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# PION DISTRIBUTION AMPLITUDE WITHIN THE INSTANTON MODEL

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# 1 Introduction

At large momentum transfer, the amplitudes of the exclusive hadron processes due to a factorization of large and short distance dynamics [1]-[3] are expressed, in leading logarithmic approximation, as the convolution of hard and soft scattering amplitudes. The first ones are calculable in perturbative QCD; they are dominated by hard one-gluon exchange diagrams. The second ones describe the soft transition of initial and final hadron states into quarks; they are determined in terms of the hadron distribution amplitudes (DA) [1]. These phenomenological functions have the meaning of the amplitude of hadron decay (in the infinite momentum system, $p_h \to \infty$) into a quark - antiquark pair (in the meson case), with momentum fractions $xp_h$ and $\bar{x}p_h$, ($\bar{x} = 1-x$) and virtuality $\mu^2$. Since the DAs depend on the dynamics at large distances, they can be calculated only by non-perturbative technique.

The first attempt to calculate hadron DAs was performed in [4]. It resulted in a two-humped form for the pion DA. However, some time ago, the applicability of this form of DA to exclusive processes at high momentum transfer was questioned [5]. It was shown that in the collinear approximation, at momentum transfer far from asymptotic region, there dominates the soft one-gluon exchange which corresponds to large values of the strong coupling constant. Moreover, the prediction based on this DA overshoots the large $Q^2$ data on the pion transition form factor published recently by CLEO Collaboration [6] (for discussions, e.g., see ref. [7]).

Later on, in ref. [8], by using refined technique to extract the hadronic DAs based on QCD sum rules with nonlocal condensates [9], it was shown that the pion DA at low energy scale is more close in form to the asymptotic one. It was also found that the form of the hadron DAs is very sensitive to the structure of the non-perturbative vacuum in terms of nonlocal condensates. Recently, in [10] and [11], the nonlocal condensates were modeled within the instanton model.

In this letter, we will use the quark-pion dynamics developed in the framework of the instanton vacuum model (see for recent review, e.g., [12]) in order to calculate the leading-twist pion DA at a low normalization point of the order of the inverse effective instanton size $\rho_c$. The instanton model of the QCD vacuum gives the dynamical mechanism of chiral symmetry breaking, provides the solution of the $U_A(1)$ problem, and leads to understanding the physics of light pseudoscalar mesons. Moreover, it dynamically generates the momentum-dependent effective quark mass $M_q$ and quark-pion vertex $g_{\pi qq}$ and, as a consequence, provides inherently a natural ultraviolet cutoff parameter in the quark loop integrals through the effective instanton size $\rho_c$.

The instanton model parameters are naturally related to basic quantities of low energy physics. The inverse effective instanton size, $\rho_c^{-1}$, directly measures the average virtuality of quarks that flow through the vacuum with momentum $k_q$, where $\langle k_q^2 \rangle \equiv \lambda_q^2 \approx 2\rho_c^{-2}$ [10] $\approx 0.5\mathrm{GeV}^2$[13]. The quark mass parameter $M_q$ is given by the Goldberger-Treiman relation $M_q = g_{\pi qq} f_\pi$, with the quark-pion coupling being fixed by the compositeness condition. Finally, the effective instanton density, $n_c$, is determined via the gap equation.

Earlier attempts [14] (see also [15]) to calculate the pion DA have been done in the framework of the model developed in ref. [16], which had further improvements. The effective action suggested in [16] is valid only in the chiral limit and was modified consistently in [17]. In [11], it was shown that the kernel of the effective instanton-induced four quark interaction can be expressed in terms of a gauge invariant quantity, nonlocal quark condensate, effectively resuming nonperturbative effects of the instanton field. What is important, in the context of the present work, is that in general within the nonlocal models the form of the conserved currents is different of the usual local currents (see, *e.g.* [19]). These points lead to the conclusion that the approach of [16] is not fully consistent with the low energy theorems. Considering these facts, some of the previous calculations also need to be revised. In particular, the approach of [16] fails to satisfy the axial Ward-Takahashi identities (WTI). As shown in [19] the local part of the axial current is modified by the nonlocal term. Physically, it means that usual local currents are defined via (free) current quarks and the modification by nonlocal terms occurs due to the transition from current to constituent quark description in effective models. These



additional terms are not suppressed by small instanton density parameter and lead to the correction of the order of 30% to the value of the pion decay constant $F_\pi$. Since the pion decay constant is an integral measure of the pion DA, the main motivation of the present work is to estimate the effect of such terms on the leading term of the wave-function.

The paper is organized as follows. In Sect. 2, we introduce the definition for the pion DA. In Sect. 3, we write the effective instanton induced action in terms of quark fields gauged by $P\exp$ phase factors. The gauge fields in the phase-factor (vector, axial-vector, etc) are in general unphysical; however, their introduction is convenient in order to generate conserved currents of the model. The results and main conclusions are presented in the last section. In an appendix we show how the axial WTI is satisfied within the nonlocal four-quark model.

## 2 Pion distribution amplitude at low energy scale

The axial projection of the pion light-cone DA $\varphi_A(x)$ defines the leading asymptotic behaviour of the pion form factor. It parameterizes the structure of the matrix element

$$< 0|J_\mu^A(z,-z)|\pi^+(p) >= ip_\mu F_\pi \int_0^1 dx e^{i(2x-1)p\cdot z}\, \varphi_A(x), \tag{1}$$

of the bilocal operator

$$J_\mu^A(z,-z) = \bar{d}(z)\gamma_\mu\gamma_5 P\exp\left(i\int_{-z}^{z} A_\mu(z)dz^\mu\right)u(-z), \tag{2}$$

where the light-cone limit is considered, $z^\mu = \lambda n^\mu$, $n^\mu$ is the light-like vector, $n^2 = 0$, normalized by $p\cdot n = 1$, $F_\pi = 130\mathrm{MeV}$ is the weak pion decay constant and the leading-twist pion light-cone DA is normalized by

$$\int_0^1 dx\varphi_A(x) = 1. \tag{3}$$

The path-ordered Schwinger phase factor is required for gauge invariance and the integration is performed along the light-like direction $z$. This factor will be neglected in the following, since the possible contribution of a classical field (instanton) produces higher-twist corrections to the DA and that of a quantum field gives the corrections in small instanton density parameter.

The bilocal current (2) is defined in terms of current quarks and the effective low energy model, which we are going to use, is described in terms of constituent quarks $U$ and $D$. Thus, to derive the matrix element (1) we consider the vertex $\left\langle 0\left|J_\mu^A(z,-z)\right|U(k)\overline{D}(k)\right\rangle$ which becomes after the extraction of the pion pole

$$\begin{aligned}&\left\langle 0\left|J_\mu^A(z,-z)\right|U(k)\overline{D}(p-k)\right\rangle = \\ &\left\langle 0\left|J_\mu^A(z,-z)\right|\pi^+(p)\right\rangle \frac{1}{m_\pi^2-p^2}\Gamma_{\pi q}^a(k,p)\,,\end{aligned} \tag{4}$$

where

$$\Gamma_{\pi q}^a(k,p) = \left\langle \pi^+(p)|U(k)\overline{D}(p-k)\right\rangle.$$

. Then, expressing the matrix element

$$\left\langle 0\left|J_\mu^A(z,-z)\right|U(k)\overline{D}(p-k)\right\rangle$$

through a loop integral, taking into account the constituent quark rescattering, selecting the pion pole, the expression for the DA is reduced to[1]

$$\begin{aligned} p^\mu F_\pi\varphi_A(x) &= 2N_c\int\frac{d^4k}{(2\pi)^4 i}\delta(x-k\cdot n) \\ &\quad tr\{\Gamma_{\pi q}^a(k,p)\,S(k)^A\Gamma^{\mu a}(k,p)S(k-p)\}, \end{aligned} \tag{5}$$

where $x$ is the fraction of the pion momentum, $p$, carried by a quark. The $\delta-$ function in (5) accumulates information about all the moments of the DA and is related to them by the Mellin transform.

In the above expression $S(k)$, ${}^A\Gamma^{\mu a}(k,p)$ and $\Gamma_{\pi q}^a(k,p)$ are the dressed quark propagator, quark-axial current and quark-pion vertices, respectively. It is the main subject of the rest of this letter to specify these functions. To this goal we shell use the covariant effective low-energy model with separable nonlocal four-quark interaction.

[1]This expression generalizes one given previously in [14] (and also in [15]). In these works the local axial current vertex, $\gamma_\mu\gamma_5$, was used instead of the dressed one, $\Gamma^\mu(k,q)$. As it will be clear this approximation is inconsistent with the axial WTI.



Moreover, the actual calculations will be done within a model with interquark interaction induced by instanton exchange. The advantages of the instanton model is that the form of the nonlocal interaction is given by quark zero modes and the parameters of the model are directly related to the fundamental low-energy constants. One can verify that the numerical dependence of the results on the pion and current quark masses is negligible and can be ignored within the following considerations: $m_\pi = 0$, $m_{curr} = 0$. However, the interplay of the effective quark mass, $M_q$, and the scale of the nonlocality of the vacuum field, $\lambda_q^2$, has an important effect on the form of the DA.

## 3 Gauged nonlocal four-fermion model and conserved currents

**1.** Let us consider the nonlocal chirally invariant action given by

$$S = S_0 + S_{4q}, \tag{6}$$

with

$$S_0 = \int d^4x d^4y\,\delta(x-y)\,\overline{Q}(x,X)i\hat{\partial}_y Q(X,y), \tag{7}$$

$$\begin{aligned} S_{4q} &= \frac{1}{2}G_I\int d^4X\int\prod_{n=1}^4 d^4x_n\,K_I(x_1,x_2,x_3,x_4) \\ &\left\{\sum_i\left[\overline{Q}_R(X-x_1,X)\Gamma_i Q_L(X,X+x_3)\right]\cdot\right. \\ &\left.\cdot\left[\overline{Q}_R(X-x_2,X)\Gamma_i Q_L(X,X+x_4)\right] + (R\leftrightarrow L)\right\}, \end{aligned} \tag{8}$$

where the matrix combinations, $\Gamma_i\otimes\Gamma_i$, are

$$1\otimes 1-\tau^a\otimes\tau^a, \qquad \frac{1}{2(2N_c-1)}(\sigma_{\mu\nu}\otimes\sigma_{\mu\nu}-\tau^a\sigma_{\mu\nu}\otimes\tau^a\sigma_{\mu\nu}), \tag{9}$$

$\tau^a$ are the Pauli matrices for the flavor space, $N_c = 3$ is the number of colors, and $Q_{R(L)}(x,y) = \frac{1\pm\gamma_5}{2}Q(x,y)$ are the gauged quark fields with definite chirality. The form of the action is motivated by the instanton vacuum model and, in the local limit, it reduces into the t'Hooft vertex. In the following we neglect the terms induced by tensor interaction in Eq. (9) since they do not contribute to the scalar channels. The action (6) effectively describes the interaction between quarks induced by instanton exchange and is a nonlocal generalization of the Nambu-Jona-Lasinio (NJL) model. The nonlocal kernel $K_I(x_1, x_2, x_3, x_4)$ is chosen in a separable approximation

$$K_I(x_1, x_2, x_3, x_4) = f(x_1)f(x_2)f(x_3)f(x_4), \tag{10}$$

where the function $f(x)$ is related to a profile function of the quark zero mode in the instanton field (see below).

In order to make the nonlocal action (6)-(8) gauge invariant with respect to external fields, the quarks are coupled by path-ordered phase factors:

$$Q(x,y) \equiv P \exp\left\{-i\int_x^y dz^\mu \, \Lambda_\mu^a(z)\frac{\tau^a}{2}\right\} q(y),$$
$$\Lambda_\mu^a(z) = V_\mu^a(z) + A_\mu^a(z)\gamma_5.$$

We use a formalism which is based on the path-independent definition of the derivative of the line path integral [21]

$$\frac{\partial}{\partial y^\mu}\int_x^y dz^\nu \, \Lambda_\nu(z) = \Lambda_\mu(y). \tag{11}$$

It means that the terms induced by non-minimal couplings are ignored. This formalism has been used in [18] (see also [22, 23]) for gauging nonlocal interactions. The incorporation of a gauge - invariant interaction with gauge fields is very relevant in order to treat correctly the hadron characteristics probed by external sources such as hadron form factors [19] and parton distribution functions [11].

**2.** The conserved currents are given by the derivatives of the action with respect to the external fields at zero. In the presence of nonlocal interaction the currents consist of local and nonlocal terms. For our purpose, it is enough to regard the vertice which involves one external isovector axial-vector current. It is given by[2]

$$^A\Gamma_{4q}^{\mu a}(k_1, k_2, k_3, k_4, q) = \gamma^\mu\gamma_5\tau^a/2 + G_I f(k_1)f(k_2)f(k_3)f(k_4)$$
$$\sum_{j=I}^{III}\sum_i [(\Gamma_i^\alpha)_{13}\,(\Omega_{i\alpha}\,)_{24}]_j^a \, H_j^\mu(k_1, k_2, k_3, k_4, q)\,, \tag{12}$$

where $k_i$ are the quark in(out)-going momenta and $q$ is the momentum flow through the current. The usual local piece of the vertex is obtained by gauging the kinetic term (7), which is equivalent to the application of a covariant derivative, $i\widehat{D}_y = i\widehat{\partial}_y + \widehat{V}(y) + \widehat{A}(y)\gamma_5$.

The nonlocal four-quark part of the current is generated from the interaction term (8). In order to expand the path-ordered exponentials entering the interaction, we use techniques described in [18] (see also [24]). This method consists first in to obtain the Fourier transform and make the Taylor expansion of the kernel $K_I(x_1, x_2, x_3, x_4)$; after, it is necessary to convert the powers of momenta into derivatives acting on the path-ordered exponentials and quark fields, to make the inverse Fourier transform, and then to resume.

There are two types of nonlocal vertices, generated from (8), which contribute to the isovector axial current: type-I and type-III. The type-I is given by

$$H_I^\mu(k_1, k_2, k_3, k_4, q) = F^\mu(k_1 + q, k_1) + F^\mu(k_3 - q, k_3)\,, \tag{13}$$

with the corresponding matrix combinations,

$$[(\Gamma_i^\alpha)_{13}\,(\Omega_{i\alpha}\,)_{24}]_I^a\,,$$
$$\epsilon^{abc}(\tau^c \otimes i\gamma_5\tau^b), \qquad -\epsilon^{abc}(i\gamma_5\tau^b \otimes \tau^c), \tag{14}$$

and the type-III is given by

$$H_{III}^\mu(k_1, k_2, k_3, k_4, q) = F^\mu(k_2 + q, k_2) + F^\mu(k_3 - q, k_3)$$
$$-F^\mu(k_1 + q, k_1) - F^\mu(k_4 - q, k_4)\,, \tag{15}$$

[2] Here we are following spin-isospin classification of currents given in [19]. The difference of our definitions and of that work is in the definition of the path integral. This difference is displayed in the form of (momentum) space non-local form factors $F_\mu(k', k)$. Still the longitudinal components of the currents are the same in both approaches as it should be.

with the matrix terms

$$(i\gamma_5\tau^a \otimes 1), \qquad -(i\gamma_5 \otimes \tau^a). \tag{16}$$

In the above expressions the nonlocal vertex function, $F^\mu(k \pm q, k)$, is defined as

$$F^\mu(k \pm q, k) = (2k \pm q)^\mu \frac{[f(k \pm q)/f(k) - 1]^2}{(k \pm q)^2 - k^2}$$

and we use the same notation for the function $f$ and its Fourier transform.The energy-momentum conservation law is implicitly given by the factor $(2\pi)^4 \delta(k_1 + k_2 + q - k_3 - k_4)$.

**3.** The vertices given in the previous subsection are bare ones. Now one needs to "dress" the model taking into account the rescattering processes. The first step is to construct the dressed quark propagator by means of the Schwinger–Dyson equation. We treat it in the ladder approximation, which is equivalent to working at leading order in the $1/N_c$ expansion. In the chiral limit, this equation is given by

$$M(p) = i2N_c G_I f^2(p) \int \frac{d^4k}{(2\pi)^4} \frac{\mathrm{tr}\left[\hat{k} + M(k)\right]}{k^2 - M^2(k)} f^2(k), \tag{17}$$

where a momentum-dependent quark mass, $M(p) = M_q \tilde{Q}(p)$, ($\tilde{Q}(0) = 1$), is defined by the dressed quark propagator:

$$S_F^{-1}(p) = \hat{p} - M_q \tilde{Q}(p). \tag{18}$$

The solution of the eq. (17) can be written simply as $M(p) = M_q f^2(p)$. From the other side, the momentum dependence of the nonperturbative part of the quark propagator in the non-perturbative vacuum, $\tilde{Q}(p)$, describes the nonlocal properties of the quark condensate, given by

$$\begin{aligned} \tilde{Q}(p) &= p^2 N_Q \int \frac{d^4x}{(2\pi)^4} \, exp(-ip \cdot x) Q(x^2), \\ Q(x) &= \langle : \bar{q}(0) E_g(0, x) q(x) : \rangle / \langle : \bar{q}(0) q(0) : \rangle, \end{aligned} \tag{19}$$

where the Schwinger factor, $E_g(0, x) = P \exp(i \int_0^x A_\mu(z) dz^\mu)$, in terms of the vacuum gluon field $A_\mu(z)$, guarantees the gauge invariance and $N_Q$ gives the normalization. Hence, through the solution of the gap equation (17), the function $f(p)$ is related to the nonperturbative scalar propagator (19)

$$f(p) = \sqrt{\tilde{Q}(p)}. \tag{20}$$

Now we specify the QCD vacuum model as given by the instanton induced interaction. In this case, the scalar part of the quark propagator is[3]

$$Q_I(x) = \frac{8\rho_c^2}{\pi} \int_0^\infty dr r^2 \int_{-\infty}^\infty dt \frac{\cos\left[\frac{r}{R}\left(\arctan(\frac{t+|x|}{R}) - \arctan(\frac{t}{R})\right)\right]}{[R^2 + t^2]^{3/2}[R^2 + (t + |x|)^2]^{3/2}}. \tag{21}$$

where $R^2 = \rho_c^2 + r^2$, and $\cos[...]$ factor, that comes from the Schwinger factor, effectively sums an infinite set of quark-instanton interaction terms. The normalization coefficient in eq. (19) is $N_{QI} = 2\pi^2\rho_c^{-2}$. To obtain the above equation the explicit expressions for the instanton field and quark zero mode is used [20, 10]. The equation (17), obtained in the chiral limit, determines the parameter $G_I$ as $G_I = M_q^2/(N_f n_c)$, where $n_c$ is the effective instanton density, and coincides with the result [16].

The pion mass[4] and quark-pion vertex are obtained using the Bethe–Salpeter equation for the quark-antiquark scattering amplitude. The pion state is manifested as a pole in the amplitude and, in the ladder approximation with a separable kernel, the quark-pion vertex near the pole is given by

$$\Gamma^a_{\pi q}(k, p) = g_{\pi q} f(p - k) f(k) i\gamma_5 T^a, \tag{22}$$

where the quark-pion coupling is defined by the compositeness condition

$$\frac{1}{g_{\pi q}^2} = \left. \frac{dJ_{PP}(p)}{dp^2} \right|_{p^2 = m_\pi^2}, \tag{23}$$

[3] The nonlocal condensate $Q(x)$ and form factor $f(k)$ is naturally defined in the Euclidean region, where they decrease rapidly. All loop integrals, like that of (17), are evaluated in Euclidean space ($k^2 \to -k_E^2, d^4k \to id^4k_E$). Physical results are then obtained by analytically continuing back to Minkowski space.

[4] In the chiral limit, that we use in the work, the pion mass is zero in accordance with Goldstone theorem and at finite current quark masses it is deduced from Gell-Mann-Oakes-Renner relation.

and

$$J_{PP}(p) = i2N_c \int \frac{d^4k}{(2\pi)^4} f^2(k_+) f^2(k_-) \mathrm{tr}\left[i\gamma_5 S_F(k_-) i\gamma_5 S_F(k_+)\right], \quad (24)$$

is the pion field polarization operator. In (24), the dressed quark propagator is used and a notation $k_\pm^\mu = k^\mu \pm p^\mu/2$ is introduced. Explicitly, the quark-pion coupling is given by [16]

$$\left(\frac{1}{g_{\pi qq}}\right)^2 = \frac{N_c}{4\pi^2} \int \frac{d^4k}{\pi^2 i} f^2(k) \frac{\left[f^2(k) - 2k^2 f(k) f'(k) + 4k^4 (f'(k))^2\right]}{\left(k^2 - M_q^2(k)\right)^2}, \quad (25)$$

where $f'(k) \equiv \partial f(k)/\partial k^2$.

**4.** The nonlocal four-quark vertices $\Gamma_{4q}$ induce the two-quark dressed vertices $\Gamma_{2q}$ if one quark line closed to a loop. The longitudinal part of the dressed two-quark axial-vector vertex resulted from (12) is given by[19]

$$^A\Gamma_{2q}^{\mu a}(k,q) = \gamma^\mu \gamma_5 \frac{\tau^a}{2} - \gamma_5 \frac{q_\mu}{q^2} \frac{\tau^a}{2} \{[M(k+q) + M(k)] - \quad (26)$$

$$-i4N_c N_f G_I f(k+q) f(k) \int \frac{d^4l}{(2\pi)^4} \frac{M(l^2)}{l^2 - M(l^2)} f(l) \left[f(l-q) + f(l+q)\right]\Big\},$$

where, to obtain the first term inside the curly-brackets, the gap equation (17) is used. This vertex is bare one and in particular it is free of singularities. To obtain the full axial-vector vertex we need to take into account the transition of the current into the constituent quarks through their rescattering in the channel with pion quantum numbers. In the Appendix we check explicitly that the full axial-vector current is given by

$$^A\Gamma_{full}^{\mu a}(k,q) = \left[\gamma^\mu - \frac{M(k+q) + M(k)}{q^2} q^\mu\right] \gamma_5 \frac{\tau^a}{2}. \quad (27)$$

It has physical singularity corresponding to pion and evidently satisfies the axial WTI:

$$q^{\mu A}\Gamma_\mu(k,q) = S_F^{-1}(k+q)\gamma_5 + \gamma_5 S_F^{-1}(k).$$



The WTI and the requirement that the vertices contain no unphysical singularities, uniquely define the longitudinal part of the vector and axial-vector vertices. The transverse part is model dependent and, in particular, within the present approach, depends on the path integral definition.

## 4 Results and conclusions

The pion DA is computed from (5) by using dressed quark propagator (18), quark-pion vertex (22) and quark-axial-vector current vertex (26). The momentum dependence of the dressed quantities is defined by the nonlocality of the quark condensate (19), which in the present approach is specified by the instanton model (21). The set of parameters that we use in the actual calculations is given by [11]: $\rho_c = 1.7\mathrm{GeV}^{-1}$, $M_q = 230$ MeV, $n_c = 0.7$ fm$^{-4}$. They are consistent with the low-energy observables as discussed in the introduction. In the calculation of integral in (5) we use the Laplace transform technique described in [11]. In the present work we do not use the constant mass approximation.

The graph of $\varphi_A(x)$ is presented in Fig. 1 in solid line, where we can observe that its form is close to the asymptotic DA $\varphi_A^{asympt}(x) = 6x\bar{x}$. The main contribution comes from the local part of vertex (dash-dotted line) and the contribution of nonlocal part (dashed line) is flat. Note that the flat form of the nonlocal contribution results as a sum of different nonlocal terms that have more complicated form.

The pion DA that we found is defined at low energy scale $\mu_0 \sim \rho_c^{-1}$, where the application of the instanton model is expected to be justified. It serves as initial data for the QCD evolution to higher momentum transfer scales $Q^2$. To obtain this relation, it is convenient to expand the DA over Gegenbauer polynomials, $C_n^{3/2}(x)$, that are the eigenfunctions of the kernel of the QCD evolution equations:

$$\varphi_A(x,\mu_F) = \varphi_A^{asympt}(x) \left[1 + \sum_{n=2,4,\ldots}^{\infty} B_n(\mu_0) \left(\frac{\alpha_s(\mu_F)}{\alpha_s(\mu_0)}\right)^{\gamma_n} C_n^{3/2}(2x-1)\right], \quad (28)$$

where $\gamma_n$ are the anomalous dimensions calculated in the leading order, in coupling constant $\alpha_s(\mu)$; and $B_n(\mu_0)$ are the coefficients of the Gegenbauer polynomial expansion. The model DA is well reproduced by the above expansion, with only the first few nonzero coefficients:

$$B_2(\mu_0) = 0.069, \qquad B_4(\mu_0) = -0.061,$$
$$B_6(\mu_0) = -0.017, \qquad B_{n\geq 8}(\mu_0) = 0. \tag{29}$$

The distribution obtained is extrapolated to higher experimentally accessible momentum scales using perturbative QCD, such that the comparison with experimental data can be made. We choose the QCD scale parameter $\Lambda^{(3)}_{\overline{MS}} = 250$ MeV. In Fig. 2 we show the pion DA evolved to the scale 1 and 10 GeV$^2$ in comparison with the initial distribution at the scale $\mu_0^2 = \lambda_q^2 = 0.5$ GeV$^2$.

Recently, new data on the pion transition form factor at rather high $Q^2$, becomes available [6]. The perturbative QCD predicts the high $Q^2$ behaviour of form factor as [3]

$$F_{\pi\gamma\gamma}\left(Q^2\right) = \frac{J}{\sqrt{2}}\frac{F_\pi}{Q^2}, \tag{30}$$

with the constant $J$ being defined in terms of the pion DA

$$J = \frac{2}{3}\int_0^1 \frac{dx}{x}\varphi_A(x).$$

At asymptotically high $Q^2$ the DA evolves to $\varphi_A^{asympt}(x)$ and $J^{asympt} = 2$. At highest presently available momenta, $Q^2 \approx 10$ GeV$^2$, this prediction is reduced by the lowest order QCD radiative corrections [25] to $J^{asympt}(10\ \mathrm{GeV}^2) = 1.6$ and fits CLEO data well. Our predictions for uncorrected $J$ is very stable with $Q^2$ evolution: $J^{model}\left(\mu_0^2\right) = 1.98$ and $J^{model}\left(10\mathrm{GeV}^2\right) = 2.01$ and thus indistinguishable from perturbative QCD predictions.

In summary, we present here some theoretical predictions for the pion distribution amplitude. The non-perturbative formalism is based on the instanton model of the QCD vacuum, expressing the hadron observables in terms of fundamental characteristics of the vacuum state. The parameters of the model are the effective instanton size $\rho_c$ and the quark-mass $M_q$. The first one is given by the average virtuality of the vacuum quarks and the second one is related to the pion decay constant through the Goldberger-Treiman relation. It is shown that the correct normalization of the DA is obtained by using the compositeness condition and the strict implementation of PCAC, improving some previous calculations given in [14] and [15].

Our calculations are restricted by the instanton vacuum model. In the extended nonlocal NJL where the other spin-flavor terms in the interaction are possible the pion DA can get contribution from the vertex with vector insertion. The contribution of this piece to $F_\pi$ is small and estimated as $-10\%$[19]. However, it would be interesting to consider its effect on the form of DA. We have also to be aware that the nonlocality in this model is not fixed by any microscopic principle.

The extracted pion DA corresponds to a low normalization scale, where the effective instanton approach is justified. We obtain the pion DA via standard perturbative evolution to higher momentum values, accessible by experiment. A reasonable agreement with the CLEO data on transition pion form factor at high momentum transfer was found. The formalism used to derive the above results constitutes a complementary approach to lattice simulations, QCD sum rules and to phenomenological fits to experimental data.

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## Appendix

For completeness, we present an explicit derivation of the full axial-vector vertex and demonstration of the axial WTI. After allowing the constituent quark rescattering in the channel with pion quantum numbers, the full vertex becomes:

$$ {}^A\Gamma^{\mu a}_{full}(k,p) = {}^A\Gamma^{\mu a}(k,p) + {}^A\Gamma^{\mu a}_{rescat}(k,p), \quad (31) $$

where ${}^A\Gamma^{\mu a}(k,p)$ is the bare vertex given in eq. (26) and

$$ {}^A\Gamma^{\mu a}_{rescat}(k,p) = \frac{p^\mu p_\nu}{p^2} J^\nu_{PA}(p) \frac{G_I}{1 - G_I J_{PP}(p)} \frac{1}{g^2_{\pi q}} \Gamma^a_{\pi q}(k,p), \quad (32) $$

with $\Gamma^a_{\pi q}(k,p)$, $J_{PP}(p)$ defined by (22), (24), correspondingly, and

$$ J^\mu_{PA}(p) = i2N_c \int \frac{d^4k}{(2\pi)^4} f(k) f(k') $$
$$ \mathrm{tr}\left\{ {}^A\Gamma^{\mu a}(k,p) \left[\hat{k} + M(k)\right]^{-1} i\gamma_5 \left[\hat{k'} + M(k')\right]^{-1} \right\}, \quad (33) $$

with $k' = k + p$. Due to the gap equation (17), the vertex (32) contains a pole at $p^2 = m^2_\pi$. Comparing the residues at the poles in (27) and (4) at $z = 0$ one gets the expression for the pion decay constant. The integral in (33) is reduced to the integral defining $g^{-2}_{\pi q}$ in (23), on pion mass surface, and the eq. (33) can be written as $J^\mu_{PA}(p) = 2i\frac{M_q}{g_{\pi q}} p^\mu$. From the other side, this matrix element defines the decay constant $J^\mu_{PA}(p) = 2F_\pi i p^\mu$. Thus, the pion decay constant $F_\pi$ is reproduced as given by the Goldberger-Treiman relation [19] $F_\pi = \sqrt{2} M_q / g_{\pi q}$. In similar way if one inserts in the integral of (33) the factor $\exp[-i(p-2k)\cdot z]$ projecting the quark with momentum $k$ along light-like direction $z$ one derives the eq. (5).

The full vertex can be rewritten in the form explicitly satisfying the WTI. The first two terms in the r.h.s. of eq. (26) clearly satisfies the WTI. In order to compensate the third term of such equation, the rescattering term ${}^A\Gamma^{\mu a}_{rescat}(k,p)$, by using eq. (26), becomes

$$ -i\frac{p^\mu}{p^2} \frac{G_I N_c N_f}{1 - G_I J_{PP}(p)} f(k') f(k) $$



$$ \Big[ G_I J_{PP}(p) \int \frac{d^4l}{(2\pi)^4} \frac{M(l^2)}{l^2 - M^2(l^2)} f(l) \left[ f(l+p) + f(l-p) \right] - $$
$$ - \int \frac{d^4l}{(2\pi)^4} \frac{\mathrm{tr}\left\{ \left[\hat{p} - M(l^2) - M(l'^2)\right] \left[\hat{l'} + M(l'^2)\right] \gamma_5 \left[\hat{l} + M(l^2)\right] \gamma_5 \right\}}{(l'^2 - M^2(l'^2))(l^2 - M^2(l^2))} $$
$$ \cdot f(l') f(l) \Big] \quad (34) $$

where $l' = l + p$. By cancelling one of the factors $l^2 - M^2(l^2)$ in the denominator of the integral with the term from the Dirac trace in the numerator and properly shifting the integration variables, the first term inside the square brackets of (32) may be rewritten in the same form as the second. This demonstrates the required cancellation and the full vertex is given by (27).

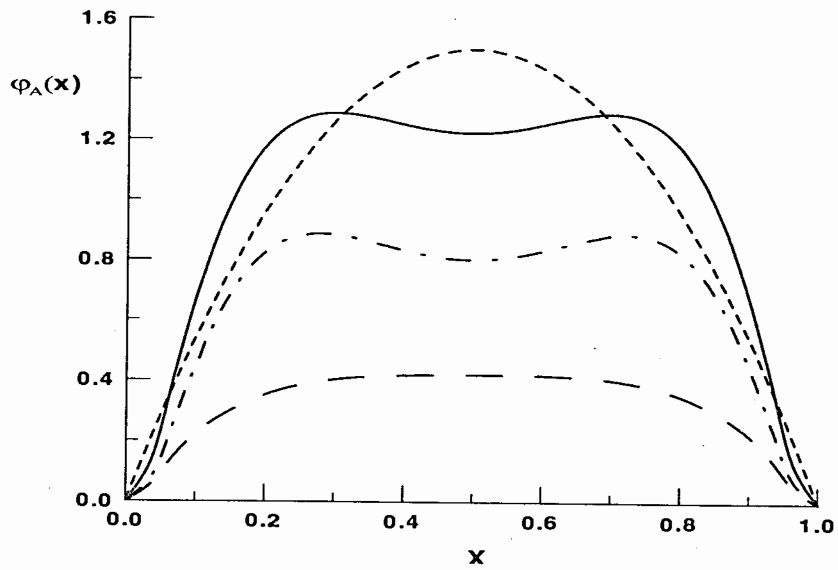


Figure 1: The axial projection of the pion distribution amplitude (solid line) at the low energy scale $\mu_0^2 = 0.5$ GeV$^2$. The contribution of the local part of vertex is shown by dash- dotted line and the contribution of nonlocal part by dashed line. The asymptotic distribution amplitude is shown by dotted line.

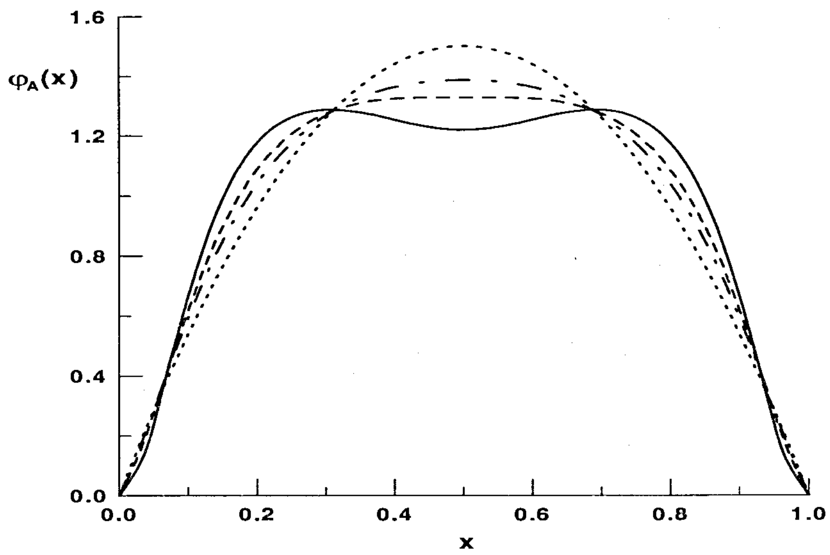


Figure 2: The axial projection of the pion distribution amplitude (solid line) at the low energy scale $\mu_0^2 = 0.5$ GeV$^2$ and its evolution to higher momentum transfers squared: $Q^2 = 1$ GeV$^2$ (dashed line), $Q^2 = 10$ GeV$^2$ (dot-dashed line). The asymptotic distribution amplitude is shown by dotted line.



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