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A.Pashnev*

MASSLESS AND SPINNING PARTICLES AS DYNAMICS IN ONE DIMENSIONAL (SUPER)DIFFEOMORPHISM GROUPS

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*E-mail: pashnev@thsun1.jinr.dubna.su

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Пашнев А. E2-99-42
Безмассовые частицы и частицы со спином как динамика
в одномерной группе (супер)диффеоморфизмов

Показано, что динамика $D+2$ элементов группы (супер)диффеоморфизмов в одной ($1+1$ для супер) размерности описывает D-мерные безмассовые релятивистские частицы со спином. Координаты этих элементов ($D+2$ айнбайна, $D+2$ связности и одна дополнительная общая координата высшей размерности) играют роль координат, импульсов и лагранжевого множителя, необходимого для явного конформно- и репараметризационно-инвариантного описания D-мерной частицы со спином в терминах $D+2$-мерного пространства-времени.

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Pashnev A. E2-99-42
Massless and Spinning Particles as Dynamics
in One Dimensional (Super)Diffeomorphism Groups

It is shown that dynamics of $D+2$ elements of the (super)diffeomorphism group in one ($1+1$ for super) dimension describes the D-dimensional (spinning) massless relativistic particles. The coordinates of this elements ($D+2$ einbeins, $D+2$ connections and 1 additional common coordinate of higher dimensionality) play the role of coordinates, momenta and Lagrange multiplier, needed for the manifestly conformal and reparametrization invariant description of the D-dimensional (spinning) particle in terms of the $D+2$-dimensional spacetime.

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1 Introduction

As is well known there exist several equivalent formulations of the massless relativistic particles. The second order and first order formalisms are examples of them. The essential ingredient of both approaches is einbein - the field describing one dimensional gravity. One more example is the conformally invariant description [1], which starts from $D+2$ dimensional spacetime. The existence of alternative approaches always sheds some new light on the nature of the physical system. In particular, the conformally invariant description from the very beginning considers the particle coordinates and einbein on the equal footings. For the extended spinning particle [2]-[7] the analogous description [8] shows that the gravitinos of the corresponding one dimensional supergravity are on the same footings as well.

In the present work we consider the natural description of the massless relativistic particle and $N=1$ spinning particle in terms of nonlinear realization of the infinite dimensional diffeomorphism group of the one dimensional space ((1,1) superspace).

We construct the first order conformally invariant formulation and show that the spacetime coordinates and one dimensional supergravity fields are realized as dilatons of one dimensional diffeomorphism group. We consider simultaneously the dynamics of $D+2$ different points in the group space, hence they contain the same number of dilatons. The corresponding $(D+2)$ components of momentum are connected with the Cristoffel symbols. One more parameter of the group having higher dimension is the same for all $(D+2)$ points. It plays the role of Lagrange multiplier and effectively reduces the number of spacetime coordinates from $D+2$ to D ones.

In the second section of the paper we describe the conformally invariant approach to relativistic particles and spinning particles. The third section is devoted to the description of spinless particle. In the fourth and fifth sections we construct the reparametrization invariant in the $(1,1)$ superspace worldvolume action for $N=1$ spinning particle. Some further possibilities of applying the developed formalism are discussed in Conclusions.

2 Conformally invariant description

In this section for convenience of reader we remind the conformally invariant description of the relativistic particle [1], $N = 1$ [1],[8] and extended [8] spinning particle.

The action for bosonic massless relativistic particle in D - dimensional spacetime can be written in terms of $D+2$ coordinates $x_{\mathcal{A}}$, $\mathcal{A}=0,1,\ldots D+1$, of the spacetime with the signature

$$\Sigma_{\mathcal{A}} = (-\underbrace{++\ldots++}_{D}-): \tag{2.1}$$

$$S = \int d\tau (\frac{1}{2}\dot{x}^2 - \frac{1}{2}\lambda x^2). \tag{2.2}$$

Besides of the $SO(D,2)$ invariance, it is gauge-invariant under the transformations

$$\delta x = \epsilon \dot{x} - \frac{1}{2}\dot{\epsilon} x, \tag{2.3}$$

$$\delta \lambda = \epsilon \dot{\lambda} + 2\dot{\epsilon}\lambda + \frac{1}{2}\dddot{\epsilon}. \tag{2.4}$$

The relation of the action (2.2) with the usual D - dimensional action is established by solving the equation of motion for the Lagrange multiplier λ

$$x^A x_A \equiv x^a x_a + 2x_+ x_- = 0; \quad a = 0, \dots, D-1; \quad x_\pm = \frac{1}{\sqrt{2}}(x_D \pm x_{D+1}). \tag{2.5}$$

In terms of new variables

$$\tilde{x} = \frac{x}{x_+}, \quad e = \frac{1}{x_+^2}, \quad (x_- = -\frac{x^a x_a}{2x_+}), \tag{2.6}$$

the Lagrangian becomes

$$L = \frac{1}{2}\frac{\dot{\tilde{x}}^2}{e}. \tag{2.7}$$

Its reparametrization invariance

$$\delta \tilde{x} = \epsilon \dot{\tilde{x}}, \quad \delta e = \dot{\epsilon} e + \epsilon \dot{e} \tag{2.8}$$

is the consequence of (2.3)-(2.4).

The modification of the action (2.2) to the case of extended spinning particle [6] is [8]

$$L = \left(\frac{1}{2}\dot{x}^2 + \frac{1}{2}i\dot{\gamma}_i \cdot \gamma_i\right) - \left(\frac{1}{2}\lambda x^2 + i\lambda_i \gamma_i \cdot x + \frac{1}{2}i\lambda_{ij}\gamma_i \cdot \gamma_j\right) \tag{2.9}$$

(for $N = 1$ spinning particle the action was constructed in [1]). Here γ_i, $i = 1, \dots N$, are Grassmann variables which become γ - matrices upon quantization. After the solution of the equations of motion for the Lagrange multipliers λ, λ_i and some redefinitions like (2.6) one can derive the usual D - dimensional action for N - extended spinning particle in the form of [6].

So, the Lagrange multipliers λ in pure bosonic case and λ, λ_i in the case of extended spinning particle play the crucial role in the conversion of the $D+2$ - dimensional actions into D - dimensional ones. Nevertheless, their geometrical meaning as well as the nature of initial $D+2$ coordinates x_A is unclear. In the next sections we will show that all this functions of τ have an interpretation in terms of parameters of diffeomorphism groups.

3 Geometrical description of the massless particle

Consider the auxiliary 1 - dimensional bosonic space with the coordinate s. The generators of the corresponding diffeomorphism group

$$L_m = i s^{m+1}\frac{\partial}{\partial s}, \tag{3.1}$$

form the Virasoro algebra without central charge

$$[L_n, L_m] = -i(n-m)L_{n+m}. \tag{3.2}$$

In what follows we will consider the subalgebra of the algebra (3.2) which is formed by the regular at the origin generators L_m, $m \geq -1$.

The most natural is the following parametrization of the group element

$$G = e^{i\tau L_{-1}} \cdot e^{iU^1 L_1} \cdot e^{iU^2 L_2} \cdot e^{iU^3 L_3} \dots e^{iUL_0}, \tag{3.3}$$

in which all multipliers with the exception of e^{iUL_0} are ordered with the help of dimensionality of the correspondent generators: $[L_m] = m$. Such structure of the group element simplifies the evaluation of the variations δU^m under the infinitesimal left action

$$G' = (1 + i\epsilon)G, \tag{3.4}$$

where $\epsilon = \sum_{m=0}^{\infty} \epsilon^m L_{m-1}$ belongs to the algebra of the diffeomorphism group. The transformation laws of the coordinates in (3.3) are [9]

$$\delta\tau = \varepsilon(\tau) \equiv \epsilon^0 + \epsilon^1\tau + \epsilon^2\tau^2 + \dots, \tag{3.5}$$

$$\delta U = \dot{\varepsilon}(\tau), \tag{3.6}$$

$$\delta U^1 = -\dot{\varepsilon}(\tau)U^1 + \frac{1}{2}\ddot{\varepsilon}(\tau), \tag{3.7}$$

$$\delta U^2 = -2\dot{\varepsilon}(\tau)U^2 + \frac{1}{6}\dddot{\varepsilon}(\tau). \tag{3.8}$$

In general U^n transforms through τ and U^m, $m < n$. At this stage it is natural to consider all parameters as the fields in one dimensional space parametrized by coordinate τ. It means the following active form of the transformations of the parameters $U(\tau), U^m(\tau)$

$$\delta U(\tau) = -\varepsilon(\tau)\dot{U}(\tau) + \dot{\varepsilon}(\tau), \tag{3.9}$$

$$\delta U^1(\tau) = -\varepsilon(\tau)\dot{U}^1(\tau) - \dot{\varepsilon}(\tau)U^1(\tau) + \frac{1}{2}\ddot{\varepsilon}(\tau), \tag{3.10}$$

$$\delta U^2(\tau) = -\varepsilon(\tau)\dot{U}^2(\tau) - 2\dot{\varepsilon}(\tau)U^2(\tau) + \frac{1}{6}\dddot{\varepsilon}(\tau). \tag{3.11}$$

One can easily verify that the functions $x = e^{U(\tau)/2}$ and $\lambda = -3U^2(\tau)$ have exactly the transformation laws (2.3)-(2.4) with $\varepsilon(\tau) = -\epsilon$. Simultaneously $U^1(\tau)$ transforms as one dimensional Cristoffel symbol.

The independence of δU^2 from U and U^1 means that one can consider more than one group elements

$$G_{\mathcal{A}} = e^{i\tau L_{-1}} \cdot e^{iU^1_{\mathcal{A}} L_1} \cdot e^{iU^2 L_2} \cdot e^{iU^3_{\mathcal{A}} L_3} \dots e^{iU_{\mathcal{A}} L_0}, \quad \mathcal{A} = 0, 1 \dots, D+1, \tag{3.12}$$

which have identical values of parameters τ and $U^2(\tau)$ and differ in the values of all other parameters $U^m_{\mathcal{A}}$. This property is valid when all of them are transformed with the same infinitesimal transformation parameter $\varepsilon(\tau)$. Note that higher parameters U^m, $m \geq 3$ can not be identical because their transformation laws include $U^1_{\mathcal{A}}$. On the other side for each integer N exists such parametrization of the group elements

$$\begin{aligned} G^N_{\mathcal{A}} &= e^{i\tau L_{-1}} \prod_{m=N+1}^{\infty} e^{iU^m L_m} \cdot \\ & \quad e^{iU^1_{\mathcal{A}} L_1} \cdot e^{iU^2_{\mathcal{A}} L_2} \cdot e^{iU^3_{\mathcal{A}} L_3} \dots e^{iU^N_{\mathcal{A}} L_N} \cdot e^{iU_{\mathcal{A}} L_0}, \end{aligned} \tag{3.13}$$

in which all parameters starting with U^{N+1} are independent from the value of the index $\mathcal{A}$. One can easily show that this property is invariant under the transformations

$$G'_{\mathcal{A}} = (1 + i\epsilon) G_{\mathcal{A}}. \tag{3.14}$$

Consider the Cartan's differential form for each value of the index $\mathcal{A}$

$$\Omega_{\mathcal{A}} = G^{-1}_{\mathcal{A}} dG_{\mathcal{A}} = i\Omega^{-1}_{\mathcal{A}} L_{-1} + i\Omega^0_{\mathcal{A}} L_0 + i\Omega^1_{\mathcal{A}} L_1 + \dots. \tag{3.15}$$

All their components $(\Omega^{-1}_{\mathcal{A}}, \Omega^0_{\mathcal{A}}, \Omega^1_{\mathcal{A}}, \dots)$ are invariant with respect to the left transformation (3.14). The explicit expressions for the components of the Ω -form are:

$$\Omega^{-1}_{\mathcal{A}} = e^{-U_{\mathcal{A}}} d\tau, \tag{3.16}$$

$$\Omega^0_{\mathcal{A}} = DU_{\mathcal{A}} - 2d\tau U^1_{\mathcal{A}}, \tag{3.17}$$

$$\Omega^1_{\mathcal{A}} = (dU^1_{\mathcal{A}} + d\tau (U^1_{\mathcal{A}})^2 - 3d\tau U^2) e^{U_{\mathcal{A}}}, \dots. \tag{3.18}$$

The first of this forms is differential one-form einbein. The covariant derivative calculated with its help is

$$D_\tau = e^U_{\mathcal{A}} \frac{d}{d\tau} \tag{3.19}$$

The most interesting is the form $\Omega^1_{\mathcal{A}}$. The following expression for the action

$$\begin{aligned} S &= -\frac{1}{2} \int \sum_{\mathcal{A}} \Sigma_{\mathcal{A}} \Omega^1_{\mathcal{A}} = \\ &= -\frac{1}{2} \int d\tau \sum_{\mathcal{A}} \Sigma_{\mathcal{A}} e^{U_{\mathcal{A}}} (\dot{U}^1_{\mathcal{A}} + (U^1_{\mathcal{A}})^2 - 3U^2), \end{aligned} \tag{3.20}$$

where $\Sigma_{\mathcal{A}}$ is the signature (2.1), is invariant under the transformation (3.14) and corresponds to the first order formalism for the action (2.2). Indeed, after the integration by parts in the first term and change of variables

$$x_{\mathcal{A}} = e^{U_{\mathcal{A}}(\tau)/2}, \quad p_{\mathcal{A}} = e^{U_{\mathcal{A}}(\tau)/2} U^1_{\mathcal{A}}, \quad \lambda = -3U^2(\tau) \tag{3.21}$$

it becomes

$$S_f = \int d\tau (\dot{x}p - \frac{1}{2}p^2 - \frac{1}{2}\lambda x^2). \tag{3.22}$$

This action is invariant under the gauge transformations

$$\delta x = \epsilon \dot{x} - \frac{1}{2}\dot{\epsilon} x, \tag{3.23}$$

$$\delta \lambda = \epsilon \dot{\lambda} + 2\dot{\epsilon}\lambda + \frac{1}{2}\dddot{\epsilon}. \tag{3.24}$$

$$\delta p = \epsilon \dot{p} + \frac{1}{2}\dot{\epsilon} p - \frac{1}{2}\ddot{\epsilon} x. \tag{3.25}$$

After the elimination of $p_{\mathcal{A}}$ with the help of its equation of motion $p_{\mathcal{A}} = \dot{x}_{\mathcal{A}}$ the action (3.22) coincides with the action (2.2).

4 $N = 1$ spinning particle in a superconformal gauge

To generalize the approach on the spinning particles we firstly consider more simple example of the $N = 1$ Superconformal Algebra (SCA)

$$[L_m, L_n] = -i(m-n) L_{m+n} \tag{4.1}$$

$$[L_m, G_s] = -i(\frac{m}{2} - s) G_{m+s} \tag{4.2}$$

$$\{G_r, G_s\} = 2L_{r+s}. \tag{4.3}$$

The indices $m, n \geq -1$ are integer and $r, s \geq -1/2$ -halfinteger. Following the considerations of preceding chapter and [9] we write the group element as

$$\begin{aligned} G_{\mathcal{A}} &= e^{i\tau L_{-1}} \cdot e^{i\theta G_{-1/2}} \cdot e^{i\Theta^{3/2} G_{3/2}} \cdot e^{iU^2 L_2} \dots \\ & \quad e^{i\Theta^{1/2}_{\mathcal{A}} G_{1/2}} \cdot e^{iU^1_{\mathcal{A}} L_1} \cdot e^{iU_{\mathcal{A}} L_0}, \quad \mathcal{A} = 0, 1 \dots, D+1. \end{aligned} \tag{4.4}$$

All parameters (Grassmann Θ-s and commuting U-s) are considered as superfunctions of τ and θ which parametrize the $(1,1)$ superspace. The variation of superspace coordinates under the left action of infinitesimal superconformal transformation can be written in terms of one bosonic superfunction

$$\delta \tau = \Lambda - \frac{1}{2}\theta D_\theta \Lambda, \tag{4.5}$$

$$\delta \theta = -\frac{i}{2} D_\theta \Lambda, \tag{4.6}$$

where

$$D_\theta = \frac{\partial}{\partial\theta} + i\theta\frac{\partial}{\partial\tau}, \tag{4.7}$$

is the flat supercovariant derivative.

To calculate the invariant differential Ω- forms one should take into account that Grassmann parity of differential of any variable is opposite to its own Grassmann parity, i.e. $d\tau$ is odd and $d\theta$ is even [10]. The general expression for Ω -form is

$$\Omega_{\mathcal{A}} = G^{-1}_{\mathcal{A}}dG_{\mathcal{A}} = i\Omega^{-1}_{\mathcal{A}}L_{-1} + i\Omega^{-1/2}_{\mathcal{A}}G_{-1/2} + i\Omega^0_{\mathcal{A}}L_0 + i\Omega^{1/2}_{\mathcal{A}}G_{1/2} + i\Omega^1_{\mathcal{A}}L_1 + \ldots, \tag{4.8}$$

where two first components

$$\Omega^\tau_{\mathcal{A}} \equiv \Omega^{-1}_{\mathcal{A}} = (d\tau - id\theta\theta)e^{-U_{\mathcal{A}}} = dx^M E_{M\mathcal{A}}^{\ \tau}, \tag{4.9}$$

$$\Omega^\theta_{\mathcal{A}} \equiv \Omega^{-1/2}_{\mathcal{A}} = \{d\theta - (d\tau - id\theta\theta)\Theta_{1/2}\}e^{-U_{\mathcal{A}}/2} = dx^M E_{M\mathcal{A}}^{\ \theta} \tag{4.10}$$

define supervielbein $(x^1 \equiv \tau, x^2 \equiv \theta)$:

$$E_{M\mathcal{A}}^{\ A} = \begin{vmatrix} e^{-U_{\mathcal{A}}} & -\Theta^{1/2}_{\mathcal{A}} \cdot e^{-U_{\mathcal{A}}/2} \\ -ie^{-U_{\mathcal{A}}} \cdot \theta & e^{-U_{\mathcal{A}}/2}(1 - i\Theta^{1/2}_{\mathcal{A}} \cdot \theta) \end{vmatrix}$$

The covariant derivatives $\mathcal{D}_{A\mathcal{A}} \equiv E_{A\mathcal{A}}^{\ M}\partial_M, (A = \tau,\ \theta)$, are defined with the help of inverse supervielbein

$$E_{A\mathcal{A}}^{\ M} = \begin{vmatrix} e^{U_{\mathcal{A}}}(1 + i\Theta^{1/2}_{\mathcal{A}} \cdot \theta) & \Theta^{1/2}_{\mathcal{A}} \cdot e^{U_{\mathcal{A}}} \\ ie^{U_{\mathcal{A}}/2} \cdot \theta & e^{U_{\mathcal{A}}/2} \end{vmatrix}$$

As a result

$$\mathcal{D}_\theta = e^{U_{\mathcal{A}}/2}D_\theta,\ \ \mathcal{D}_\tau = e^{U_{\mathcal{A}}}(D_\tau + \Theta^{1/2}_{\mathcal{A}}D_\theta),\ \ D_\tau \equiv \frac{\partial}{\partial\tau}. \tag{4.11}$$

The invariant integration measure is

$$dV_{\mathcal{A}} = d\tau\underline{d\theta}Ber(E_{M\mathcal{A}}^{\ A}), \tag{4.12}$$

where $\underline{d\theta}$ is the Berezin differential and

$$Ber(E_{M\mathcal{A}}^{\ A}) = e^{-U_{\mathcal{A}}/2}. \tag{4.13}$$

Note, that all considered quantities, supervielbein, covariant derivatives and integration measure, depend on the index $\mathcal{A}$.

To construct the action for $N = 1$ spinning particle consider the component $\Omega^1_{\mathcal{A}}$ and express it through the full system of invariant differential forms $\Omega^\tau_{\mathcal{A}}$ and $\Omega^\theta_{\mathcal{A}}$

$$\Omega^1_{\mathcal{A}} = \Omega^\tau_{\mathcal{A}}Y_{\mathcal{A}} + \Omega^\theta_{\mathcal{A}}\Gamma_{\mathcal{A}}. \tag{4.14}$$

The coefficients are also invariant. In particular, $\Gamma_{\mathcal{A}}$ is odd and can be used for the construction of invariant action

$$S = \frac{i}{2}\int\sum_{\mathcal{A}} dV_{\mathcal{A}}\Sigma_{\mathcal{A}}\Gamma_{\mathcal{A}} = \tag{4.15}$$

$$\frac{i}{2}\int d\tau\underline{d\theta}\sum_{\mathcal{A}}\Sigma_{\mathcal{A}}e^{U_{\mathcal{A}}}(D_\theta U^1_{\mathcal{A}} - iD_\theta\Theta^{1/2}_{\mathcal{A}}\Theta^{1/2}_{\mathcal{A}} + 2i\Theta^{1/2}_{\mathcal{A}}U^1_{\mathcal{A}} - 2i\Theta^{3/2}_{\mathcal{A}}). \tag{4.16}$$

After the introduction of new variables

$$X = e^{U/2},\ \ \Pi = e^{U/2}U^1,\ \ \Xi = e^{U/2}\Theta^{1/2},$$

integration by parts in first term and omitting the index $\mathcal{A}$ the action becomes

$$S = \frac{i}{2}\int d\tau\underline{d\theta}(-2\Pi D_\theta X - iD_\theta\Xi \cdot \Xi + 2i\Theta^{3/2} \cdot X^2). \tag{4.17}$$

The equation of motion for Π gives $\Xi = -iD_\theta X$. Making use from this equation and identity $D^2_\theta = i\partial_\tau$ gives the final result for the action in terms of even superfields

$$X_{\mathcal{A}} = x_{\mathcal{A}} + i\theta\gamma_{\mathcal{A}} \tag{4.18}$$

and odd one

$$\Theta^{3/2}_{\mathcal{A}} = -\frac{1}{2}(\lambda_{odd} - \theta\lambda), \tag{4.19}$$

$$S = -\frac{i}{2}\int d\tau\underline{d\theta}(\dot{X}D_\theta X + 2\Theta^{3/2}X^2) \tag{4.20}$$

After the Berezin integration over θ it coincides with the manifestly conformal component action for the $N = 1$ spinning particle (2.9).

5 Reparametrization invariant $N = 1$ spinning particle

The diffeomorphism group of the superspace with one even and one odd coordinates s and η is generated by two families of even operators L_n, $n \geq -1$, and M_m, $m \geq 0$

$$L_n = is^{n+1}\frac{\partial}{\partial s},\ \ M_n = is^n\eta\frac{\partial}{\partial\eta} \tag{5.1}$$

and two families of odd operators P_r, ,Q_r, $r \geq -1/2$

$$P_{n-1/2} = is^n\frac{\partial}{\partial\eta},\ \ Q_{n-1/2} = is^n\eta\frac{\partial}{\partial s}. \tag{5.2}$$

. Their algebra is [9]

$$[L_m, L_n] = -i(m-n)L_{m+n}, \tag{5.3}$$
$$[L_m, M_n] = inM_{m+n}, \tag{5.4}$$
$$[L_m, P_s] = i(s+\frac{1}{2})P_{m+s}, \tag{5.5}$$
$$[L_m, Q_s] = -i(m-s+\frac{1}{2})Q_{m+s}, \tag{5.6}$$
$$[M_m, P_s] = -iP_{m+s}, \tag{5.7}$$
$$[M_m, Q_s] = iQ_{m+s}, \tag{5.8}$$
$$\{P_r, Q_s\} = iL_{r+s} + i(r+\frac{1}{2})M_{m+s}. \tag{5.9}$$

It is convenient to write the group element as

$$\begin{aligned} G_{\mathcal{A}} \;=\;& e^{i\tau L_{-1}} \cdot e^{i\theta P_{-1/2}} \cdot e^{i\psi Q_{-1/2}} \cdot e^{i\Theta^{3/2}P_{3/2}} \cdot e^{i\Psi^{3/2}Q_{3/2}} \ldots \\ & e^{i\Theta^{1/2}_{\mathcal{A}}P_{1/2}} \cdot e^{i\Psi^{1/2}_{\mathcal{A}}Q_{1/2}} \cdot e^{iU^1_{\mathcal{A}}L_1} \cdot e^{iV^1_{\mathcal{A}}M_1} \cdot e^{iU_{\mathcal{A}}L_0} \cdot e^{iV_{\mathcal{A}}M_0}, \quad \mathcal{A} = 0, 1\ldots, D+1. \end{aligned} \tag{5.10}$$

All parameters (odd Θ, Ψ and even U, V) again are considered as superfunctions of τ and θ which parametrize the $(1,1)$ superspace. The left infinitesimal transformation leads to the reparametrization of superspace coordinates

$$\delta\tau = a(\tau,\theta), \quad \delta\theta = \xi(\tau,\theta) \tag{5.11}$$

and to the following variation of ψ [9]

$$\delta\psi = -\partial_\theta a + \dot{a}\psi - \partial_\theta\xi\psi. \tag{5.12}$$

One can show that such gauge freedom is enough to choose gauge

$$\psi = -i\theta. \tag{5.13}$$

Before going to such gauge one can calculate all invariant quantities - supervielbein, covariant derivatives and integration measure:

$$E_M{}^A_{\mathcal{A}} = \begin{vmatrix} e^{-U_{\mathcal{A}}} & -\Theta^{1/2}_{\mathcal{A}} \cdot e^{-V_{\mathcal{A}}} \\ e^{-U_{\mathcal{A}}} \cdot \psi & e^{-V_{\mathcal{A}}}(1+\Theta^{1/2}_{\mathcal{A}} \cdot \psi) \end{vmatrix}$$

$$\mathcal{D}_\theta \;=\; e^{V_{\mathcal{A}}}D_\theta, \quad D_\theta = \partial_\theta - \psi\partial_\tau, \tag{5.14}$$
$$\mathcal{D}_\tau \;=\; e^{U_{\mathcal{A}}}(\partial_\tau + \Theta^{1/2}_{\mathcal{A}}D_\theta), \tag{5.15}$$
$$Ber(E_M{}^A_{\mathcal{A}}) = e^{-U_{\mathcal{A}}+V_{\mathcal{A}}}. \tag{5.16}$$

Denoting the expansion of Ω-forms in terms of the basic system of one-forms $\Omega^A_{\mathcal{A}}$

$$\Omega(L)^1_{\mathcal{A}} = \Omega^\tau_{\mathcal{A}} \cdot Y(L)^1_{\mathcal{A}} + \Omega^\theta_{\mathcal{A}} \cdot \Gamma(L)^1_{\mathcal{A}}, \tag{5.17}$$

where additional letter L means that the corresponding component $\Omega(L)^1_{\mathcal{A}}$ is the coefficient at the generator L_1, one can write the invariant action in the form

$$S \;=\; \frac{i}{2}\int\sum_{\mathcal{A}} d\tau\underline{d\theta} \cdot e^{-U_{\mathcal{A}}+V_{\mathcal{A}}}\Sigma_{\mathcal{A}}\{\Gamma(L)^1_{\mathcal{A}} + m(\Gamma(L)^0_{\mathcal{A}} - 2\Gamma(M)^0_{\mathcal{A}})\} = \tag{5.18}$$
$$\frac{i}{2}\int d\tau\underline{d\theta}\sum_{\mathcal{A}}\Sigma_{\mathcal{A}} \cdot e^{2V_{\mathcal{A}}}(D_\theta U^1_{\mathcal{A}} + D_\theta\Theta^{1/2}_{\mathcal{A}} \cdot \Psi^{1/2}_{\mathcal{A}} + +\Psi^{3/2}_{\mathcal{A}} + D_\theta\psi \cdot \Theta^{3/2}_{\mathcal{A}} \tag{5.19}$$
$$\Psi^{1/2}_{\mathcal{A}} \cdot U^1_{\mathcal{A}} - D_\theta\psi \cdot \Theta^{1/2}_{\mathcal{A}}U^1_{\mathcal{A}}) +$$
$$m\frac{i}{2}\int d\tau\underline{d\theta}\sum_{\mathcal{A}}\Sigma_{\mathcal{A}}e^{-U_{\mathcal{A}}+2V_{\mathcal{A}}}\{D_\theta(U_{\mathcal{A}} - 2V_{\mathcal{A}}) + \Psi^{1/2}_{\mathcal{A}} - D_\theta\psi\Theta^{1/2}_{\mathcal{A}}\}. \tag{5.20}$$

In this action without loss of any information one can choose the gauge (5.13). Indeed, when we calculate the equation of motion for ψ and choose this gauge, they are consequence of the equations of motion, which follow from the gauge fixed action.

The parameter m in the action is arbitrary parameter of dimension of mass. On mass shell it effectively disappears, because the last term (5.20) in the action is auxiliary. Its first term in a gauge (5.13) is a total derivative and the equation of motion for $U_{\mathcal{A}} - 2V_{\mathcal{A}}$ leads to the connection between $\Psi^{1/2}_{\mathcal{A}}$ and $\Theta^{1/2}_{\mathcal{A}}$:

$$\Psi^{1/2}_{\mathcal{A}} = -i\Theta^{1/2}_{\mathcal{A}}. \tag{5.21}$$

This last equation reduces the action (5.18) to the action for $N=1$ superconformal group (4.15).

So, the action (5.18), which is invariant under the transformations of the whole diffeomorphism group of the $(1,1)$ superspace describes the $N=1$ spinning particle.

6 Conclusions

In the framework of nonlinear realizations of infinite - dimensional diffeomorphism groups of one dimensional bosonic space and $(1,1)$ superspace we have constructed the conformally and reparametrization invariant actions for massless particle and $N=1$ spinning particle in arbitrary dimension D. It is achieved by simultaneous consideration of several group elements. The parameters of corresponding group points include simultaneously the coordinates and momenta. The interaction between coordinates is obliged to parameters with higher dimensions, which are the same for all considered points on the group space.

It would be interesting to apply the method developed here and in [9] to other infinite dimensional symmetries, such as diffeomorphism groups of extended superspaces and higher dimensional spaces, W-algebras and so on

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