# 05ъЕДИНЕННЫЙ 

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## POMERON FUSION AND CENTRAL $\eta$ <br> AND $\eta^{\prime}$ MESON PRODUCTION

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Рассчитан вклад померонного слияния в сечение рождения $\eta$ - и $\eta^{\prime}$-мезонов в двойном дифракционном рассеянии в рамках модели померона Доннахью-Ландшоффа. Показано, что механизм двойного померонного обмена не объясняет полного набора экспериментальных данных, полученных коллаборацией WA102. Тем не менее, указанный механизм позволяет объяснить экспериментальные данные для рождения $\eta^{\prime}$-мезона.

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The contribution of pomeron fusion to the cross section of $\eta$ and $\eta^{\prime}$ productions in double-diffractive scattering has been calculated within the Donnachie-Landshoff model of pomeron. It is shown that the double pomeron exchange mechanism does not explain the full set of the recent data of WA102 Collaboration, though it might not be inconsistent with $\eta^{\prime}$ productions.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

The structure of the pomeron is now widely under discussion [1]. One of the interesting ways to investigate this structure is the double diffractive process(DDP). Recently, the precise experimental data on central productions of pseudoscalar mesons, $\pi^{0}, \eta$ and $\eta^{\prime}$, have been published by WA102 Collaboration [2]. The most spread wisdom is that the main contribution to the DDP cross section should be related with the double pomeron exchange (DPE)[3]. The kinematics of DDP corresponds to a small momentum transfer region and in this region the contribution from other processes such as photon-photon and vector meson-vector meson fusion is also possible [4]. However, the experimental data give the value of the cross section of pseudoscalar meson productions which are several orders of magnitude larger than one can expect, for example, from $\gamma \gamma$ fusion [2]. One of the interesting feature of the data is unusual azimuthal angle dependence of the cross section. Different mechanisms, which may responsible for an enhancement of the production cross section in the kinematical region where the azimuthal angle between the $p_{T}$ vectors of two final protons is 90 degrees, have been discussed in recent papers [4],[5], [6].

The subject of our paper is to estimate the DPE contribution to the cross section of central $\eta, \eta^{\prime}$ productions in WA 102 kinematics.

A DPE diagram contributing to central $\eta, \eta^{\prime}$ productions, is presented in Fig.1. In order to


Figure 1: The DPE contribution to the central pseudoscalar meson production
estimate this contribution, the Donnachie-Landshoff(DL) model of pomeron [7] will be used. In this model the quark-quark interaction through the pomeron exchange is similar to the photon exchange with modified propagator

$$
\begin{equation*}
M_{\text {int }}=i \beta_{0}^{2} \bar{q}\left(p_{1}^{\prime}\right) \gamma_{\mu} q\left(p_{1}\right) \bar{q}\left(p_{2}^{\prime}\right) \gamma_{\mu} q\left(p_{2}\right)\left(S / S_{0}\right)^{\alpha P(t)-1}, \tag{1}
\end{equation*}
$$

where $S=\left(p_{1}+p_{2}\right)^{2}, t=\left(p_{1}-p_{1}^{\prime}\right)^{2}, \beta_{0} \approx 1.8 \mathrm{GeV}^{-1}, S_{0} \approx 1 \mathrm{GeV}$ and $\alpha_{P}(t)=1+\varepsilon+\alpha^{\prime} t$ is the pomeron trajectory with $\epsilon \approx 0.085$ and $\boldsymbol{\alpha}^{\prime}=0.25 \mathrm{GeV}^{-2}$. To calculate the contribution of the diagram in Fig.1, one should take into account the form factors in pomeron-proton and pomeron-pomeron-pseudoscalar meson vertices. For the pomeron-proton vertex, the electromagnetic form factor of the nucleon

$$
\begin{equation*}
F_{p}(t)=\frac{4 m_{p}^{2}-2.79 t}{\left(4 m_{p}^{2}-t\right)(1-t / 0.71)}, \tag{2}
\end{equation*}
$$

where $m_{p}$ is the proton mass, is usually used [7]. This form factor gives a rather good description of the $t$ dependence of the elastic $p p$ cross section at high energy [7]. Unfortunately, the pomeron-pomeron-pseudoscalar meson form factor is not known well. Within the DL model, the property of the pomeron-quark vertex is similar to the quark-photon vertex [7] and thus, for this form factor we expect the same momentum transfer dependence as for the transition pseudoscalar meson form factor $\gamma^{*} \gamma^{*} \rightarrow \eta, \eta^{\prime}$. For a small momentum transfer which we consider here in

DPE, the Brodsky-Lepage formula [8]

$$
\begin{equation*}
F_{P P M}\left(t_{1}, t_{2}\right)=\frac{1}{\left(1-t_{1} / 8 \pi^{2} f_{P S}^{2}\right)\left(1-t_{2} / 8 \pi^{2} f_{P S}^{2}\right)} \tag{3}
\end{equation*}
$$

can be used for this form factor, where $f_{P S}$ is a meson decay constant which is related to partial width $\Gamma_{\gamma \gamma}$ (see [9])

$$
\begin{equation*}
f_{P S}=\frac{\alpha}{\pi} \sqrt{\frac{M^{3}}{64 \pi \Gamma_{\gamma}}} . \tag{4}
\end{equation*}
$$

The cross section of the meson production in the reaction

$$
\begin{equation*}
p\left(p_{1}\right)+p\left(p_{2}\right) \rightarrow p\left(p_{1}^{\prime}\right)+p\left(p_{2}^{\prime}\right)+M\left(p_{M}\right) \tag{5}
\end{equation*}
$$

is given by the formula

$$
\begin{equation*}
d \sigma=\frac{d P S^{3}}{4 \sqrt{\left(p_{1} \cdot p_{2}\right)^{2}-m_{p}^{4}}} \sum_{\text {spin }}|T|^{2} \tag{6}
\end{equation*}
$$

## where

$$
\begin{equation*}
d P S^{3}=\frac{d^{4} p_{1}^{\prime} d^{4} p_{2}^{\prime} d^{4} p_{M}}{(2 \pi)^{9}}(2 \pi)^{4} \delta\left(p_{1}^{\prime}-m_{p}^{2}\right) \delta\left(p_{2}^{\prime}-m_{p}^{2}\right) \delta\left(p_{M}^{2}-M^{2}\right) \delta\left(p_{1}+p_{2}-p_{1}^{\prime}-p_{2}^{\prime}-p_{M}\right) \tag{7}
\end{equation*}
$$

is the 3 -body phase space volume: $M$ is the meson mass, $T$ is the matrix element for the DPE reaction and $\sum_{\text {spin }}$ stands for spin summation and spin average for the final and initial proton states, respectively.

At high energies and small momentum transfers, the four-momenta of initial and final protons in the center of mass system are given as

$$
\begin{align*}
p_{1} & \approx\left(P+m_{p}^{2} / 2 P, \overrightarrow{0}, P\right), \quad p_{2} \approx\left(P+m_{p}^{2} / 2 P, \overrightarrow{0},-P\right) \\
p_{1}^{\prime} & \left.\approx\left(x_{1} P+\left(m_{p}^{2}+\vec{p}_{1 T}^{2}\right) / 2 x_{1} P, \vec{p}_{1 T}, x_{1} P\right), \quad p_{2}^{\prime} \approx\left(x_{2} P+\vec{p}_{2 T}^{2}\right) / 2 x_{2} P, \vec{p}_{2 T},-x_{2} P\right), \tag{8}
\end{align*}
$$

where $P=\sqrt{S} / 2, S=\left(p_{1}+p_{2}\right)^{2}$. By using the result of Ref.[10] for the high energy phase space volume at small momentum transfers, we obtain

$$
\begin{equation*}
d^{4} p_{1,2}^{\prime} \delta\left(p_{1,2}^{2}-m_{p}^{2}\right) \approx \frac{1}{4} d t_{1,2} d x_{1,2} d \Phi_{1,2} \tag{9}
\end{equation*}
$$

where $\Phi_{i}$ are azimuthal angles of final protons and $t_{1,2}=-q_{1,2}^{2}=\left(p_{1,2}-p_{1,2}^{\prime}\right)^{2}$. Then, one can obtain

$$
\begin{equation*}
d P S^{3}=\frac{1}{2^{8} \pi^{4}} d t_{1} d t_{2} d x_{1} d x_{2} d \Phi \delta\left(S\left(1-x_{1}\right)\left(1-x_{2}\right)-M^{2}\right) \tag{10}
\end{equation*}
$$

for the phase space volume in the DPE reaction. The matrix element is given by

$$
\begin{equation*}
T=-9 \beta_{0}^{2} F_{p}\left(t_{1}\right) F_{p}\left(t_{2}\right) \bar{u}\left(p_{1}^{\prime}\right) \gamma_{\mu} u\left(p_{1}\right) \bar{u}\left(p_{2}^{\prime}\right) \gamma_{\mu} u\left(p_{2}\right)\left(S_{1} / S_{0}\right)^{\alpha_{p}\left(t_{1}\right)-1}\left(S_{2} / S_{0}\right)^{\alpha p\left(t_{2}\right)-1} p_{M}^{\tau} T_{\mu \nu \tau}, \tag{11}
\end{equation*}
$$

where $S_{1,2}=\left(q_{2,1}+p_{1,2}\right)^{2}, S_{0} \approx 1 \mathrm{GeV}$ and $T_{\mu \nu \tau}$ is the matrix element for two pomeron fusion into pseudoscalar mesons through flavor singlet axial vector current. By using the pomeronphoton analogy we can connect this matrix element with the matrix element of the meson decay $M \rightarrow \gamma \gamma$. This decay is determined by axial anomaly originated from a triangle graph (see Fig.1). Therefore, by taking into account only difference in the photon and pomeron coupling constants with quarks in the triangle graph, we get the result

$$
\begin{equation*}
p_{M}^{\tau} T_{\mu \nu \tau}=i \lambda F_{P P M}\left(t_{1}, t_{2}\right) \epsilon_{\mu \nu \rho \sigma} q_{1}^{\rho} q_{2}^{\sigma} \tag{12}
\end{equation*}
$$

where

$$
\begin{equation*}
\lambda=\frac{18 D \beta_{0}^{2}}{D^{\prime} \alpha} \sqrt{\frac{2 \Gamma_{T r}}{\pi M^{3}}} . \tag{13}
\end{equation*}
$$

Here $\Gamma_{\eta \rightarrow \gamma \gamma}=0.46 \times 10^{-6} \mathrm{GeV}, \Gamma_{\eta \rightarrow \gamma \gamma}=4.28 \times 10^{-6} \mathrm{GeV}$ and factors $D$ and $D^{\prime}$ are related with the wave functions of the $\eta$ and $\eta^{\prime}$

$$
\begin{gather*}
\dot{\eta}=-\sin \Theta \eta_{0}+\cos \Theta \eta_{8} \quad \eta^{\prime}=\cos \Theta \eta_{0}+\sin \Theta \eta_{8},  \tag{14}\\
D_{\eta}=-\sin \Theta, \quad D_{\eta^{\prime}}=\cos \Theta  \tag{15}\\
D_{\eta}^{\prime}=2 \sqrt{2} \cos \Theta-\sin \Theta, \quad D_{\pi^{\prime}}^{\prime}=2 \sqrt{2} \cos \Theta+\sin \Theta, \tag{16}
\end{gather*}
$$

where $\Theta=-19.5^{\circ}$ is a singlet-octet mixing angle. At high energies we have

$$
\begin{equation*}
\bar{u}\left(p_{1,2}^{\prime}\right) \gamma_{\mu} u\left(p_{1,2}\right) \approx\left(p_{1,2}+p_{1,2}^{\prime}\right)_{\mu} \tag{17}
\end{equation*}
$$

and thus, the matrix element (11) becomes

$$
\begin{equation*}
T=i 36 \beta_{0}^{2} \lambda\left(S_{1} / S_{0}\right)^{\alpha \rho}\left(t_{1}\right)-1\left(S_{2} / S_{0}\right)^{\alpha \rho\left(t_{2}\right)-1} \epsilon_{\mu \nu \rho \sigma} p_{1}^{\mu} p_{2}^{\nu} p_{1}^{\prime \rho} p_{2}^{\prime \sigma} \tag{18}
\end{equation*}
$$

By using (10) and (18) and the equations

$$
\begin{equation*}
\vec{p}_{1,2 T}^{2}=-\dot{x}_{1,2} t_{1,2}-\left(1-x_{1}\right)^{2} m_{p}^{2} \tag{19}
\end{equation*}
$$

which follow from (8), the cross section is finally described by

$$
\begin{align*}
\frac{d \sigma}{d t_{1} d t_{2} d x_{F} d \Phi}= & \frac{3^{4} \beta_{0}^{4} \lambda^{2} F_{p}^{2}\left(t_{1}\right) F_{p}^{2}\left(t_{2}\right) F_{P P M}^{2}\left(t_{1}, t_{2}\right)}{2^{9} \pi^{4} \sqrt{x_{F}^{2}+4 M^{2} / S}}\left(x_{1} t_{1}+\left(1-x_{1}\right)^{2} m_{p}^{2}\right)\left(x_{2} t_{2}+\left(1-x_{2}\right)^{2} m_{p}^{2}\right) \\
& \left(S_{1} / S_{0}\right)^{2\left(\alpha_{P}\left(t_{1}\right)-1\right)}\left(S_{2} / S_{0}\right)^{2\left(a_{p}\left(t_{2}\right)-1\right)} \sin ^{2} \Phi \tag{20}
\end{align*}
$$

where $x_{1}$ and $x_{2}$ are related with $x_{F}=x_{2}-x_{1}$ by the equation $\left(1-x_{1}\right)\left(1-x_{2}\right)=M^{2} / S$ and

$$
\begin{equation*}
S_{1,2}=S\left(1-x_{1,2}\right)+m_{p}^{2}+2 t_{1,2} . \tag{21}
\end{equation*}
$$

The kinematical limits for the phase space integration in (20) are given from the positivity of $\vec{p}_{1,2 T}^{2}$ (19) and the condition $S_{1,2} \geq\left(M+m_{p}\right)^{2}$.

It should be mentioned that the cross section (20) has a specific azimuthal angle dependence which is related with the Lorenz structure of the matrix element of axial anomaly (12). The cross section has a maximum at $90^{\circ}$. The same dependence has been observed by WA102 Collaboration [2].
The cross sections for the $\eta$ and $\eta^{\prime}$ production were calculated for WA102 kinematics at $\sqrt{S}=29.1 \mathrm{GeV}$. The total cross sections calculated in the interval $0 \leq x_{F} \leq 0.1$ are *

$$
\begin{equation*}
\sigma(\eta)=49 n b, \quad \sigma\left(\eta^{\prime}\right)=422 n b, \tag{22}
\end{equation*}
$$

which should be compared with the experiment data [2]

$$
\begin{equation*}
\sigma(\eta)^{e x p}=1295 \pm 16 \pm 120 \mathrm{nb}, \quad \sigma\left(\eta^{\prime}\right)^{e x p}=588 \pm 18 \pm 60 \mathrm{nb} . \tag{23}
\end{equation*}
$$

[^0]Considering uncertainty of the pomeron-quark coupling constant and experimental errors, the DPE model seems to be not inconsistent with data of $\eta^{\prime}$ productions. On the contrary, the situation is hopeless for $\eta$ productions: DPE cannot explain at all so big experimental data of cross sections. It should be noticed that the small DPE contribution to $\eta$ meson productions is related to the structure of its wave function (14) which contains only a small flavor singlet component that can contributes to DPE. It is impossible to obtain $\sigma(\eta)>\sigma\left(\eta^{\prime}\right)$ for any mechanism of these mesons productions which are sensitive only to the flavor singlet component of their wave function.

As for $\eta^{\prime}$ productions, the DPE model seems to work rather well. The $x_{F}$ dependence of the DPE cross section for $\eta^{\prime}$ productions is in qualitative agreement with experimental data, though the cross section is decreasing faster than the data at large $x_{F}$ regions, as shown in Fig.2. The differential cross section of $\eta^{\prime}$ productions is also presented in Fig. 3 as a function of the momentum transfer from one of the proton vertices. This dependence is in agreement with experimental data which shows fast decreasing of the production at large $t^{*}$.


Figure 2: $x_{F}$-dependence of the DPE contribution to $\eta^{\prime}$ central production in comparison with experimental data of WA102 Collaboration. The experimental data have been normalized to the total DPE cross section.

The energy dependence of the $\eta$ production was measured by WA102 Collaboration [11] and was found that it decreased with energy. This fact is also in contradiction with prediction of slightly increasing of the DPE cross section with energies (20).
In summary, although the DPE model seems to work rather well for $\eta^{\prime}$ productions, it docs not explain the $\eta$ productions. The situation is rather complicated: the full set of the WA102 data at $\sqrt{S}=29.1 \mathrm{GeV}$ cannot be explaincd by the DPE model alone and we need some other mechanisms. One of them can be related with the contribution of the nouperturbative fluctuation of gluon fields, i.e. instantons to the central meson productions [6].

[^1] (3).

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[^0]:    * We also have performed the exact phase space integration for the DPE reaction without using high energy approximation (10). The final result is slightly smaller than (22).

[^1]:    * The $t$ - dependence of $\eta$ and $\eta^{\prime}$ productions is very sensitive to the form factor in the pomeron-pomeronmeson vertex. The statement that DPE should have $e^{-b \mid t}$ belhavior [2] is not correct, because one should also take into account an additional $t$-dependence connected with nonlocality of the pomeron-pomeron-meson vertex

