ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

ДУБНА

E2 - 9913

B.Z.Kopeliovich, Ch.Tseren

INCLUSIVE REACTIONS WITH PION DOUBLE CHARGE EXCHANGE AT HIGH ENERGIES

1976

E2 - 9913

B.Z.Kopeliovich, Ch.Tseren

INCLUSIVE REACTIONS WITH PION DOUBLE CHARGE EXCHANGE AT HIGH ENERGIES

Submitted to ЯФ

I. INTRODUCTION

The reactions with exotic quantum numbers in the crossing channel, including the double charge exchange pion processes, occupy a special place in high energy physics. On the Regge scheme this is a source of direct information on the properties of j-plane branching points /1/.

In connection with the increasing common interest in the investigation of inclusive reactions a series of experiments devoted to the study of the double charge exchange pion inclusive processes have been carried out in last years /2/:

$$\pi^- + \beta \to \pi^+ + X, \qquad (1a)$$

$$\pi^+ + \beta \to \pi^- + X. \qquad (1b)$$

Here β denotes a target - a nucleon or a nucleus; X is the arbitrary set of produced particles.

The purpose of the present paper is to study the mechanism of high energy reactions (1a) and (1b) in the hard part of the momentum spectra of produced pions. The calculation has been carried out in the multiperipheral approach, using the one-pion exchange (OPE) model. In order to perform all the calculations analytically and to observe immediately the energy and x-dependence (where x is the Feynman invariant variable) of the various mechanism contributions some of approximations have been made and the region of the small momentum transferred only has been considered. The analysis showed that OPE model corresponds well to the available experimental data.

Reactions (1a) and (1b) essentially differ from analogous binary processes. Although the binary processes underline the inclusive reactions, the energy, at which they proceed, is not equal to the total energy of reactions (1). It changes within wide limits but the largest effect arises from the low energy region, where the resonance production is dominant. Thus, the inclusive resonance production (ρ, f, g) with a subsequent decay into the $\pi^+\pi^-$ pair is responsible for reactions (1).

The paper is arranged as follows: In Sect. 2 we consider the OPE model. The underlying formula for the inclusive cross section of processes (1) is obtained. It appears to be bound with the amplitudes of $\pi\pi$ and $\pi\beta$ elastic scattering.

The separation of different contributions to these amplitudes corresponds to the division of the inclusive cross-section into the parts having various energy and x-dependences.

In Sect. 3, the energy-independent part of the cross-section is calculated. It corresponds to the separation of a Pomeron part from the total cross-section of $\pi\beta$-interaction.

In Sect. 4 the corrections to the inclusive spectra due to the $\pi\beta$-scattering cross-section contribution which vanish with the rise of energy are calculated. The Deck diagram contribution to the cross-section is also taken into account.

II. THE OPE-MODEL

In the multiperipheral approach, one takes into consideration that any inclusive process $\alpha+\beta\to\gamma+X$ in the fragmentation region of the particle α is a result of the interaction of α with one of the particles of the "comb", emitted by β but not with β itself. This case, as is shown below, leads to the important difference of the inclusive reaction with exotic t-channel quantum numbers from binary one. We make a natural assumption that multiperipheral comb's particle, interacting with α, is a pion. It is the basic assumption of the OPE-model, formulated by Amati, Fubini and Stangellini/3/ and developed by Boreskov, Kaidalov and Ponomarev/4/ who improved it by taking into account the Reggeization of the pion exchange.

The graph shown in fig. 1 corresponds to the amplitude of reaction (1a) in this model.

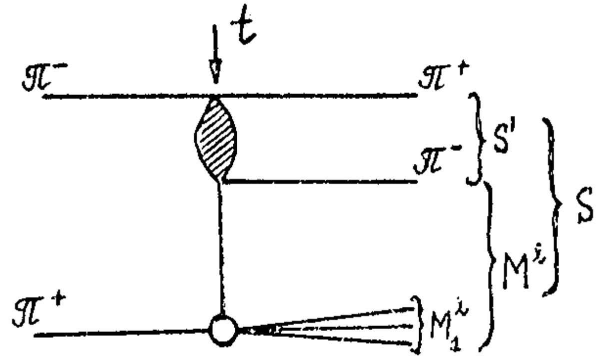


Fig. 1

In fig. 1 s', M^2 and M_1^2 denote the effective masses squared of the $\pi^+\pi^-$ pair, the beam in reaction (1a) and the part of this beam without a π^--meson, respectively. s is the square of the total c.m. energy, t is the 4-momentum transfer squared for π^+ and π^- mesons in reaction (1a) and u is 4-momentum squared of the virtual pion.

The inclusive cross-section for the reaction (1a) is described by graph in fig. 2, where the dotted line means the absorption part and the cross shows the particle on the mass shell. It is obvious that inclusive cross-section is bound with the amplitude $T^{\pi\pi}(s,t,u)$ of the reaction $\pi^- + \pi^+ \to \pi^+ + \pi^-$ and the total cross-section for $\pi\beta$-interaction $\sigma^{\pi\beta}(M_1^2, u)$.

The pion virtuality is taken into account by the u-dependence of both functions. This connection is given by the following relation:

$$\sigma_{inv}(s,t,x) = \frac{1-x}{2^8\pi^6}\int_{-\infty}^{0}\frac{e^{R^2u}\,du}{(\mu^2-u)^2}\int_0^{z_m}\sigma^{\pi\beta}(M_1^2)\,z\,dz\int_0^{\pi}|T^{\pi\pi}(s',t,u)|^2 d\phi. \qquad (2)$$

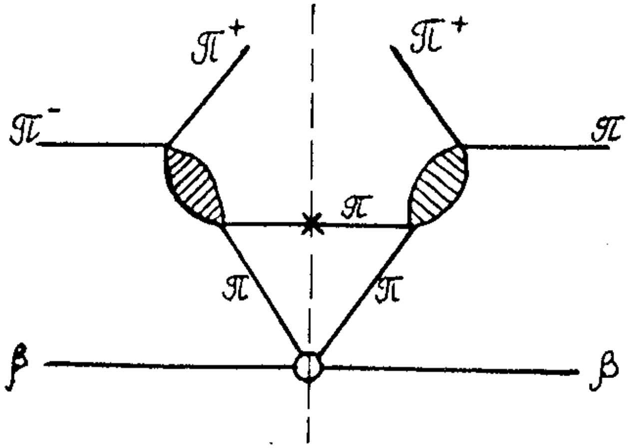


Fig. 2

Here $\sigma_{inv} = Ed^3\sigma/d^3p$ is the invariant cross-section of reaction (1); $x = 2p_{||}/\sqrt{s}$ is the Feynman invariant variable; μ is the pion mass. The factor e^{R^2u} takes into account the u-dependence of $T^{\pi\pi}$ and $\sigma^{\pi\beta}$; $z = M_1^2/M^2$ and ϕ is the azimuthal angle between the transverse momentum transferred $-\vec{p}_\perp$ in the reaction $\pi^- + \pi^+ \to \pi^+ + \pi^-$ and the $\vec{k}_\perp$ - transverse component of the virtual pion momentum.

The Reggeization of pion exchange is neglected here, since in the different versions, considered below, either the region of small energy, coming into π-exchange $(z \sim z_m)$, is important or this region is so narrow that the Reggeization may be taken into account by choosing the value of R^2 in (2).

The expression for s' is obtained in Appendix 1, and is as follows:

$$s' = (1-x)^{-1}\{\mu^2 - u - t + 2tz + 2|t|^{1/2}\sqrt{-z(\mu^2-u-t) - u - tz^2}\cos\phi\}. \qquad (3)$$

The integral over z in (2) has upper limit z_m, which is received in appendix 2 and is equal to:

$$z_m = \frac{1}{2|t|}\{\mu^2 - u - t + \sqrt{(\mu^2-u-t)^2 - 4ut}\}. \qquad (4)$$

The result of the integration of (2) depends on the concrete form of $T^{\pi\pi}$ and $\sigma^{\pi\beta}$. Some cases are considered below.

III. INCLUSIVE CROSS-SECTION AT ASYMPTOTIC ENERGIES

Let us consider the reaction (1a) in the energy region, where one can neglect the contribution to the cross-section, which vanishes as a power energy. The remaining part of the cross-section may be calculated by using formula (2), if one takes into account only $\sigma_0^{\pi\beta}$ -energy independent part of $\sigma^{\pi\beta}(M_1^2)$.

$$\sigma^{\pi^-N}(M_1^2) = \sigma_0^{\pi^-N} + \sigma_1^{\pi^-N}(M_1^2). \tag{5}$$

For the sake of definiteness, in this and the following sections we discuss reactions (1a) and β is identified with the nucleon. The vanishing with rise of M_1^2, term-$\sigma_1^{\pi^-N}(M_1^2)$ in (5) is considered in the following section. The substitution $\sigma_0^{\pi^-N}$ into (2) leads to the fact that σ_{inv} really does not depend on s at the fixed values of t and $x \cong 1-M^2/s$. To calculate the cross-section it is necessary to know the amplitude $T^{\pi\pi}(s',t)$. As already have been noted the calculation does not claim for high precision. Therefore, we take for $T^{\pi\pi}(s',t)$ the following approximate expression:

$$T^{\pi\pi}(s',t) = T_{res}^{\pi\pi}(s',t) + T_{cut}^{\pi\pi}(s',t), \tag{6}$$

where $T_{res}^{\pi\pi}$ is the direct channel resonance contribution, which reproduces well the scattering amplitude at small values of s'; $T_{cut}^{\pi\pi}$ is the cut contribution to the amplitude which is important at large values of s'.

1. The Resonance Contribution

The main role in the low energy $\pi\pi$-scattering amplitude play the resonances ρ, f, g. The relevant contribution equals to:

$$|T_R^{\pi\pi}(s',t)|^2 = \Phi_R^2(t)\frac{(\Gamma_R^{2\pi})^2}{(s'-m_R^2)^2 + m_R^2\Gamma_R^2}. \tag{7}$$

Here m_R is the resonance mass, Γ_R and $\Gamma_R^{2\pi}$ are the full and partial widths of the resonance decay; $\Phi_R(t)$ is the factor containing the different normalysing coefficients which determines the angle distribution $|T_R^{\pi\pi}(s',t)|^2$ depending on the resonance spin

$$\Phi_\rho(t) = \frac{24\pi m_\rho^2}{p_\rho}\cos\theta, \tag{8}$$

$$\Phi_f(t) = \frac{20\pi m_f^2}{p_f}(3\cos^2\theta - 1), \tag{9}$$

$$\Phi_g(t) = \frac{84\pi m_g^2}{p_g}\cos\theta(\frac{5}{3}\cos^2\theta - 1). \tag{10}$$

Here $p_R = m_R/2$ is the momentum of pions, produced by R-decay in its rest system; θ is the scattering angle in the c.m.s. of the reaction $\pi^- + \pi^+ \to \pi^+ + \pi^-$ and $\cos\theta \cong 1 + t/2p_R^2$. Since the decay widths are much smaller than the interval between resonance masses, then one may neglect interference of the resonance contributions to the cross-section for $\pi\pi$-scattering and obtains:

$$|T_{res}^{\pi\pi}(s',t)|^2 \cong \sum_R |T_R^{\pi\pi}(s',t)|^2. \tag{11}$$

Due to the small value Γ_R we can also make the replacement:

$$\frac{1}{(s'-m_R^2)^2+m_R^2\Gamma_R^2} \to \frac{\pi}{m_R\Gamma_R}\delta(s'-m_R^2). \qquad (12)$$

By integrating over ϕ in (2) one obtains:

$$\sigma_{inv}(s,t,x)=\sum_R \frac{\sigma_0^{\pi^- N}}{2^8\pi^5}\,\frac{(1-x)(\Gamma_R^{2\pi})^2}{m_R\Gamma_R}\,\Phi_R^2(t)\int_{-\infty}^{0}\frac{e^{R^2u}\,du}{(\mu^2-u)^2}\int_0^{z'_m} z\,dz\left(\frac{ds'}{d\phi}\right)^{-1}, \qquad (13)$$

where

$$\frac{ds'}{d\phi}=2|t|^{1/2}\sqrt{z(u+t-\mu^2)-t(1+z^2)}\,(1-x)\sin\phi, \qquad (14)$$

and the $\sin\phi$ is found from (3) at $s'=m_R^2$. At the fixed value of $s'=m_R^2$ the upper limit of integration over z in (13) equals not z_m from (4) but z'_m, which is found from (3) at $s'=m_R^2$ and $|\cos\phi|=1$.

$$z'_m=\frac{m_R^2}{4t}(1-x)+\frac{u}{m_R^2(1-x)}-\frac{\mu^2-u-t}{2t}-\frac{(\mu^2-u-t)^2}{4m_R^2t\,(1-x)}. \qquad (15)$$

By integrating over z in (13) and taking into account (14) and (15) we have

$$\sigma_{inv}(s,t,x)=\sum_R \frac{\sigma_0^{\pi^- N}}{3\cdot 2^9\pi^5}\cdot\frac{(\Gamma_R^{2\pi})^2}{m_R^3\Gamma_R\,t^2}\,\Phi_R^2(t)\,\times \qquad (16)$$

$$\times\int_{\lambda_1}^{\lambda_2} e^{R^2(\mu^2-\lambda)}\frac{d\lambda}{\lambda}(-a_R+2\lambda b_R+\lambda^2 c_R)^{3/2},$$

where

$$\lambda=\mu^2-u,\quad a_R=[m_R^2(1-x)+t]^2-4\mu^2t,$$

$$b_R=m_R^2(1-x)-t,\quad c_R=-1. \qquad (17)$$

The integration limits $\lambda_{1,2}$ are found from (2) at $z=0$ and $\cos\phi=\pm 1$.

$$\lambda_{1,2}=\mu^2+(\sqrt{m_R^2(1-x)-\mu^2}\mp\sqrt{|t|})^2. \qquad (18)$$

We note that the interval $(\lambda_2-\lambda_1)$ narrows at $t\to 0$. Therefore, for the small values of t, which are of interest for us, the factor $\exp[R^2(\mu^2-\lambda)]$ may be taken out behind the integral sign, putting in it $\lambda=m_R^2(1-x)$. After this the integration can be performed and one finds:

$$\sigma_{inv}(s,t,x)=\sum_R \frac{\sigma_0^{\pi^- N}}{2^{11}\pi^4}\,\frac{(\Gamma_R^{2\pi})^2}{m_R^5\Gamma_R}\,e^{R^2[\mu^2-m_R^2(1-x)]}\left[\frac{\Phi_R(t)}{t}\right]^2(b_R-\sqrt{a_R})^2. \qquad (19)$$

At $t=0$ this expression becomes simplier and has the following form:

$$\sigma_{inv}(s,t=0,x)=\sum_R \frac{\sigma_0^{\pi^- N}}{2^9\pi^4}\,\frac{(\Gamma_R^{2\pi})^2}{m_R^5\Gamma_R}\,e^{R^2[\mu^2-m_R^2(1-x)]}\,\times \qquad (20)$$

$$\times\,\Phi_R^2(0)\left(1-\frac{\mu^2}{m_R^2(1-x)}\right)^2,$$

Quantity (20) is shown graphically in fig. 7. The value of R^2 is chosen (see below) from the comparison with the experimental data and is equal to $R^2=3(\mathrm{GeV}/c)^{-2}$. Obviously the cross-section at $t=0$ does not practically depend on x.

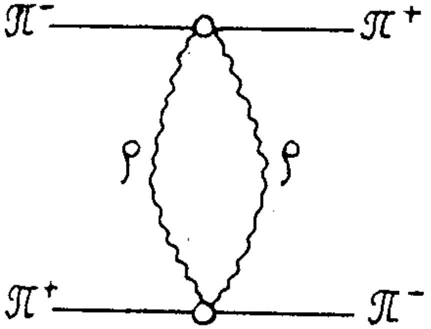


Fig. 3

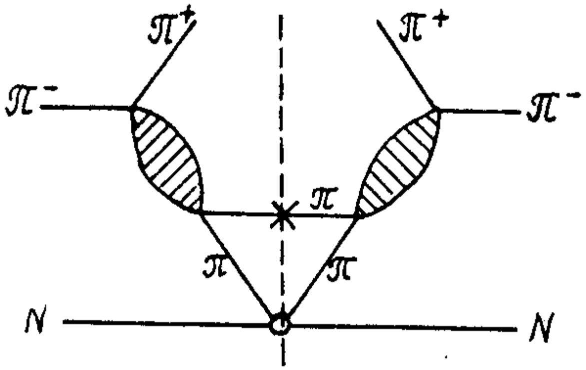


Fig. 4

2. The Cut Contribution

We consider now the 2nd term in the right-hand part of (6). The graph shown in fig. 3 corresponds to it.

In this case the graph in fig. 2 may be drawn again as is shown in fig. 4. The aim of the following calculations is to show that the contribution of this graph to the inclusive cross-section is negligibly small. Therefore, for simplicity, we take $t = 0$.

The amplitude $T^{\pi\pi}(s',0)$ in the eikonal approximation has the form.

$$T^{\pi\pi}(s',0) = \frac{1}{8\pi s_0} \frac{g^4_{\pi\pi\rho}(0)}{R_1^2 + \alpha'_\rho \ln(s'/s_0) - i(\pi/2)\alpha'_\rho} . \qquad (21)$$

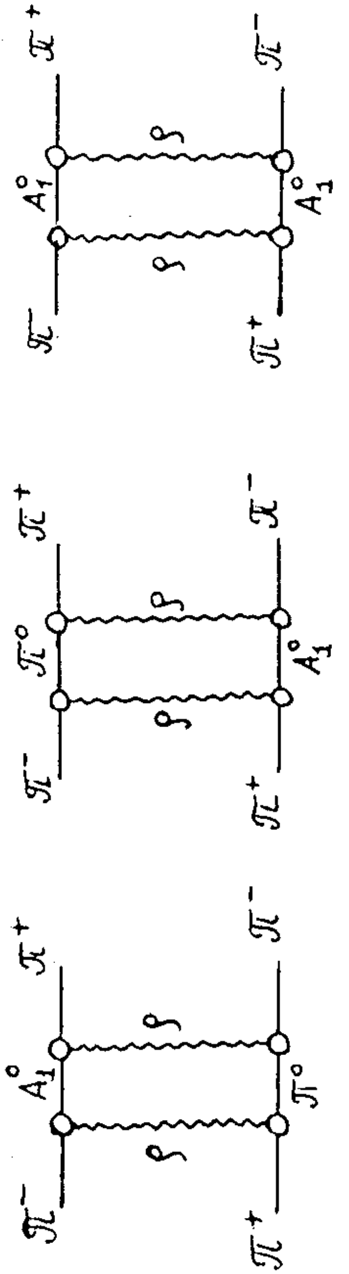
Here $g_{\pi\pi\rho}(t) = g_{\pi\pi\rho}(0)\cdot e^{R_1^2 t/2}$ is the emission vertex of ρ reggeon by the pion; $\alpha_\rho(t) = 1/2 + \alpha'_\rho t$ is the trajectory of the ρ-pole.

After the substitution of (21) into (2) all the integrations are easily performed and one obtains

$$\sigma_{inv}(s,t=0,x) = \frac{\sigma_0^{\pi^- p}\, g^8_{\pi\pi\rho}(0)\,(1-x)}{2^{17}\pi^7 3\mu^2 s_0^2 [R_1^2 - \alpha'_\rho \ln(1-x)]^2} . \qquad (22)$$

The parameters $g_{\pi\pi\rho}(0)$ and R_1^2 can be found from the data on elastic πN scattering and the pn-charge exchange reaction and are equal to $g_{\pi\pi\rho}(0) = 6.3$ and $R_1^2 = 4(\mathrm{GeV/c})^{-2}$ /5/. The contribution of (22) to the inclusive cross-section is shown in fig. 7. It is seen that it is approximately two orders smaller than the resonance contribution. Nevertheless, the existence of the considerable increase of the eikonal contribution is possible due to the inelastic intermediate states shown in fig. 3. The corrections arising from the A_1-meson production are most important. The corresponding graphs are shown in fig. 5.

The contribution of the graph in fig. 5 has the same energy dependence as the eikonal one, only in the case, when $A_1 \to \rho\pi$ coupling constant corresponds to d-wave decay. The d-wave coupling /6/ is known and the calculation of the graph in fig. 5a shows that its contribution is approximately twice larger than the eikonal one. In this case the value of cross-section (22) must be multiplied by $3^4 = 81$. But the precision of measurement of the d-wave vertex for the decay $A_1 \to \rho\pi$ is not very high and since it enters the cross-section to the 8th power, then within this



a b c

Fig. 5

precision the cross-section changes in such a wide interval that the value of the cut contribution become questionable. One may say definitely only that the cut contribution to the cross-section cannot be of the order of 100%, since in this case its magnitude in the amplitude of $\pi\pi$ scattering is so great that it cannot be "sewed" together with the resonance contribution at small values of s'. Below the cut contribution to the cross-section is not taken into account.

IV. CROSS-SECTION CONTRIBUTION DECREASING WITH ENERGY

We return to expression (5) and consider the correction to the cross-section from the 2nd term - $\sigma_1^{\pi^- N}(M_1^2)$, involving non-Pomeron contributions. At low values of M^2, the main effect comes from the resonances in the direct channel, from which we take into account $\Delta(1236)$. At the large value of M_1^2, $\sigma^{\pi^- N}$ is well described by the Reggeon f, ρ contribution.

$$\sigma_1^{\pi N}(M_1^2) \cong \sigma_\Delta^{\pi N}(M_1^2) + \sigma_{Regge}^{\pi N}(M_1^2). \qquad (23)$$

1. Δ isobar

The first term in (23) equals to

$$\sigma_\Delta^{\pi N}(M_1^2) = \frac{32\pi m_\Delta}{p_\Delta} \; \frac{\Gamma_\Delta^{\pi N}\Gamma_\Delta}{(M_1^2 - m_\Delta^2)^2 + m_\Delta^2 \Gamma_\Delta^2}. \qquad (24)$$

Here $p_\Delta = [(m_\Delta^2 - m_N^2 - \mu^2)^2 - 4m_N^2\mu^2]^{1/2}/2m_\Delta$ is the momentum of π and N in the rest system of Δ;

Γ_Δ and $\Gamma_\Delta^{\pi^- N}$ are the total and partial widths of the decay $\Delta(\Gamma_\Delta^{\pi^- p} = \frac{1}{3}\Gamma_\Delta^{\pi^+ p})$; m_Δ is the Δ-isobar mass.

Using the smallness of Γ_Δ, one can carry out following replacement in (2):

$$\sigma_\Delta^{\pi^- N}(M_1^2) \approx \frac{32\pi^2 m_\Delta^2}{p_\Delta M_1^2 M^2}\Gamma_\Delta^{\pi^- N}\,\delta\left(z - \frac{m_\Delta^2}{M^2}\right) F, \qquad (25)$$

where

$$F = \frac{1}{\pi}\left[\operatorname{arctg}\left(\frac{M_0^2 - m_\Delta^2}{m_\Delta \Gamma_\Delta}\right) + \operatorname{arctg}\left(\frac{m_\Delta^2 - (m_N+\mu)^2}{m_\Delta \Gamma_\Delta}\right)\right].$$

Here F takes into account the final limits of the integration over M_1^2 from $(m_N+\mu)^2$ to M_0^2 - the value, from which the cross-section for the πN scattering can be described by the Regge poles. We adopt $M_0^2 = 4\ \text{GeV}^2$.

By substitution (25) into (2) and integration over ϕ analogously to (13) and over z, one obtains

$$\sigma_{inv}(s,t,x) = \sum_R \frac{m_\Delta^2 \Gamma_\Delta^{\pi^- N}(\Gamma_R^{2\pi})^2 \Phi_R^2(t) F}{(2\pi)^3 s^2 m_R \Gamma_R p_\Delta} \times$$

$$\times \int_{\lambda_1^{(\Delta)}}^{\lambda_2^{(\Delta)}} \frac{d\lambda}{\lambda^2} e^{R^2(\mu^2-\lambda)}(-a_R^{(\Delta)} + 2\lambda b_R + \lambda^2 c_R)^{-1/2}, \qquad (26)$$

$$a_R^{(\Delta)} = a_R + 4tm_R^2 m_\Delta^2/s, \qquad (27)$$

a_R, b_R and c_R are determined in (17). The integration limits $\lambda_{1,2}^{(\Delta)}$ have been found from (3) at $s' = m_R^2$ and $z = m_\Delta^2/M_0^2$. So

$$\lambda_{1,2}^{(\Delta)} = m_R^2(1-x) - t \mp 2|t|^{1/2}\sqrt{m_R^2(1-x) - (m_R^2 m_\Delta^2/s) - \mu^2}. \qquad (28)$$

Performing the integration in (26) we have

$$\sigma_{inv}(s,t,x) = \sum_R \frac{1}{(4\pi)^2}\,\frac{m_\Delta^2 \Gamma_\Delta^{\pi^- N}(\Gamma_R^{2\pi})^2 F \Phi_R^2(t)}{s^2 m_R \Gamma_R p_\Delta} \times$$

$$\times\, e^{R^2[\mu^2 - m_R^2(1-x)]}\,\frac{2b_R}{(a_R^{(\Delta)})^{3/2}}. \qquad (29)$$

It is seen that contribution to the cross-section decreases with increasing energy as s^{-2}. It forms the peak in the region $x \cong 1$ at small $|t|$. Indeed, taking $t=0$ one obtains

$$\sigma_{inv}(s,t=0,x) = \sum_R \frac{1}{(4\pi)^2}\,\frac{m_\Delta^2 \Gamma_\Delta^{\pi^- N}(\Gamma_R^{2\pi})^2 \Phi_R^2(0) F}{s^2 m_R^5 \Gamma_R p_\Delta (1-x)^2}\, e^{R^2[\mu^2 - m_R^2(1-x)]}.$$

The value of the right-hand part of (30) with $R^2 = 3(\text{GeV/c})^{-2}$ for $p_L = 16$ GeV/c is shown graphically in fig. 7.

2. The Reggeon Contribution to $\sigma_1^{\pi^- N}(M_1^2)$

The ρ-Reggeon contribution is neglectgible in comparison with f in the amplitude of πN-scattering, so

$$\sigma_f^{\pi^- N}(M_1^2) = \gamma_f^{\pi^- N}\cdot(M_1^2/s_0)^{-1/2}. \qquad (31)$$

where $\gamma_f^{\pi N} = 30$ mb is the residue of f-Reggeon in the πN forward scattering amplitude. After substituting (31) into (2) and the computation of the integrals at $t=0$ we find

$$\sigma_{inv}(s,t=0,x) = \sum_R \frac{\gamma_f^{\pi N}\cdot \Phi_R^2(0)\cdot(\Gamma_R^{2\pi})^2\cdot e^{[\mu^2 - m_R^2(1-x)]R^2}}{3\cdot 2^7\cdot \pi^4 m_R^5 \Gamma_R \sqrt{M_0^2/s_0}}\, y(1-y^3) \qquad (32)$$

here

$y = \sqrt{M_0^2/s(1-x)}$.

The expression (32) has the maximum at $y^3 = 1/4$, i.e., at $s(1-x) \cong 10$, so this contribution to the cross-section in the hard part of the spectrum at $x = 0.7 \div 0.9$ in fact is not decreased with energy but rises to energy in few tens GeV. Fig. 7 is the plot of the magnitude of (32) at $p_L = 16$ GeV/c. It is seen that its contribution is small in comparison with the total value of the cross-section.

3. The Deck Diagram

In the $\sigma^{\pi^- N}(M_1^2, u)$ there is also the nucleon pole, which corresponds to the graph shown in fig. 6. In this case the calculation can be made similarly to the computation of the Λ-isobar contribution.

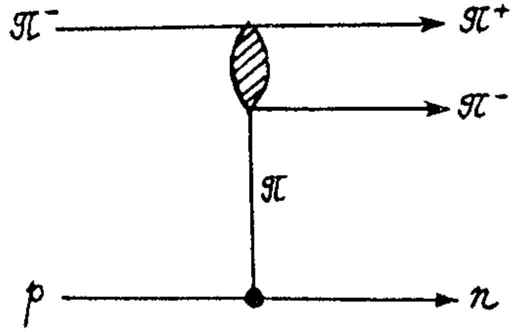


Fig. 6

In formula (2) instead of $\sigma^{\pi N}(M_1^2)$ one should substitute

$$\sigma^{\pi N}(M_1^2) \to -\frac{2\pi g^2}{M^2 M_1^2} u \delta\left(z - \frac{m_N^2}{M_1^2}\right). \qquad (33)$$

Then, instead of (26) one obtains the expression

$$\sigma_{inv}(s,t,x) = \sum_R \frac{g^2 (\Gamma_R^{2\pi})^2 \Phi_R^2(t)}{2^7 \pi^4 m_R \Gamma_R s^2} \times$$
$$\times \int_{\lambda_1^{(N)}}^{\lambda_2^{(N)}} \frac{\lambda - \mu^2}{\lambda^2} e^{-R_2^2(\lambda - \mu^2)} d\lambda \left(-a_R^{(N)} + 2\lambda b_R + \lambda^2 c_R\right)^{-1/2}. \qquad (34)$$

Here

$$a_R^{(N)} = a_R + 4t m_N^2 m_R^2 / s$$

$$\lambda_{1,2}^{(N)} = m_R^2(1-x) - t \mp 2|t|^{1/2} \sqrt{m_R^2(1-x) - m_R^2 m_N^2/s - \mu^2}. \qquad (35)$$

After performing integration in (34) one obtains

$$\sigma_{inv}(s,t,x) = \sum_R \frac{g^2 (\Gamma_R^{2\pi})^2 \Phi_R^2(t)}{2^9 \pi^3 s^2 m_R \Gamma_R} e^{R_2^2[\mu^2 - m_R^2(1-x)]} \times$$
$$\times \left[(a_R^{(N)})^{-1/2} - b_R \mu^2 (a_R^{(N)})^{-3/2}\right]. \qquad (36)$$

At $t=0$ this expression has the following form:

$$\sigma_{inv}(s,t=0,x) = \sum_R \frac{g^2 (\Gamma_R^{2\pi})^2 \Phi_R^2(0)}{2^9 \pi^2 s^2 (1-x) \Gamma_R m_R^3} e^{R_2^2[\mu^2 - m_R^2(1-x)]} \left[1 - \frac{\mu^2}{m_R^2(1-x)}\right]^2. \qquad (37)$$

From (37) it is seen that the contribution of the Deck diagram increases at $x \to 1$, though not so fast as the Λ-isobar contribution from (30). The contribution of the Deck diagram at $t=0$ to the inclusive cross-section at $p_L = 16$ GeV/c and $R_2^2 = 5\ (\mathrm{GeV/c})^{-2}$ /4/ is shown in fig. 7.

V. DISCUSSION OF THE RESULTS

From fig. 7 it is evident that the energy independent or scaling part in the cross-

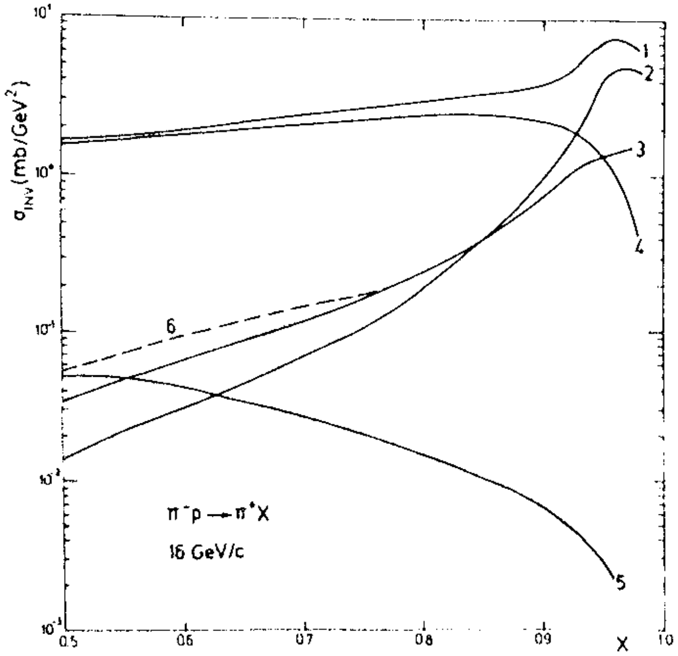


Fig. 7. Calculation of the inclusive cross-section for pion double charge exchange at $t=0$ and $p_L = 16$ GeV/c. 1 the total contribution to the cross-section for all mechanisms. Scaling contributions: 4 - expression (20); 5 - expression (22); Non-scaling contributions: 2 - expression (30); 3 - expression (37); 6 - expression (32).

section of reaction (1a) dominates already at 16 GeV. Since all the dependence of this contribution from a sort of the target β is contained in the factor $\sigma_0^{\pi\beta}$ then the factorization must occur, i.e., the relation

$$\sigma_{inv}^{\pi\beta}(s,t,x)/\sigma_0^{\pi\beta} \qquad (38)$$

should not depend on type of particle β. At the same time the cross-section contribution vanishing with energy violates factorization properties. For example, the value $\Gamma_\Lambda^{\pi^+p}$ is 3 times larger than $\Gamma_\Lambda^{\pi^-p}$ in (26) and the Deck diagram gives the contribution to the cross-section of reaction (1a) only, if β is a proton. Consequently the deviation from the universality of (38) rises with the decrease of the incident pion energy and at $x \to 1$ (the latter is seen from fig. 7). In the case of heavy nuclei the factorization is also violated due to the absorption of produced pions inside the nucleus. Note that in the case of nuclear interaction the contribution to the double charge exchange cross-section can result from two successive charge exchange reactions of pions on two different nucleons of the nucleus ($\pi^- \to \pi^0$ and $\pi^0 \to \pi^+$).

However, one can believe that at high energies the contribution of this mechanism is of the same order as the cut contribution, i.e., it is small.

The total contribution to the cross-section for all mechenisms considered above is compared with the experimental data at $p_\perp^2 = 0.03$ (GeV/c) and different energies (see fig. 8). The good agreement is observed. In the region $x \simeq 1$ as it is seen from the above computations the value of the cross-section is not sensitive to the parameter R^2. We have found the optimum value $R^2 = 3(\text{GeV/c})^{-2}$.

In conclusion we note that the reactions considered do not give any information on the cuts because of the large contribution of the mechanism connected with meson resonance production. For the purpose of studying the branching points the double charge

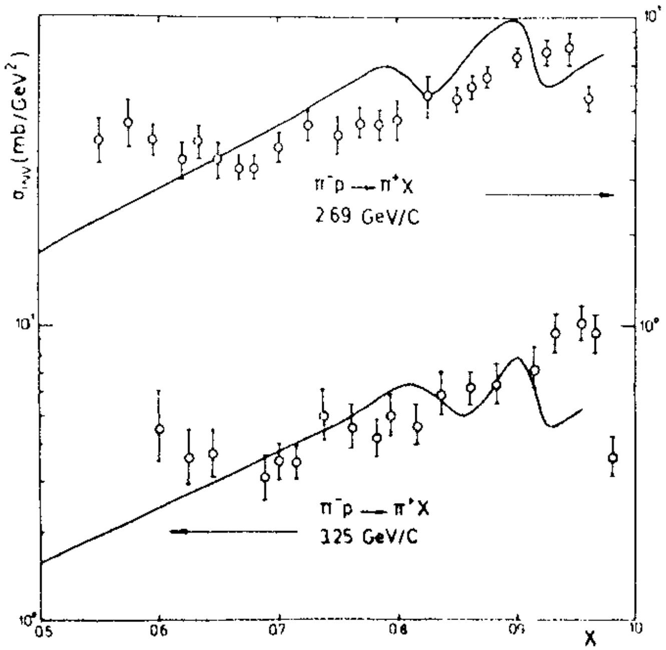


Fig. 8a. Calculation by (19), (29) and (36). Comparison with experimental data/3/ at $p_\perp^2 = 0.03$ $(\mathrm{GeV}/c)^2$ and $p_L = 2.65$ and 3.25 (GeV/c).

exchange of K-mesons is a more suitable reaction. Because of the great mass of K-mesons, the $s^*, \phi, \ldots$ resonances cannot give the contribution to the region $x \cong 1$, i.e., the cut contribution must be observed here in the pure form.

The authors are grateful to L.I.Lapidus for many helpful discussions and L.A.Ponomarev for valuable remarks.

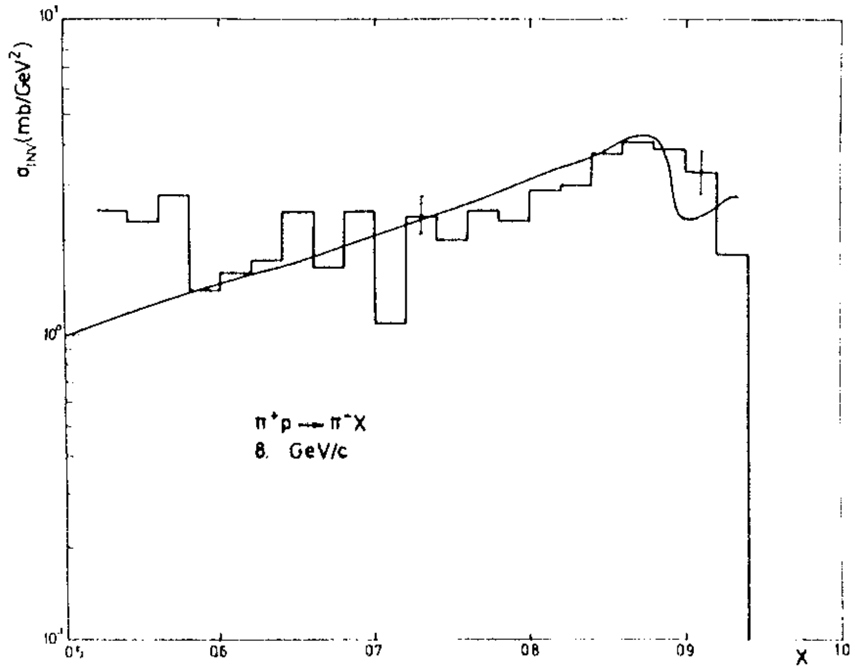


Fig. 8b. Comparison with experimental data/3/ at $p_\perp^2 = 0.03$ $(\mathrm{GeV}/c)^2$ and $p_L = 8$ (GeV/c).

APPENDIX I

Let us find the connection of s' with $M^2/s \cong 1-x$, M_Y^2/M^2, u and t. In the multiperipheral kinematics it is easy to obtain that

$$s' = \frac{\mu^2 + \kappa_\perp^2}{(1-x)(1-z)}, \qquad \text{(A.1)}$$

where $\vec{\kappa}_\perp$ is the normal component of the produced pion momentum (see fig. 1). For $\vec{\kappa}_\perp$ one may write the following equality:

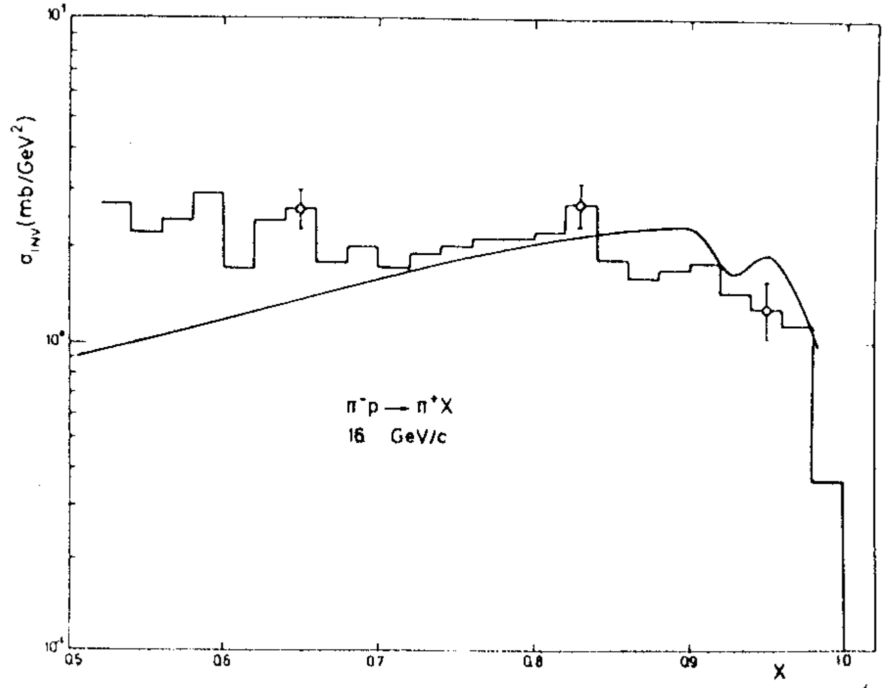


Fig. 8c. Comparison with experimental data[3] at $p_\perp^2 = 0.03$ (GeV/c)^2 and $p_L = 16$ GeV/c.

$$\vec{\kappa}_\perp = \vec{p}_\perp + (1-z)\vec{k}_\perp, \qquad (A.2)$$

where $\vec{p}_\perp$ and $\vec{k}_\perp$ are the transverse components of π^+ meson and the virtual pion momenta, respectively. $\vec{p}_\perp$ is connected apparently with t:

$$p_\perp^2 \simeq -t \qquad (A.3)$$

$k_\perp^2$ can be expressed through u

$$k_\perp^2 = (1-z)(u_{min} - u), \qquad (A.4)$$

where u_{min} is the value of u at $k_\perp = 0$

$$u_{min} = -\mu^2 \frac{z}{1-z} + zt. \qquad (A.5)$$

Thus, from (A.2) - (A.5) we have

$$\kappa_\perp^2 = (\mu^2 - u - t)(1-z) - \mu^2 + 2tz(1-z) + 2(1-z)\sqrt{-z(\mu^2 - u - t) - u - tz^2}\cos\phi, \qquad (A.6)$$

where $\cos\phi = (\vec{p}_\perp \cdot \vec{k}_\perp)/(|\vec{p}_\perp||\vec{k}_\perp|)$. From (A.1) and (A.6) we obtain formula (3). It is worth noting that the expression has been obtained in a different, more complicated way in ref.[7].

APPENDIX II

The maximum value of z at the fixed t and u can be obtained from (A.5), because $u = u_{min}$ when $z = z_m$

$$u = -\mu^2 \frac{z_m}{1 - z_m} + tz_m. \qquad (A.7)$$

From here we obtain (4).

REFERENCES

1. V.Barger. Rapporteur talk. Proc. London Conf., 1974.
2. B.M.Abramov et al. Yad.Fiz., 22, 1178 (1975); P.Bossetti et al. Nucl. Phys., B54, 141 (1973); V.Sredhar et al. Nucl. Phys., B88, 202 (1975); F.T.Go et al. Phys. Rev., D11, 3092 (1975); P.Bozzatta et al. Nuovo Cim., 15A, 45 (1973).

3. D.Amati et al. Nuovo Cim., 26, 826 (1962).
4. K.G.Boreskov et al. Yad.Fiz., 15, 361 (1972). Preprint ITEP, 43, 1973.
5. K.G.Boreskov et al. Yad.Fiz., 21, 825 (1975); Yad.Fiz., 14, 814 (1971);

6. J.Ballam et al. Phys.Rev., 1D, 94 (1970).
7. H.D.I.Abarbanel et al. Ann. of Phys., 73, 156 (1972).

Received by Publishing Department
on June 29, 1976.