{1}{m-n} \sum_{i=1}^{m} \frac{1}{p_i} (\sigma_l(s_i, A, a) - \sigma_l^{EXPT}(s_i))^2,$$

where m is the total (for all processes) number of measurements $(m = 312)$, n is the number of unknown parameters $(n = 16)$ and p_i is the corresponding diagonal element of the weighting matrix $P(x^*)$, is equal to 1.02.

Figure 2 represents the comparison of the obtained model with experiment. The errors are the sum of the statistical error found experimentally and systematic error equal to one percent of the experimental value. On the horizontal axis the values of the invariant energy squared are plotted.

III. DISCUSSION OF THE RESULTS

In the obtained concrete model for the interaction effective radius, the coefficient of the logarithmic increase is independent of quantum numbers.

A slight change of the power of the logarithmic rising from process to process and

Table I

$A_1 = .9[illegible]4 \pm .008$	$A_4 = -.131 \pm .00[illegible]$	$A_8 = .496 \pm .003$	$A_{12} = .090 \pm .001$	$A_{13} = .922 \pm .003$
$A_2 = -.101 \pm .001$	$A_5 = .554 \pm .003$	$A_9 = .0364 \pm .0006$		$A_{14} = .0096 \pm .0007$
$A_3 = .68[illegible] \pm .003$	$A_6 = .3[illegible]8 \pm .002$	$A_{10} = -.030 \pm .002$		$A_{15} = .016 \pm .002$
	$A_7 = -.5[illegible] \pm .024$	$A_{11} = .068 \pm .005$		$A_{16} = .012 \pm .002$
$R_1 = A_1 + A_2[illegible] + A_3 J(J+1)$	$R_2 = A_4 B^2 + A_5 J(J+1) + A_6 I(I+1) + A_7 (Y/2)^2$	$R_3 = A_8 + A_9 I3 + A_{10} Y + A_{11} S$	$R_4 = A_{12}$	$R_5 = A_{13} + A_{14} I3 + A_{15} Y + A_{16} S$

Table II

	$R_1 \pm \Delta R_1$	$R_2 \pm \Delta R_2$	$R_3 \pm \Delta R_3$	$R_4 \pm \Delta R_4$	$R_5 \pm \Delta R_5$	m	χ^2
$\bar{p}p$	[illegible].012±.010	1.844±.007	.49[illegible]±.003	.090±.001	.922±.003	38	5.18
pp		.80 ± .04	.472±.005		.903±.004	89	16.41
$\pi^- p$	1.392±.008	1.53 ± .01	.448±.004		.933±.003	7[illegible]	2.51
$\pi^+ p$		1.53 ± .01	.520±.004		.952±.003	85	.77
$K^- p$	1.30[illegible]±.009	1.010±.008	.428±.00[illegible]		.910±.004	58	2.65
$K^+ p$		.50 ± .03	.540±.007		.975±.005	26	.55

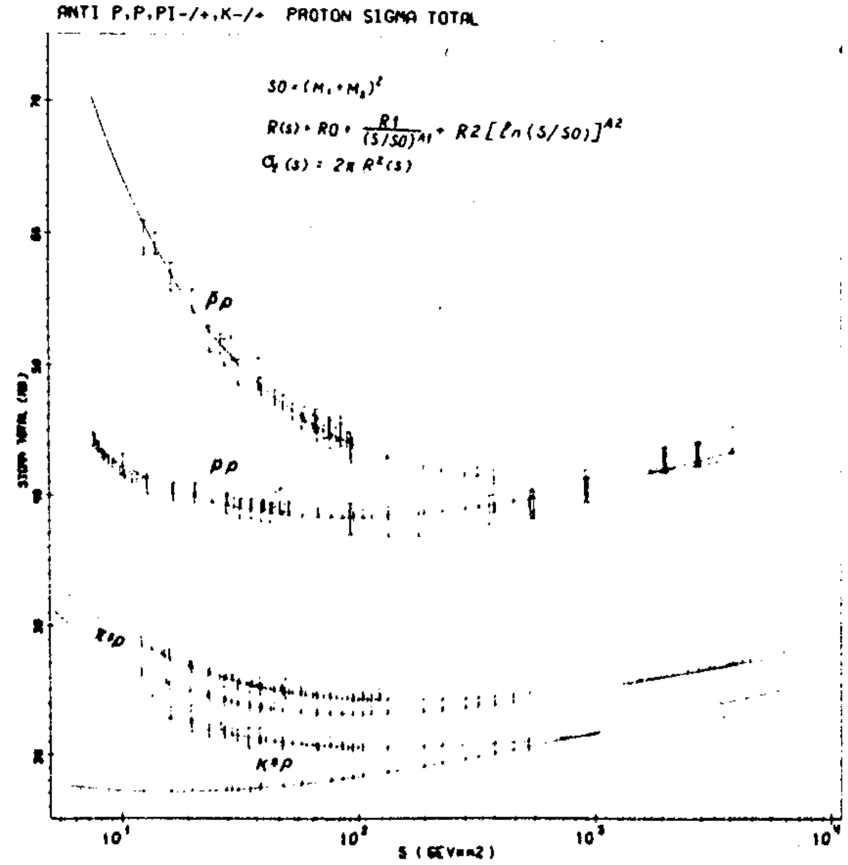

Fig. 2

its difference from unity is due to the fact that the influence of the resonance range is represented in the model by a single term only with power decrease in energy.

The consideration of the total cross-sections $\sigma_t(ab)$, but not $\sigma_t(ab)$ and $\sigma_t(a\bar{b})$ simultaneously causes the contradiction of the obtained model with the CPT-invariance condition.

As is seen from fig. 2 one can predict the cross-over of the total cross-section curves of the processes $\bar{p}p$, pp and $\pi^- p$, $\pi^+ p$ about E_L = 1000 GeV, and for the processes $K^- p$ and $K^+ p$ at the end of the energy interval of the Batavia accelerator.

Figure 3 (left) represents the experimental data on total cross-section of $\bar{p}$, p, $\pi^{\mp}$, $K^{\mp}$ on deutron and neutron. The theoretical curves obtained by inserting the quantum numbers of neutron and deutron (J = 1, I = 1) into formulae (11) are given in fig. 3 (right).

CONCLUSION

On the basis of the hypothesis about the quasipotential nature of strong interactions at high energies one obtaines the dependence of hadron-hadron total cross-sections on energy and quantum numbers.

The cross-over of $\bar{p}p$ and pp, $\pi^{\mp}p$ and $K^{\mp}p$ total cross-sections is predicted.

There appears the possibility to discover analogous dependences for other measured quantities - the differential cross-sections and so on. The necessary condition for performing such a program is the availability of more full than now experimental data on hadron scattering both in energy and, especially, in momentum transfer squared.

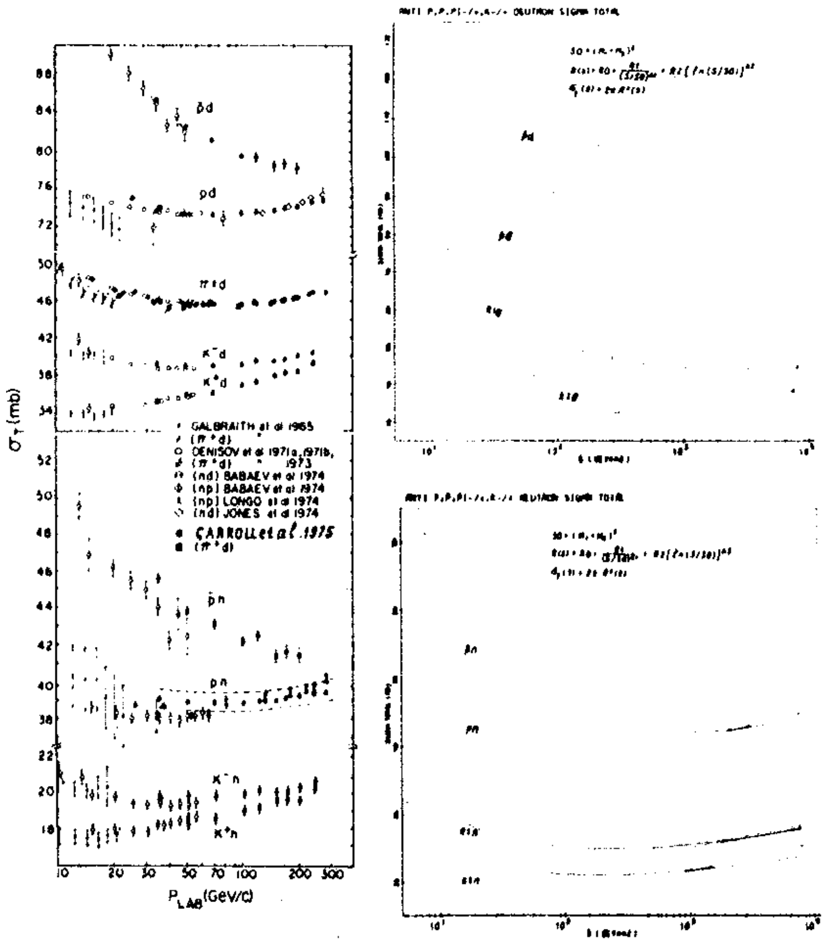

Fig. 3.

ACKNOWLEDGEMENT

The authors are thankful to Prof. V.G.Kadyshevsky, V.A.Meshcheryakov, M.A.Mestvirishvili for maintenance and interest in our work, Drs. M.D.Mateev, R.M.Mir-Kasimov, A.N.Sissakian, L.A.Slepchenko, V.I.Zhuravlev for many critical discussions.

REFERENCES

1. Н.Н.Боголюбов. Теория симметрии элементарных частиц, лекции на Ялтинской школе (1966), в сборнике "Физика высоких энергий и теория элементарных частиц", Наукова думка, Киев (1967).

2. A.A.Logunov, A.N.Tavkhelidze. Quasi-Optical Approach in Quantum Field Theory, Nuovo Cim., 29, 380 (1963).
   В.Г.Кадышевский, А.Н.Тавхелидзе. Квазипотенциальный подход в релятивистской задаче двух тел, в сборнике "Проблемы теоретической физики", Москва, Наука, (1969).
3. V.G.Kadyshevsky. Quasipotential Type Equation for the Relativistic Scattering Amplitude. Nucl.Phys., B86, 125 (1968); M.D.Mateev, V.G.Kadyshevsky. On a Relativistic Quasi-Quasipotential Equation in the Case of Particles with Spin, Nuovo Cim., 55A, 275 (1967).
   В.Г.Кадышевский, Р.М.Мир-Касимов, Н.Б.Скачков. Трехмерная формулировка релятивистской проблемы двух тел, ЭЧАЯ, том 2, вып. 3, стр. 637 (1972).

4. S.Cht.Mavrodiev. Some Consequences of the Fourier Analysis on the Lorentz Group for the Relativistic Quantum Mechanics, Comm. of JINR, E2-7910, Dubna, 1974.
   С.Ш.Мавродиев. Некоторые физические приложения гармонического анализа на группе Лоренца, Автореферат диссертации, ОИЯИ, 2-8487, Дубна, 1974.
5. V.Barger. Reaction Mechanisms at High Energy. Plenary Session Talk at the XVII Int.Conf. on High Energy Phys., London (1974).
6. Л.Александров. Регуляризованные вычислительные процессы Ньютона-Канторовича. ЖВМ и МФ., том 11, 1 (1971); Л.Александров. Авторегуляризованные итерационные процессы Ньютона-Канторовича. Сообщение ОИЯИ, Р5-5511, Дубна, 1970.
7. Л.Александров. Программа для решения нелинейных систем уравнений на основе регуляризованных процессов типа Гаусса-Ньютона. Сообщение ОИЯИ, Р5-7259, Дубна, 1973.

8. S.P.Denisov et al. Differences of Total Cross-Sections for Momenta up to 65 GeV/c, Nucl.Phys., B65, 1 (1973).
9. K.Egert, B.Betev et al. A Measurement of the Proton-Proton Cross-Section of the CERN-ISH, Nucl.Phys., B98, 93 (1975).
10. A.S.Carrol et al. Total Cross-Section of p, $\bar{p}$, $\pi^{\pm}$ and $K^{\pm}$ on Protons and Deuterons between 23 and 280 GeV/c, Preprint Fermilab-PUB-75+exp 7100-104 (1975).
11. E.Bracci et al. Compilation of Cross-Sections I, II, III, CERN/HERA 72-1, 72-2, 73-3.

12. А.А.Логунов, М.А.Мествиришвили, О.А.Хрусталев. Ограничения на поведение упругих и неупругих процессов при высоких энергиях, I и II , ЭЧАЯ, том 3, вып. 1, стр. 514 (1972).
13. Л.Д.Соловьев, А.В.Щелкачёв. ЭЧАЯ, том 6, вып. 3, стр. 571 (1975).
14. В.И.Саврин, Н.Е.Тюрин, О.А.Хрусталев. Метод U - матрицы в теории сильных взаимодействий, ЭЧАЯ, том. 7, вып. 1, стр. 21 (1976).
15. H.J.Lipkin. New Systematics in Hadron Total Cross-Section, Phys.Rev., D11, 1827 (1975).
16. В.П.Гердт, В.А. Мещеряков. Униформизация амплитуды рассеяния вперед в кварковой модели, ОИЯИ, Р2-9572, Дубна, 1976.
17. Б.Р.Десай, В.А.Царев. Комплексные полюса Редже, ЭЧАЯ, том. 5, вып. 3, стр. 693 (1974).
18. С.Щ.Мавродиев. Модель рассеяния адронов при высоких энергиях. Доклад на Международном семинаре по глубоконеупругим и инклюзивным процессам. Сухуми 1975. Препринт ОИЯИ, Р2-8897, Дубна, 1975.
19. S.Cht.Mavrodiev, D.Karajov. Relativistic Fourier Analysis, Quasipotential Approach and Proton-Proton Elastic Scattering, JINR, E2-8896, Dubna, 1975.
20. Л.Александров, П.Ангелеску, Ф.Никитиу, И.В.Фаломкин, Ю.А.Щербаков. Зависящий от энергии фазовый анализ π^+ He упругого рассеяния, ОИЯИ, Р1-8328, Дубна, 1974.

Received by Publishing Department
on July 2, 1976.