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DECONFINEMENT PHASE TRANSITION IN ROTATING NONSPHERICAL COMPACT STARS

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Деконфайнмент-фазовый переход
во вращающихся несферических компактных звездах

Сформулированы самосогласованные уравнения гравитационного поля и его источников для аксиально-симметричного случая для применения к вращающимся компактным звездам. Развита теория возмущений по угловой скорости и определены физические характеристики звезд, такие как масса, форма, момент инерции и энергия. Метод позволяет рассмотреть возможность изменения внутренней структуры звезды во время вращения, а также разделить вычисление углового момента в зависимости от разных компонент в коррекции к моменту инерции. Численные решения произведены при использовании уравнения состояния, определяющего деконфайнмент-фазовые переходы, сконструированные при сохранении числа барионов и электрического заряда. Во время эволюции замедления нейтронных звезд, при значениях угловой скорости ниже критической может появляться кварковое ядро, что может быть зарегистрировано как характерный сигнал во временной эволюции пульсаров. Показано, что при сценарии замедления в результате дипольно-магнитного излучения смещение тормозного индекса от значения $n = 3$ не только сигнализирует о появлении, но и позволяет определить размеры кваркового ядра в пульсарах. Предложен также другой сценарий эволюции, связанный с аккрецией массы на звезду, во время которого возможен переход замедления в ускорение вращения, что будет сигналом к деконфайнмент-фазовым переходам в быстро вращающихся компактных объектах. Возможными кандидатами в такие объекты являются рентгеновские парные звезды малой массы с кГц квазипериодическими осцилляциями (КПО).

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Deconfinement Phase Transition in Rotating Nonspherical Compact Stars

We formulate the self-consistent set of equations for the gravitational field and its sources for the case of axial symmetry relevant for the application to rotating compact stars. We develop a perturbation theory with respect to angular velocity and define physical quantities such as mass, shape, momentum of inertia and total energy of the star. This method allows an investigation of the change of the internal structure of the star due to rotation as well as a separate evaluation of the angular velocity dependence of the different contributions to the moment of inertia. Numerical solutions have been performed using an equation of state describing the deconfinement phase transition as constrained by the conservation of total baryon number and electric charge. During the spin down evolution of the rotating neutron star, below critical values of angular velocity a quark matter core can appear which might be detected as a characteristic signal in the pulsar timing. We show that in the spin-down scenario due to magnetic dipole radiation the deviation of the breaking index from $n = 3$ could signal not only the occurrence but also the size of a quark core in the pulsar. We propose also another scenario where due to mass accretion onto the star a spin-down to spin-up transition might signal a deconfinement transition in the rapidly rotating compact object. Possible candidates of such stars might be found among the recently discovered low-mass X-ray binaries with kHz QPO's.

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I. INTRODUCTION

In many recent astrophysical applications of the theory of dense matter it is necessary to investigate the properties of rapidly rotating compact objects within general relativity theory. The reason of this development is the hope that changes in the internal structure of the dense matter, e.g. during phase transitions, could have observable consequences for the dynamics of the rotational behavior of these objects. Particular examples are the observations of glitches and postglitch relaxation in pulsars which are discussed as signals for superfluidity in nuclear matter [1] and the suggestion that the braking index is remarkably enhanced when a quark matter core occurs in the centre of a pulsar during its spin-down evolution [2]. Further constraints for the nuclear equation of state come from the observation of quasi-periodic brightness oscillations (QPO's) in low-mass X-ray binaries which entail mass and radius limits for rapidly rotating neutron stars [3].

The problem of rotation in the general relativity theory was and remains one of the central and complicated problems [4]. Besides of the modern methods of numerical solutions of this problem the method of perturbation theory [5] is physically the most systematic approach for the solution of the problem for stationary gravitational fields and their sources.

From the practical point of view for definitions of the integral characteristics of the astrophysical objects it is important to analyze the asymptotical expansion of the metric tensor at large distances from the stars, to be able to compare the results with observational data. One can of course introduce the physical parameters of the configuration using the symmetry properties of the object and the gravitational field by expressing them in terms of conserved charges. In this work we are using the definition of the J_z projection of the angular momentum of the nonspherical rotating star (z is the axis of the star's rotation and symmetry of the gravitational field) as a conserved integral of motion. It is a well known integral of the non diagonal element of the energy-momentum tensor in the frame of spherical coordinates.

Using the method of perturbation theory we are going to calculate the total mass, angular momentum and shape deformation from the iterative solution of the gravitational field equations in case of hydrodynamical, thermodynamical and chemical equilibrium for given total baryon number and angular velocity Ω of the object. The perturbation method allows to solve the problem for all possible angular velocities, since the expansion parameter is the ratio of the rotational and gravitational energy for which it has been shown, that for compact objects in the stationary rotating regime without matter flux, the first two terms of the series expansion give a sufficiently good approximation.

The evolution of the rotating stars could have a different origins and scenarii. Our aim in this work is to discuss possible signals for a deconfinement phase transition during the evolution of rotating compact object on the basis of solutions for the Ω dependence of the moment of inertia.

II. SELF-CONSISTENT SET OF FIELD EQUATIONS FOR STATIONARY ROTATING STARS

A. Einstein equations for axial symmetry

The general form of the metric for an axial symmetric space-time manifold is

$$ds^2 = e^\nu dt^2 - e^\lambda dr^2 - r^2 e^\mu (d\theta^2 + \sin^2\theta (d\phi + \omega dt)^2) \quad , \tag{2.1}$$

written in a spherical symmetric coordinate system in order to obtain as a limiting case the Schwarzschild solution. This line element is time-translational and axial-rotational invariant; all metric functions are dependent on the coordinate distance from the coordinate center r and azimuthal angle θ between the radius vector and the axis of symmetry.

Reversal symmetry of the time and polar angle ϕ requires that all metric coefficients except ω must be even functions of the angular velocity

$$\Omega = \frac{d\phi}{dt} \tag{2.2}$$

of the star, the gravitational field of which is described by the Eq. (2.1). The physical characteristics of the rotating object depend on the centrifugal forces in the local inertial frame of the observer. In general relativity due to the Lenz-Thirring law rotational effects are described by $\bar{\omega}$ the difference of the frame dragging frequency $-\omega$ and the angular velocity Ω

$$\bar{\omega} \equiv \Omega + \omega(r, \theta). \tag{2.3}$$

The energy momentum tensor of stellar matter can be approximated by the expression of the energy momentum tensor of an ideal liquid

$$T^\nu_\mu = (\varepsilon + p) u_\mu u^\nu - p \delta^\nu_\mu, \tag{2.4}$$

where u^μ is the 4-velocity of matter, p the pressure and ε the energy density.

We assume that the star due to high viscosity (ignoring the super-fluid component of the matter) rotates stationary as a solid body with an angular velocity Ω that is independent of the spatial coordinates. The time scales for changes in the angular velocity which we will consider in our applications are well separated from the relaxation times at which hydrodynamical equilibrium is established, such that the assumption of a rigid rotator model is justified.

Therefore there are only two non-vanishing components of the velocity

$$\begin{aligned} u^\phi &= \Omega\, u^t \\ u^t &= 1/\sqrt{e^\nu - r^2 e^\mu \bar{\omega}^2 sin^2\theta} \end{aligned} \tag{2.5}$$

Once the energy-momentum tensor (2.4) is fixed by the choice of the equation of state for stellar matter, the unknown metric functions ν,λ, μ,$\bar{\omega}$ can be determined by the set Einstein field equations of which we use the following four combinations. The

equation of state which we will use for our investigation of the deconfinement transition in rotating compact stars will be introduced in Sect. III.

There are three Einstein equations for the determination of the diagonal elements of the metric tensor

$$\begin{aligned} G^r_r - G^t_t &= 8\pi G (T^r_r - T^t_t) \ , \\ G^\theta_\theta + G^\phi_\phi &= 8\pi G (T^\theta_\theta + T^\phi_\phi) \ , \\ G^r_\theta &= 0 \ , \end{aligned} \tag{2.6}$$

and one for the determination of the non diagonal element

$$G^t_\phi = 8\pi G T^t_\phi \ . \tag{2.7}$$

Here G is the gravitational constant and G^ν_μ the Einstein tensor.

We use also one equation for the hydrodynamical equilibrium (Euler equation)

$$H(r,\theta) \equiv \int \frac{dp'}{p' + \varepsilon'} = \frac{1}{2} \ln[u^t(r,\theta)] + \text{const}, \tag{2.8}$$

where the gravitational enthalpy H thus introduced is a function of the energy and/or pressure distribution.

The parameters of the theory are the angular velocity of the rotation Ω and the central energy density $\varepsilon(0)$ of the star configuration.

B. Perturbative approach to the solution

The problem of the rotation can be solved iteratively by using a perturbation expansion of the metric tensor and the physical quantities in a Taylor series with respect to the angular velocity. As a small parameter for this expansion we use the ratio of the rotational energy to the gravitational one of a homogeneous Newtonian star $E_{rot}/E_{grav} = (\Omega/\bar{\Omega})^2$, where $\bar{\Omega}^2 = 8\pi G\rho(0)$ with the mass density $\rho(0)$. This expansion gives sufficiently correct solutions already at $O(\Omega^2)$, since the expansion parameter is limited to values $\Omega/\bar{\Omega} \ll 1$ by the condition of mechanical stability of the rigid rotation. This can easily be seen by considering as an upper limit for attainable angular velocities the so called Kepler one $\Omega_K = \sqrt{GM/R_e^3}$ with M being the total mass and R_e the equatorial radius. For homogeneous Newtonian spherical stars $\Omega < \Omega_K = \bar{\Omega}/\sqrt{6}$.

The expansion of the metric tensor is given by

$$g_{\mu\nu}(r,\theta;\Omega) = \sum_{j=0}^{\infty} \left(\frac{\Omega}{\bar{\Omega}}\right)^j g^{(j)}_{\mu\nu}(r,\theta) \ . \tag{2.9}$$

According to the symmetries of the metric coefficients introduced in Eq. (2.1) we have even orders $j = 0, 2, ...$ for the diagonal elements*

---

*Notation corresponds to the works [5].

$$
\begin{aligned}
e^{-\lambda(r,\theta;\Omega)} &= e^{-\lambda^{(0)}(r)}(1+(\Omega/\bar{\Omega})^2 f(r,\theta)) + O(\Omega^4) \ , \\
e^{\nu(r,\theta;\Omega)} &= e^{\nu^{(0)}(r)}(1+(\Omega/\bar{\Omega})^2 \Phi(r,\theta)) + O(\Omega^4) \ , \\
e^{\mu(r,\theta;\Omega)} &= r^2(1+(\Omega/\bar{\Omega})^2 U(r,\theta)) + O(\Omega^4) \ ,
\end{aligned} \tag{2.10}
$$

and odd orders only for the frame dragging frequency ω

$$
\omega(r,\theta;\Omega) = \frac{\Omega}{\bar{\Omega}}\, q(r,\theta) + O(\Omega^3). \tag{2.11}
$$

The distributions of pressure, energy density and "enthalpy" introduced in Eq.(2.8) are also included in the scheme of this perturbation expansion

$$
\begin{aligned}
p(r,\theta;\Omega) &= p^{(0)}(r) + (\Omega/\bar{\Omega})^2 p^{(2)}(r,\theta) + O(\Omega^4) \ , \\
\varepsilon(r,\theta;\Omega) &= \varepsilon^{(0)}(r) + (\Omega/\bar{\Omega})^2 \varepsilon^{(2)}(r,\theta) + O(\Omega^4), \\
H(r,\theta;\Omega) &= H^{(0)}(r) + (\Omega/\bar{\Omega})^2 H^{(2)}(r,\theta) + O(\Omega^4).
\end{aligned} \tag{2.12}
$$

All functions with superscript (0) denote the solution of the static configuration and therefore they are only functions of r, the others are the corrections corresponding to the rotation.

This series expansion allows to transform the Einstein equations into a coupled set of equations for the coefficient functions which can be solved by recursion. At zeroth order we recover the nonlinear problem of the static spherically symmetric star configuration (Tolman-Oppenheimer-Volkoff equations), see the next subsection II C. The first recursion step is to solve Eq. (2.7) in order to obtain the dragging frequency in $O(\Omega)$ and to define the moment of inertia for the sperically symmetric configuration. In subsection II E we will consider the second order contribution in the Ω- expansion (2.10), (2.12) where the $O(\Omega^2)$ corrections to the moment of inertia in can be found. The next terms in the expansion which are of $O(\Omega^3)$ correspond to corrections of the frame dragging frequency and will be neglected since they go beyond the approximation scheme adopted in the present paper.

C. Zeroth order: Static spherically symmetric star models

The functions of the spherically symmetric solution can be found from Eq. (2.6) and Eq. (2.8) in zeroth order of the Ω- expansion.

That is the solution of the following equations (Tolman-Oppenheimer-Volkoff)

$$
\frac{dp^{(0)}(r)}{dr} = -G(p^{(0)}(r) + \varepsilon^{(0)}(r))\frac{m(r) + 4\pi p^{(0)}(r) r^3}{r(r-2Gm(r))} \ , \tag{2.13}
$$

where $m(r)$ is the distribution of accumulated mass

$$
m(r) = 4\pi \int_0^r \varepsilon^{(0)}(r') r'^2 dr' \ . \tag{2.14}
$$

within a sphere of radius r. For the gravitational potentials we have

$$
\lambda^{(0)}(r) = -\ln(1 - \frac{2Gm(r)}{r}) \tag{2.15}
$$

$$
\nu^{(0)}(r) = -\lambda^{(0)}(R_0) - 2G\int_r^{R_0} \frac{m(r') + 4\pi p^{(0)}(r') r'^3}{r'(r' - 2Gm(r'))} dr' \ . \tag{2.16}
$$

R_0 is the spherical radius of the star, which is defined by $P^{(0)}(R_0) = 0$. The set of Eq.(2.13) and Eq.(2.14) fulfills the following conditions at the center of the configuration: $\varepsilon^{(0)}(0) = \varepsilon(0)$ and $m(0) = 0$. The central energy density $\varepsilon(0)$ is the parameter of the spherical configuration. The total mass of the spherically distributed matter in the selfconsistent gravitational field is $M_0(\varepsilon^{(0)}(0)) = m(R_0)$.

D. Moment of inertia

In the first order of the approximation we are solving Eq.(2.7), where the unknown function $q(r,\theta)$ defined by Eq. (2.11) is independent of the angular velocity. Using the static solutions Eqs.(2.13)-(2.15), and the representation of $q(r,\theta)$ by the series of the Legendre polynomials

$$
q(r,\theta) = \sum_{m=0}^{\infty} q_m(r) \frac{dP_{m+1}(\cos\theta)}{d\cos\theta} \tag{2.17}
$$

we find the equations for the coefficients $q_m(r)$. It is proved that $q(r,\theta)$ is a function of the distance r only, i.e. that $q_m(r) = 0$ for $m > 0$ [4,5].

Let us write down the equations for $\bar{\omega}(r) = \Omega(1 + q_0(r)/\bar{\Omega})$, which is more suitable for the solution of the resulting equation in first order

$$
\frac{1}{r^4}\frac{d}{dr}(r^4 j(r)\frac{d\bar{\omega}(r)}{dr}) + \frac{4}{r}\frac{dj(r)}{dr}\bar{\omega}(r) = 0 \ , \tag{2.18}
$$

which corresponds to Ref. [4], where it was obtained using a different representation of the metric. Here we use the notation $j(r) \equiv e^{-(\nu^{(0)}(r) + \lambda^{(0)}(r))/2}$, for which outside of configuration holds $j(r) = 1 \ , \quad r > R_0$.

By definition, the angular momentum of the star in the case of stationary rotation is a conserved quantity and can be expressed in invariant form

$$
J = \int T_\phi^t \sqrt{-g} dV \ , \tag{2.19}
$$

where $\sqrt{-g}dV$ is the invariant volume and $g = \det ||g_{\mu\nu}||$. For the case of slow rotation where the shape deformation of the rotating star can be neglected and using the definition of the moment of inertia $I_0(r) = J_0(r)/\Omega$ accumulated in the sphere with radius r, we obtain from Eq. (2.19)

$$
\frac{dI_0(r)}{dr} = \frac{8\pi}{3} r^4 (\varepsilon^{(0)}(r) + p^{(0)}(r)) e^{(-\nu^{(0)(r)} + \lambda^{(0)}(r))/2} \frac{\bar{\omega}(r)}{\Omega} \ . \tag{2.20}
$$

Using this equation one can reduce the second order differential equation (2.18) to the first order one

$$\frac{d\bar{\omega}(r)}{dr} = \frac{6GJ_0(r)}{r^4 j(r)} . \tag{2.21}$$

and solve (2.18) as a coupled set of first order differential equations, one for the moment of inertia (2.20) and the other (2.21) for the frame dragging frequency $\bar{\omega}(r)$.

This system of equations is valid inside and outside the matter distribution. In the center of the configuration holds $I_0(0) = 0$ and $\bar{\omega}(0) = \bar{\omega}_0$. The finite value $\bar{\omega}_0$ has to be defined such that the dragging frequency $\bar{\omega}(r)$ smoothly joins the outer solution

$$\bar{\omega}(r) = \Omega\left(1 - \frac{2GI_0}{r^3}\right) . \tag{2.22}$$

at $r = R_0$, and approaches Ω in the limit $r \to \infty$. In the external solution (2.22) the constant $I_0 = I_0(R_0)$ is the total moment of inertia of the slowly rotating star and $J_0 = I_0\Omega$ is the corresponding angular momentum. In this order of approximation, I_0 is a function of the central energy density or the total baryon number only. Explicit dependences of the moment of inertia on the angular velocity occur in the second order of approximation.

E. Second order corrections to moment of inertia

Due to the rotation in Ω^2-approximation the shape of the star is an ellipsoid, and each of the equal-pressure (isobar) surfaces in the star is an ellipsoid as well. All diagonal elements of the metric and energy-momentum tensors could be represented as a series expansion in Legendre polynomials

$$g^{(2)}_{\mu\nu}(r,\theta) = \sum_{l=0}^{\infty} (g_{\mu\nu})_l(r) P_l(\cos\theta) . \tag{2.23}$$

It has been shown that only the solutions with $l = 0, 2$ obeying the continuity conditions on the surface are non trivial.

The deformation of the isobaric surfaces due to the rotation can be parametrized by the shift $R(r,\theta) - r = \Delta(r,\theta)$ which describes the deviation from the spheric distribution as a function of the radius r in the given polar angle θ and is completely determined by

$$R(r,\theta) = r + \left(\frac{\Omega}{\bar{\Omega}}\right)^2 (\Delta_0(r) + \Delta_2(r) P_2(\cos\theta)), \tag{2.24}$$

since the expansion coefficients of the deformation $\Delta_l(r)$ can be calculated from the pressure corrections

$$\Delta_l(r) = -\frac{p_l(r)}{dp^{(0)}(r)/dr} . \tag{2.25}$$

$l \in \{0, 2\}$ is the polynomial index in the angular expansion. The function $R(R_0, \theta)$ is the distance of the star surface from the center of the configuration in the direction with the angle θ to the polar axis. In particular, we can define the equatorial radius

$R_e = R(R_0, \theta = \pi/2)$ and the polar radius $R_p = R(R_0, \theta = 0)$ and the excentricity $\epsilon = \sqrt{1 - (R_p/R_e)^2}$.

The correction to the momentum of inertia $\Delta I(r) = I(r) - I_0(r)$ can be represented in the form

$$\Delta I = \Delta I_{Redistribution} + \Delta I_{Shape} + \Delta I_{Field} + \Delta I_{Rotational\ Energy} . \tag{2.26}$$

The first three contributions can be expressed by integrals of the angular averaged modifications of the matter distribution, the shape of the configuration and the gravitational fields, in the form

$$\Delta I_\alpha = \int_0^{I_0(R_0)} dI_0(r) \{W_0^{(\alpha)}(r) - \frac{W_2^{(\alpha)}(r)}{5}\} , \tag{2.27}$$

where

$$W_l^{(Field)}(r) = \left(\frac{\Omega}{\bar{\Omega}}\right)^2 (2U_l(r) - (f_l(r) + \Phi_l(r))/2) , \tag{2.28}$$

$$W_l^{(Shape)}(r) = \left(\frac{\Omega}{\bar{\Omega}}\right)^2 \frac{d\,\Delta_l(r)}{dr} , \tag{2.29}$$

$$W_l^{(Redistribution)}(r) = \left(\frac{\Omega}{\bar{\Omega}}\right)^2 \frac{p_l(r) + \varepsilon_l(r)}{p^{(0)}(r) + \varepsilon^{(0)}(r)} , \tag{2.30}$$

respectively, which have to be determined from the Eq. (2.6) in second order approximation. The contribution of the change of the rotational energy to the moment of inertia

$$\Delta I_{Rotating\ Energy} = \frac{4}{5} \int_0^{I_0(R_0)} dI_0(r) r^2 \bar{\omega}^2(r) e^{-\nu_0(r)}\} . \tag{2.31}$$

includes the frame dragging contribution. In this expansion we neglect the influence of the change of the frame dragging frequency, since it corresponds to the next order of the perturbative expansion $\sim O(\Omega^3)$. A more detailed description of the solutions of the field equations in the $\sim O(\Omega^2)$ approximation is beyond the scope of this article, so that we give only a short summary in Appendix A and refer to works of Hartle [4] and Chubarian [5].

III. MODEL EOS WITH DECONFINEMENT PHASE TRANSITION

For the investigation of the deconfinement phase transition expected to occur in neutron star matter at densities above the nuclear saturation density $n_0 = 0.16$ fm^{-3} several approaches to quark confinement dynamics have been discussed, see e.g. [10,12,11] which lead to interesting conclusions for the properties of quark matter at high densities. Most of the approaches to quark deconfinement in neutron star matter, however, use a thermodynamical bag-model for the quark matter and employ a standard two-phase description of the equation of state (EOS) where the hadronic phase and the quark matter phase are modeled separately and the resulting EOS is obtained by imposing

Gibbs' conditions for phase equilibrium with the constraint that baryon number as well as electric charge of the system are conserved [13,14]. Since the focus of our work is the elucidation of qualitative features of signals for a possible deconfinement transition in the pulsar timing, we will consider here such a rather standard, phenomenological model for an EOS with deconfinement transition.

The total pressure $p(\{\mu_i\},T)$ as a thermodynamical potential is given by

$$p(\{\mu_i\},T) = (1-\chi)p_H(\{\mu_h\},T) + \chi p_Q(\{\mu_q\},T) + p_L(\{\mu_l\},T) \quad , \tag{3.1}$$

where

$$p_H(\{\mu_h\},T) = \sum_{h=n,p} p_h^{id}(\mu_h^*, T; m_h^*) \tag{3.2}$$

is the EOS of the relativistic $\sigma-\omega$ mean-field model (Walecka model) for nuclear matter and

$$p_Q(\{\mu_q\},T) = \sum_{q=u,d} p_q^{id}(\mu_q, T; m_q) - B \tag{3.3}$$

is the pressure for two-flavor quark matter within a bag model EOS with the phenomenological bag pressure $B = 75$ MeVfm^{-3} that enforces quark confinement and the transition to nuclear matter at low densities. In a neutron star, these hadronic phases of matter are in $\beta-$ equilibrium with electrons and muons which contribute to the pressure balance with

$$p_L(\{\mu_l\},T) = \sum_{l=e^-,\mu^-} p_l^{id}(\mu_l, T; m_l) \quad . \tag{3.4}$$

In the above expressions $p_i^{id}(\mu_i,T;m_i) = p_i^-(\mu_i,T;m_i) + p_i^+(\mu_i,T;m_i)$ is the partial pressure of the Fermion species i as a sum of particle and antiparticle contributions defined by

$$p_i^{\pm}(\mu_i,T;m_i) = \gamma_i \int_0^\infty \frac{d^3k}{(2\pi)^3} T \ln[1 + \exp(\sqrt{k^2+m_i^2}/T \pm \mu/T)] \quad . \tag{3.5}$$

In the relativistic mean-field model, the masses and chemical potentials have to be renormalized by the mean-values of the $\sigma-$ and $\omega-$ fields [15,16] $m_h^* = m_h - g_\sigma\bar{\sigma}$, $\mu_h^* = \mu_h - g_\omega\bar{\omega}_0$.

All the other thermodynamic quantities of interest can be derived from the pressure (3.1) as, e.g., the partial densities of the species

$$n_j(\{\mu_i\},T) = \frac{\partial p(\{\mu_i\},T)}{\partial \mu_j} \quad . \tag{3.6}$$

The chemical equilibrium due to the direct and inverse β- decay processes imposes additional constraints on the values of the chemical potentials of leptonic and baryonic species [14,17] such that only two independent chemical potentials remain according to the corresponding two conserved charges of the system, the total baryon number N_B as well as electrical charge Q

$$n_B = \frac{N_B}{V} = (1-\chi)n_H(\{\mu_h\},T) + \chi n_Q(\{\mu_q\},T) \tag{3.7}$$

$$0 = \frac{Q}{V} = (1-\chi)q_H(\{\mu_h\},T) + \chi q_Q(\{\mu_q\},T) + q_L(\{\mu_l\},T) \quad . \tag{3.8}$$

The deconfinement transition is obtained following the construction of Glendenning [13,14], which obeys the global conservation laws and allows to find the volume fraction of the quark matter phase $\chi = V_Q/V$ in the mixed phase where $p_H(\{\mu_h\},T) = p_Q(\{\mu_q\},T)$, such that at given n_B and T the total pressure under the conditions (3.7), (3.8) is a maximum.

In Fig. 1 we show the composition of the hybrid star matter as a function of the total baryon density at $T = 0$. Solving the Tolman-Oppenheimer-Volkoff equations (2.13)-(2.15) for the hydrodynamical equilibrium of static spherically symmetric relativistic stars with the above defined EOS, we find that a configuration at the stability limit could have a quark matter core with a radius as large as $\sim 75\%$ of the stars radius.

What implications this phase transition for rotating star configurations might have will be investigated in the next section by applying the method developed in Sect. II for the above EOS.

IV. RESULTS AND DISCUSSION

The results for the stability of rotating neutron star configurations with possible deconfinement phase transition according to the EOS described in the previous section are shown in Fig. 2, where the total baryon number, the total mass, and the moment of inertia are given as functions of the equatorial radius (left panels) and in dependence on the central baryon number density (right panels) for static stars (solid lines) as well as for stars rotating with the maximum angular velocity $\Omega_{\max}$ (dashed lines). The dotted lines connect configurations with fixed total baryon numbers $N_B/N_\odot = 1.3, 1.55, 1.8, 2.14$ and it becomes apparent that the rotating configurations are less compact than the static ones. They have larger masses, radii and momenta of inertia at less central density such that for suitably chosen configurations a deconfinement transition in the interior can occur upon spin-down.

In Fig. 3 we show the critical regions of the phase transition in the inner structure of the star configuration as well as the equatorial and polar radii in the plane of angular velocity Ω versus distance from the center of the star. It is obvious that with the increase of the angular velocity the star is deformating its shape. The maximal excentricities of the configurations with $N_B = 1.3\ N_\odot$, $N_B = 1.55\ N_\odot$ and $N_B = 1.8\ N_\odot$ are $\epsilon(\Omega_{\max}) = 0.7603$, $\epsilon(\Omega_{\max}) = 0.7655$ and $\epsilon(\Omega_{\max}) = 0.7659$, respectively. Due to the changes of the central density the quark core could disappear above a critical angular velocity.

In Fig. 4 we display the dependence of the moment of inertia on the angular velocity for configurations with the same total baryon number $N_B = 1.55\ N_\odot$ together with the different contributions to the total change of the moment of inertia. As it is shown the most important contributions come from the mass redistribution and the shape deformation. The relativistic contributions due to field and rotational energy are less important. In the same Fig. 4 we show the decrease of the spherical moment of inertia due to the decrease of the central density for high angular velocities which tends to

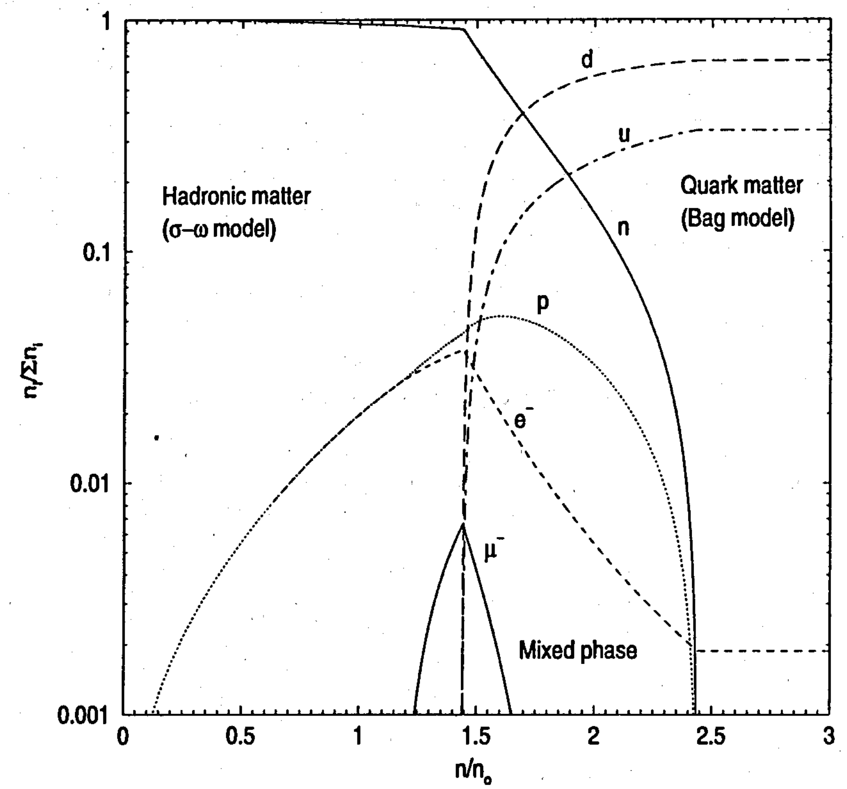

FIG. 1. Composition of hybrid star matter in β-equilibrium as a function of baryon number density. The hadronic equation of state is a relativistic mean-field model ($\sigma - \omega$), the quark matter one is a two-flavor bag model with $B = 75$ MeV fm^{-3}.

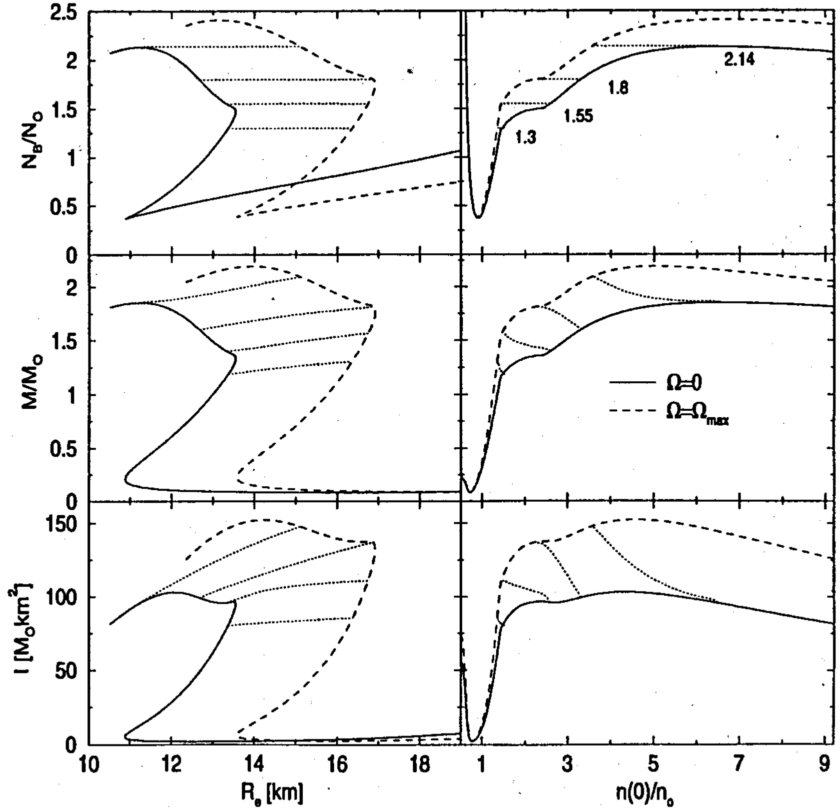

FIG. 2. Baryon number N, mass M, and moment of inertia I as a function of the equatorial radius (left panels) and the central density (right panels) for neutron star configurations with a deconfinement phase transition. The solid curves correspond to static configurations, the dashed ones to those with maximum angular velocity $\Omega_{\rm max}$. The lines between both extremal cases connect configurations with the same total baryon number $N_B/N_\odot = 1.3, 1.55, 1.8, 2.14$.

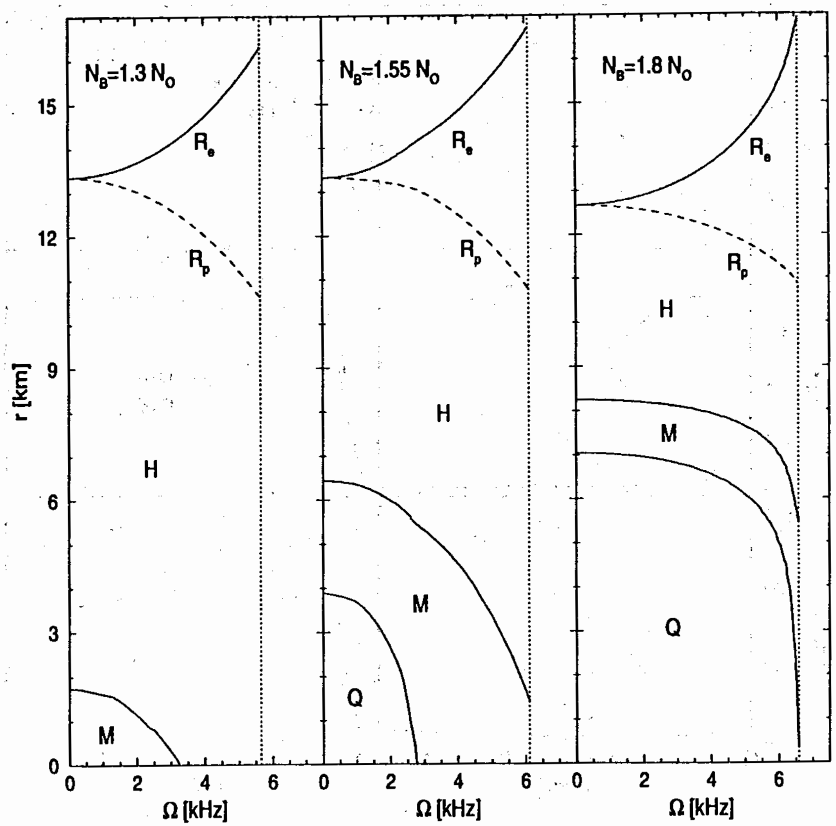

FIG. 3. Phase structure of rotating hybrid stars in equatorial direction in dependence of the angular velocity Ω for stars with different total baryon number: $N_B/N_\odot = 1.3, 1.55, 1.8$.

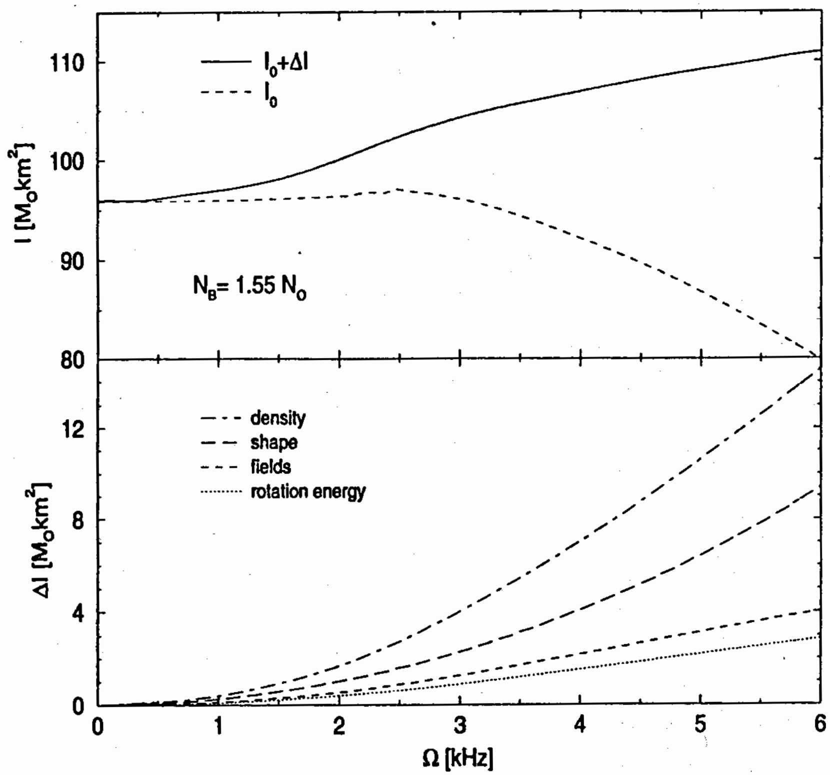

FIG. 4. Contributions to the dependence of the moment of inertia on the angular velocity

partially compensate the further increase of the total moment of inertia for large Ω. There is no dramatic change in the slope of $I(\Omega)$ at $\Omega_{\rm crit} = 2.77$ kHz.

Fig. 5 shows the dependence of the moment of inertia as a function of the angular velocity. It is demonstrated that the behavior of $I(\Omega)$ for a given total number of baryons N_B strongly depends on the presence of a pure quark matter core in the center of the star. If the core does already exist or it does not appear when the angular velocity increases up to the maximum value $\Omega_{\rm max}$ then the second order derivative of the moment of inertia $I(\Omega)$ does not change its sign. For the configuration with $N_B = 1.55\ N_\odot$ the critical value for the occurence of the sign change is $\Omega_{\rm crit} = 2.77$ kHz while for $N_B = 1.8\ N_\odot$ it is close to $\Omega_{\rm max} = 6.6$ kHz.

In order to point out possible observable consequences of such a characteristic behaviour of $I(\Omega, N_B)$ we consider two possible scenarios for changes in the pulsar timing: (A) dipole radiation and the resulting dependence of the braking index on the angular velocity as suggested in Ref. [2] and (B) mass accretion onto rapidly rotating neutron stars.

A. Dipole radiation

Due to the energy loss by dipole radiation the star has to spin-down and the resulting change of the angular velocity can be parametrized by a power law

$$\frac{\dot{\Omega}}{\Omega} = -K\Omega^{n(\Omega)-1}, \tag{4.1}$$

where K is a constant and $n(\Omega)$ is the braking index

$$n(\Omega) = \frac{\ddot{\Omega}\,\Omega}{\dot{\Omega}^2} = 3 - \frac{3I'\Omega + I''\Omega^2}{2I + I'\Omega}, \tag{4.2}$$

where we used the notation $I' = (\partial I(\Omega, N_B)/\partial\Omega)|_{N_B={\rm const}}$, with the corresponding definition of I'', see also [2].

In Fig. 6 we display the result for the braking index $n(\Omega)$ for a set of configurations with fixed total baryon numbers ranging from $N_B = 1.55\ N_\odot$ up to $N_B = 1.9\ N_\odot$, the region where during the spin-down evolution a quark matter core could occur for our model EOS. We observe that only for configurations within the interval of total baryon numbers $1.4 \le N_B/N_\odot \le 1.9$ a quark matter core occurs during the spin-down as a consequence of the increasing central density, see also Fig. 3, and the braking index shows variations. The critical angular velocity $\Omega_{\rm crit}(N_B)$ for the appearance of a quark matter core can be found from the minimum of the braking index Eq. (4.2). As can be seen from Fig. 6, all configurations with a quark matter core have braking indices $n(\Omega) < 3$ and braking indices significantly larger than 3 can be considered as precursors of the deconfinement transition. The magnitude of the jump in $n(\Omega)$ during the transition to the quark core regime is a measure for the size of the quark core.

Is it feasible to observe such a jump within a reasonable time interval?

It would be sufficient to observe the maximum of the braking index $n_{\rm max}$ in order to infer not only the onset of deconfinement from the fact that $n(\Omega) \neq 3$ but also the

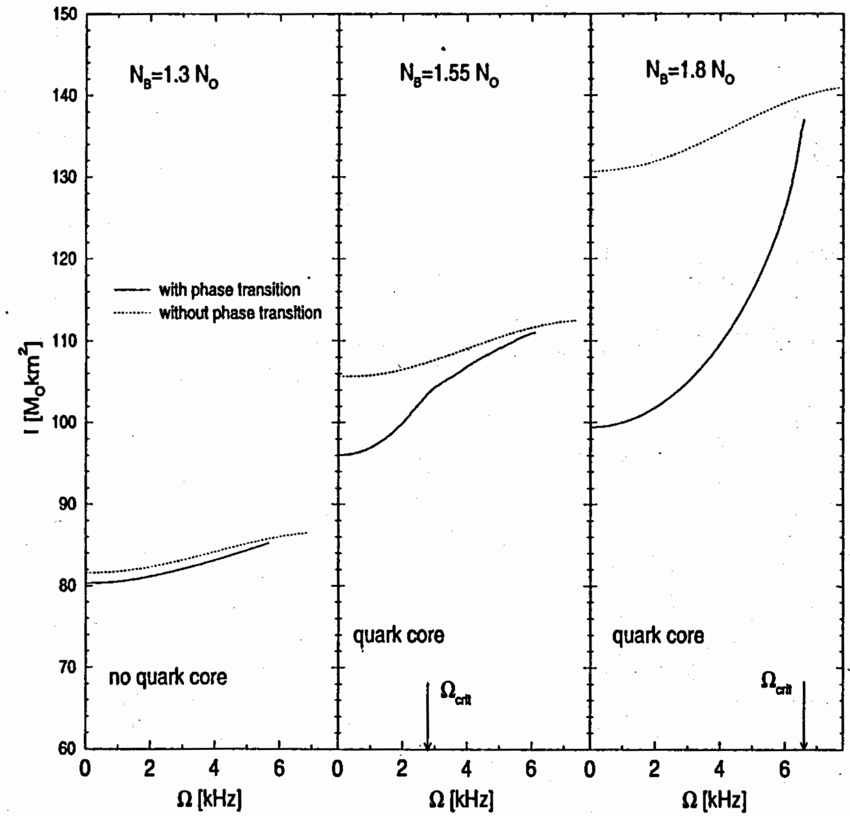

FIG. 5. Momentum of inertia as a function of angular velocity with (solid lines) and without (dashed lines) deconfinement phase transition for fixed total baryon number $N_B/N_\odot = 1.3$ (left panel), $N_B/N_\odot = 1.55$ (middle panel), $N_B/N_\odot = 1.8$ (right panel).

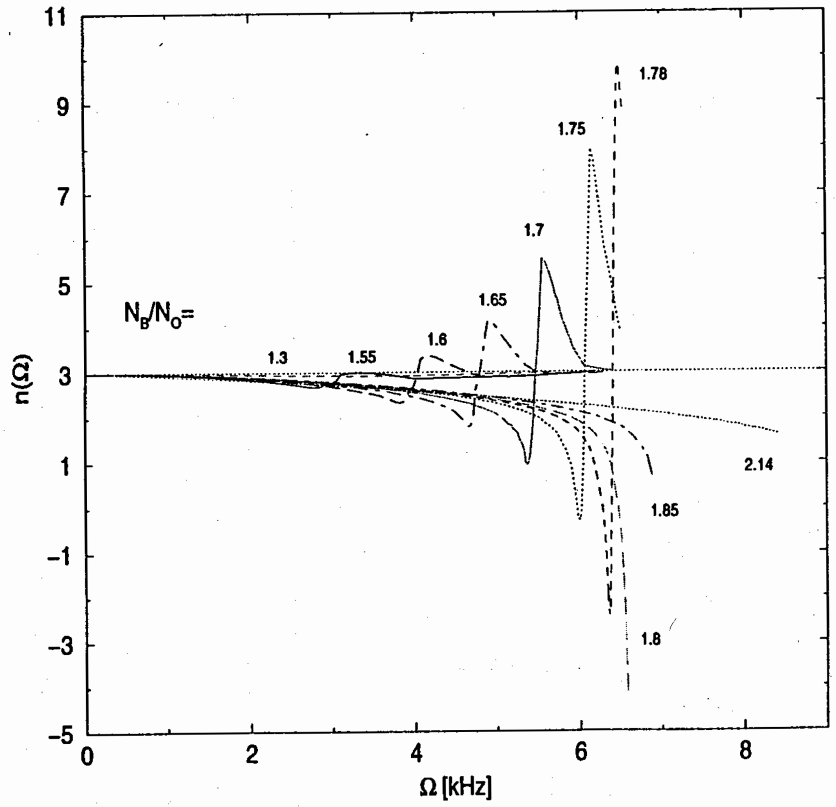

FIG. 6. Braking index due to dipole radiation from fastly rotating isolated pulsars as a function of the angular velocity. The minima of $n(\Omega)$ indicate the appearance/disappearance of quark matter cores.

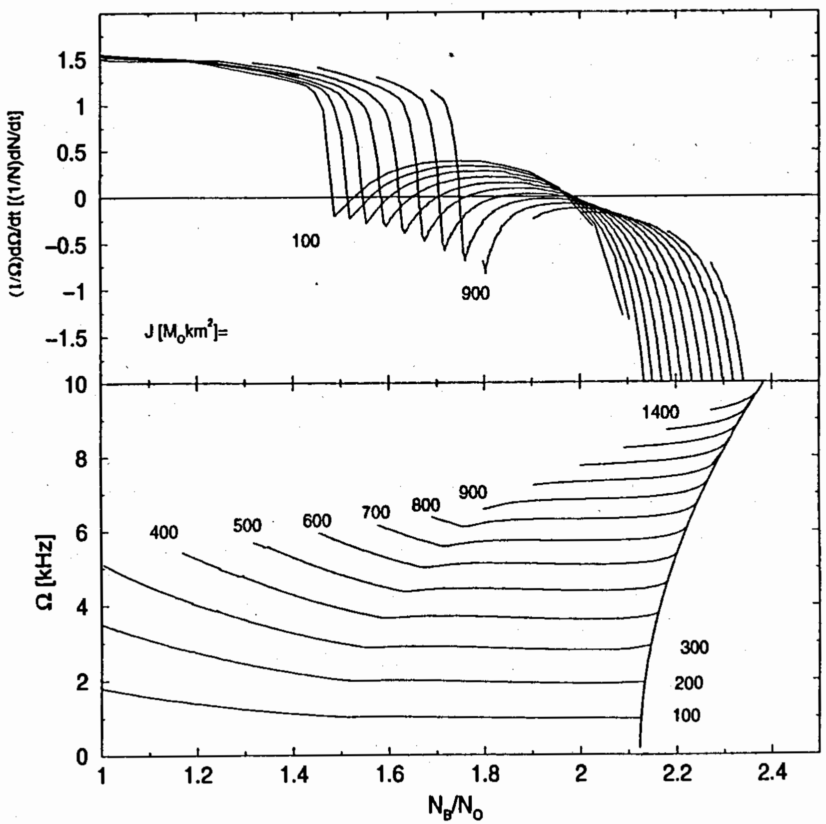

FIG. 7. Total baryon number dependence of the spin-down rate $\dot{\Omega}/\Omega$ in units of the baryon number accretion rate ($\dot{N}_B/N_B$) and the corresponding angular velocity (lower panel) for different (conserved) total angular momenta $J[M_\odot km^2] = 100, 200, \ldots, 1400$. The suggested signal for a deconfinement transition in rapidly rotating neutron stars with baryon number accretion is a transition from a spin-down to a spin-up regime, i.e. a zero in the spin-down rate.

size of the quark core to be developed during further spin-down. Our calculations show that a significant enhancement of the breaking index does only occur for pulsars with periods $P < 1.5$ ms (corresponding to $\Omega > 4$ kHz) which, however, have not been observed in nature.

B. Mass accretion

A higher spin-down rate might be possible for rotating neutron stars with mass accretion, where at high rotation frequency the angular momentum transfer from accreting matter and the influence of magnetic fields can be neglected [18] such that the evolution of the angular velocity is determined by the dependence of the moment of inertia on the total mass, i.e. baryon number,

$$\frac{\dot{\Omega}}{\Omega} = -\left(\frac{N_B}{I}\frac{\mathrm{d}I}{\mathrm{d}N_B}\right)\Big|_{J=\mathrm{const}}\frac{\dot{N}_B}{N_B}\,, \tag{4.3}$$

where $J = I\,\Omega =$ const has been assumed. In Fig. 7 we consider the change of the pulsar timing due to mass accretion with a constant accretion rate $\dot{N}_B/N_B$ for fixed total angular momentum as a function of the total baryon number. Here the change from spin-down to spin-up behaviour during the pulsar evolution signals the deconfinement transition. When the pulsar has developed a quark matter core then the change of the moment of inertia due to further mass accretion is negligible and does no longer influence on the pulsar timing. However, in this quark matter core regime the transfer of angular momentum from the accreting matter might dominate and can lead to a continuation of the spin-up. It is interesting to investigate in future research whether e.g. low-mass X-ray binaries with mass accretion for which recently quasi-periodic brightness oscillations (QPO's) with frequencies up to ~ 1200 Hz have been observed [3] might be discussed as possible candidates for rapidly rotating neutron stars for which consequences of the transition to the quark core regime due to mass accretion might be observed.

V. CONCLUSIONS

On the example of the deconfinement transition from hadronic to quark matter we have demonstrated that the rotational characteristics of neutron stars (braking index, spin-down rate) are sensitive to changes of their inner structure and can thus be investigated in order to detect structural phase transitions.

The theoretical basis for the present work was a perturbation method for the solution of the Einstein equations for axial symmetry which allows to calculate the contribution of different rotational effects to the change of the moment of inertia. This quantity can be used as a tool for the investigation of the changes in the rotation timing for different scenarios of the neutron star evolution.

The deviation of the braking index from the value $n = 3$ (magnetic dipole radiation) as a function of the angular velocity has been suggested as a possible signal for the deconfinemnt transition and the occurence of a quark matter core in pulsars. We have

reinvestigated this signal within our approach and could show that the magnitude of this deviation is correlated with the size of the quark core.

For neutron stars with mass accretion we have suggested that under the assumption of total angular momentum conservation a flip from spin-down to spin-up behaviour signals the appearence of a quark matter core. A more detailed investigation is necessary in order identify possible candidates of rotating compact objects with mass accretion (see e.g. [3]) for which the suggested deconfinement signal could be relevant.

ACKNOWLEDGMENT

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APPENDIX A: FUNCTIONS FOR THE CALCULATION OF ΔI

In the integral representations for the contributions to the correction of the moment of inertia we have introduced the set of functions $\Phi_l(r), f_l(r), U_l(r)$ for $l = 0, 2$. They can all be expressed by the solutions of the zeroth and first order equations and by the unknown functions $H_0(r)$, $L_2(r)$ and $S_2(r)$, where

$$\Phi_0(r) = -H_0(r) + K(r) + \Phi(0), \tag{A1}$$

$$\Phi_2(r) = -H_2(r) - K(r), \tag{A2}$$

$$H_2(r) = B_2\,L_2(r) + S_2(r) \tag{A3}$$

$$\begin{aligned} f_0(r) = {} & \left[-1 + 8\pi G(p^{(0)} + \varepsilon^{(0)})r^2\right] H_0(r) + \left[1 - 16\pi G(p^{(0)} + \varepsilon^{(0)})r^2\right] K(r) \\ & - \frac{2}{3}r^4\left(\frac{dq_0(r)}{dr}\right)^2 e^{-\lambda^{(0)}-\nu^{(0)}} - 16\pi G r^2 p_0(r), \end{aligned} \tag{A4}$$

$$\begin{aligned} f_2(r) = {} & -\left[1 + 4\pi G(p^{(0)} + \varepsilon^{(0)})r^2\right] H_2(r) - \left[1 + 8\pi G(p^{(0)} + \varepsilon^{(0)})r^2\right] K(r) \\ & + \frac{1}{3}r^4\left(\frac{dq_0(r)}{dr}\right)^2 e^{-\lambda^{(0)}-\nu^{(0)}} + 8\pi G r^2 p_2(r), \end{aligned} \tag{A5}$$

and

$$K(r) = \frac{2}{3}r^2(q_0(r) + \bar{\Omega})^2 e^{-\nu^{(0)}}\,. \tag{A6}$$

The integration constants $\Phi(0)$, B_2 could be defined by the continuity conditions with the external solutions for the functions $\Phi_0(r)$ and $\Phi_2(r)$, see Refs. [4,5].

The functions $U_0(r)$ and $U_2(r)$ one can define by integrating the following equations

$$\begin{aligned} \frac{dU_0}{dr} &= \frac{1}{r}(H_0(r) - f_0(r) + K(r)) - \frac{dK(r)}{dr} + \frac{dH_0(r)}{dr} + \frac{1}{2}\frac{d\nu^{(0)}}{dr}(H_0(r) - f_0(r) - K(r)), \\ \frac{dU_2}{dr} &= -\frac{1}{r}(H_2(r) + f_2(r) + K(r)) + \frac{dK(r)}{dr} + \frac{dH_2(r)}{dr} + \frac{1}{2}\frac{d\nu^{(0)}}{dr}(H_2(r) - f_2(r) + K(r))\,. \end{aligned} \tag{A7}$$

For the unknown functions $H_0(r)$, $L_2(r)$ and $S_2(r)$ there are second order differential equations which can be found in Ref. [5]), and which we do not repeat here.

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