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REPARAMETRIZATION INVARIANCE AND THE SCHRÖDINGER EQUATION

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1 Introduction

Time plays a central and peculiar role in Hamiltonian quantum mechanics. In the standard non-relativistic quantum mechanics one can describe the motion of a system by using the canonical variables which are only functions of time. The scalar product specifies a direct probability of observation at one instant of time [1]. Time is the sole observable assumed to have a direct physical significance, but it is not a dynamical variable itself. It is an absolute parameter differently treated from the other coordinates, which turn out to be operators and observables in quantum mechanics.

In the cases of non-relativistic and relativistic point particle mechanics generally covariant systems may be obtained by promoting the time t to a dynamical variable [1, 2, 3, 4, 5, 6, 7, 8]. The idea behind this transformation is to treat symmetrically the time and the dynamical variables of the system. This is achieved by taking the time t as a function of an arbitrary parameter τ (label time) in Dirac's approach [2]. The arbitrariness of the label time τ is reflected in the invariance of the action under the time reparametrization.

In this work we give the two-stage procedure for constructing generally covariant systems. Using additional gauge variables we rewrite the original action of the system in the reparametrization invariant form [2, 3]. The structure of the reparametrization transformations leads to zero Hamiltonian (first-class constraint) associated to the original action [3, 6]. At the quantum theory this constraint imposes condition on the state vector, which becomes time-independent Schrödinger equation [3, 8]. After that we consider an additional action invariant under the time reparametrization, which does not change the equations of motion of the original theory, but modifies only the first-class constraint, becomes now the time-dependent Schrödinger equation [3, 5]. In the case of different versions of supersymmetric quantum mechanics [9, 10, 11] such a procedure finds its application, when the transformations of reparametrization belong to a wider group of local transformations arising from the construction of the generally covariant systems. In this case, the set of auxiliary gauge variables are components of the world-line supergravity multiplet [19].

Here we construct a local supersymmetric action for $n = 2$, $d = 1$ supersymmetric quantum mechanics, in which the first-class constraints become time-independent Schrödinger equation, supercharges and the fermion number operator. However, there exists an additional supersymmetric invariant action, which permits the generalization of the above local supersymmetric quantum theory. Hence, we obtain the square root representation of the time-dependent

Schrödinger equation.

The plan of this work is as follows: in section 2, applying the canonical quantization procedure to reparametrization invariant action, we obtain the time-dependent Schrödinger equation. In section 3 the same procedure is applied to relativistic case. The extension to supersymmetric model is performed in section 4. Finally, section 5 is devoted to final remarks.

2 Non-relativistic parametrized particle dynamics

In this section the central idea is illustrated with the aid of a simple model of parametrized dynamics. We start by considering the theory of a non-relativistic particle moving in the D-dimensional space with dynamical variables x_i ($i = 1, 2, , D$) and with t denoting the ordinary physical time parameter. The action for this simplest model may be written as

$$S_0 = \int \left\{ \frac{1}{2} m \dot{x}_i^2(t) - V(x_i) \right\} dt, \tag{2.1}$$

where m is the mass of the particle, $\dot{x}_i = \frac{dx_i}{dt}$ is its velocity and $V(x_i)$ is the potential. The action (2.1) is invariant under the global translation of the time

$$t' \to t + c, \qquad c = \text{constant}. \tag{2.2}$$

We see, that the Lagrangian is non-degenerate in the sense that the relation between the momentum and the velocity is one to one

$$p_i = \frac{\partial L}{\partial \dot{x}^i} = m\dot{x}_i. \tag{2.3}$$

The Hamiltonian for this model has the form

$$H_0 = \frac{p_i^2}{2m} + V(x_i). \tag{2.4}$$

In the action (2.1) time t is an absolute parameter, differently treated from the other coordinates which turn out to be operators and observables in quantum mechanics. On the other hand, it is well known, that in non-relativistic point particle mechanics generally covariant systems may be obtained by promoting the time t to a dynamical variable [2, 3]. The same procedure has been applied to relativistic particle case [6, 7]. So, having in mind the application of the

procedure to the supersymmetric case we will proceed as follows. First of all, we will rewrite the action (2.1) in the parametrized form

$$\tilde{S} = \int \left\{ \frac{m\dot{x}_i^2(\tau)}{2N(\tau)} - N(\tau)V[x_i(\tau)] \right\} d\tau, \tag{2.5}$$

where the dot denotes derivative with respect to the parameter τ. $N(\tau)$ is the so called "lapse function" and relates the physical time t with the arbitrary parameter τ through $dt = N(\tau)d\tau$. This canonical variable is a pure gauge variable and it is not dynamical. $N(\tau)$ in (2.5) defines the scale on which the time is measured, and in the "gauge" $N(\tau) = 1$ the time parameter τ is identified as the "classical" time t and (2.5) becomes (2.1). On the other hand, $N(\tau)$ can be viewed as one dimensional gravity field, then the action (2.5) describes the interaction between "matter" $x_i(\tau)$ and the gravity field $N(\tau)$ [12]. The action (2.5) is invariant under the local time transformation

$$\tau' = \tau + a(\tau), \tag{2.6}$$

if $N(\tau)$ and $x_i(\tau)$ transform as

$$\delta N(\tau) = \frac{d}{d\tau}(aN), \qquad \delta x_i(\tau) = a\dot{x}_i(\tau). \tag{2.7}$$

Varying the action (2.5) with respect to $x_i(\tau)$ and $N(\tau)$ one obtain the classical equations of motion for $x_i(\tau)$ and the constraint, respectively.

Now we consider the Hamiltonian analysis of this simple constrained system. We define the canonical momentum p^i conjugated to the dynamical variable x_i as

$$p^i = \frac{\partial \tilde{L}}{\partial \dot{x}_i} = \frac{m}{N}\dot{x}^i, \tag{2.8}$$

and the classical Poisson brackets between x_i and p^j by

$$\{x_i, p^j\} = \delta_i^j. \tag{2.9}$$

The momentum conjugate to $N(\tau)$ is

$$P_N = \frac{\partial \tilde{L}}{\partial \dot{N}} = 0, \tag{2.10}$$

this equation merely constrains the variable $N(\tau)$ (primary constraint). The canonical Hamiltonian can be calculated in the usual way, it has the form $\tilde{H}_c = NH_0$, and the total Hamiltonian is

$$\tilde{H}_T = NH_0 + u_N P_N, \tag{2.11}$$

where u_N is the Lagrange multiplier associated to the constraint $P_N = 0$ in (2.10) and H_0 is the Hamiltonian of the system defined in (2.4). The canonical evolution of the constraint P_N is given by the Poisson bracket with the total Hamiltonian. Thus, we have

$$\dot{P}_N = \{P_N, \tilde{H}_T\} = -H_0 = 0, \tag{2.12}$$

leading to the secondary constraint, which is the first-class constraint [5]. In the quantum theory the first-class constraint associated with the invariance of the action (2.5) under the reparametrizations (2.6) becomes condition on the wave function ψ. Any physical state must obey the following quantum constraint

$$H_0(\hat{p}^i, x_i)\psi(x_l) = 0, \tag{2.13}$$

which is nothing but the time-independent Schrödinger equation.

Now we have to stress, that the physical meaning of the action (2.5) is different from that of the starting action (2.1). Indeed the equation (2.13) leads to the zero value of the energy of the system. To correct the situation and to get a time-dependent Schrödinger equation for the parametrized system (2.5) we will proceed as follows. We consider the following invariant action

$$S_r = \int p_t \left\{ \frac{dt(\tau)}{d\tau} - N(\tau) \right\} d\tau. \tag{2.14}$$

Now (t, p_t) is a pair of dynamic conjugated variables, p_t is the momentum corresponding to t. The action (2.14) is invariant under the reparametrization (2.6), if

$$\delta p_t = a\dot{p}_t, \qquad \delta t = a\dot{t}, \qquad \delta N = \frac{d}{d\tau}(aN). \tag{2.15}$$

So, adding the action (2.14) to the action (2.5) we obtain in the first order form the total action $\tilde{\tilde{S}} = \tilde{S} + S_r$

$$\tilde{\tilde{S}} = \int \left\{ p_i\dot{x}^i - NH_0(p,x) + p_t(\dot{t} - N(\tau)) \right\} d\tau. \tag{2.16}$$

The action (2.16) is invariant under the local transformation (2.6), if N, x, p_t and t transform according to (2.7, 2.15). Physically, the action (2.14) ensures that $N = \frac{dt}{d\tau}$. The total action (2.16) can be symply obtained from the starting action (2.1) following the Dirac approach [2]. Indeed, taking t to be a function of local time τ, $t = t(\tau)$ from (2.1) we get

$$\tilde{S}_0 = \int \left\{ \frac{1}{2}m\frac{\dot{x}_i}{\dot{t}} - V(x_i)\dot{t} \right\} d\tau, \tag{2.17}$$

where the dots now stand for $\frac{d}{d\tau}$. Due to the definitions of momenta there exists the constraint

$$p_t + \frac{1}{2}p_i^2 + V(x_i) = 0. \tag{2.18}$$

The canonical Hamiltonian is zero and the Lagrangian in the first-order form coinsides with (2.16) where N is a Lagrange multiplier. The canonical Hamiltonian obtained from the action $\tilde{\tilde{S}}$ in (2.16) has the form

$$\tilde{\tilde{H}}_c = N(p_t + H_0), \tag{2.19}$$

and the total Hamiltonian is

$$\tilde{\tilde{H}}_T = N(p_t + H_0) + u_N P_N. \tag{2.20}$$

For the consistency of the theory the constraint P_N must be conserved in time

$$\dot{P}_N = \{P_N, \tilde{\tilde{H}}_T\} = -(p_t + H_0) = 0. \tag{2.21}$$

This equation is a first-class constraint. So, the Hamiltonian equations of motion then are

$$\dot{x}_i = \{x_i, \tilde{\tilde{H}}_T\} = \frac{Np_i}{m}, \tag{2.22}$$

$$\dot{p}_i = \{p_i, \tilde{\tilde{H}}_T\} = -N\frac{dV}{dx_i}, \tag{2.23}$$

$$\dot{N} = \{N, \tilde{\tilde{H}}_T\} = u_N, \tag{2.24}$$

$$\dot{t} = \{t, \tilde{\tilde{H}}_T\} = N, \tag{2.25}$$

$$\dot{p}_t = \{p_t, \tilde{\tilde{H}}_T\} = 0. \tag{2.26}$$

The first two equations (2.22) and (2.23) are the equations of motion for the physical degrees of freedom. The action (2.16) contains one extra canonical pair (t, p_t) over (2.1), but also contains the constraint (2.21). This constraint, being the only one, is of the first-class. Furthemore, the action (2.16) describes the same number of independent degrees of freedom as the action in (2.1). The equation (2.24) shows that $N(\tau)$ is an arbitrary function playing the role of gauge field of the reparametrization symmetry. If we take the gauge condition $N(\tau) = 1$, then as it follows from (2.25), we have $t = \tau$. On the level of the equations of motion the action S_r is zero, and inserting $N = \dot{t}$ in the action $\tilde{S}$ in (2.5), we can exclude the auxiliary gauge $N(\tau)$ and obtain Dirac's approach for reparametrization invariant action in the case of non-relativistic systems (2.17) [2, 7, 8].

At the quantum level Dirac's brackets must be replaced by the commutator

$$[t, \hat{p}_t] = i\{t, p_t\}^* = i, \tag{2.27}$$

and the classical momentum p_t by the operator $\hat{p}_t$ with the representation $-i\frac{\partial}{\partial t}$ (we assume units in which $\hbar = c = 1$). Following the Dirac's canonical quantization the first-class constraints must be imposed on the wave function $\psi(x, t)$. So, the constraint (2.21) may be written as

$$i\frac{d\psi(x^l, t)}{dt} = H_0(-i\frac{d}{x^l}, x_m)\psi(x^l, t). \tag{2.28}$$

Hence, the inclusion in (2.5) of an additional reparametrization invariant action (2.14) does not change the equations of motion (2.22, 2.23), but only the constraint (2.13), which becomes (2.21). Thus, canonical quantization procedure applied to the parametrized theory (2.16) yields the correct equation for the wave function ψ (2.28), which is just the conventional time-dependent Schrödinger equation.

In the following two sections it will be shown, that the same procedure without any difficulties can be extended to the relativistic and supersymmetric cases.

3 Relativistic Point Particle

In this section we will consider a free relativistic particle. The action in this case has the form

$$S = -m\int \sqrt{1 - \dot{x}_i^2(t)}\, dt, \tag{3.1}$$

where m, t and x_i $(i = 1, 2, 3)$ are, respectively, the mass, proper time and the position of the particle. After parametrization $dt = N(\tau)d\tau$ the action (3.1) becomes

$$\tilde{S} = -m\int \sqrt{N^2(\tau) - \dot{x}_i^2(\tau)}\, d\tau. \tag{3.2}$$

This action is invariant under the local time reparametrization (2.6), if $N(\tau)$ and $x_i(\tau)$ transform as (2.7). The canonical Hamiltonian in this case has the form

$$\tilde{H}_c = NH_0 = N\left(\sqrt{p_i^2 + m^2}\right), \tag{3.3}$$

where

$$p_i = \frac{\partial \tilde{L}}{\partial \dot{x}_i} = \frac{m}{N}\frac{\dot{x}_i}{\sqrt{1 - \frac{\dot{x}_i}{N^2}}}$$

is the canonical momentum conjugated to dynamical variable x_i. So, we will rewrite the action (3.2) by considering (2.14) in the first order form, we get

$$\tilde{\tilde{S}} = \int \left\{p_i\dot{x}_i + p_0(\dot{x}^0 - N) - N\sqrt{p_i^2 + m^2}\right\} d\tau, \tag{3.4}$$

where $p_0 \equiv p_t$ and $x^0 \equiv t$. The relativistic canonical Hamiltonian is

$$\tilde{\tilde{H}}_c = NH = N\left(\sqrt{p_i^2 + m^2} + p_0\right), \tag{3.5}$$

where H is the classical relativistic constraint corresponding to the action (3.4). At the quantum level this constraint becomes condition on the wave function ψ

$$\left(-i\frac{d}{dx_0} + \sqrt{\hat{p}_i^2 + m^2}\right)\psi(x_0, x_i) = 0, \tag{3.6}$$

which is the time-dependent Schrödinger equation for the relativistic free massive particle. Note, that if we take the lapse function as

$$N(\tau) = e(\tau)\frac{\sqrt{p_i^2 + m^2} - p_0}{2}, \tag{3.7}$$

and insert it in (3.5) we have then

$$\tilde{\tilde{H}}_c = \frac{e(\tau)}{2}\left(\sqrt{p_i^2 + m^2} + p_0\right)\left(\sqrt{p_i^2 + m^2} - p_0\right) = \frac{e(\tau)}{2}\left(p_i^2 + m^2 - p_0^2\right). \tag{3.8}$$

Using the relations (3.7), (2.7) and (2.15) for the $N(\tau), p_i(\tau)$ and $p_0(\tau)$, it is easy to show, that $e(\tau)$ transforms as

$$\delta e = \frac{d}{d\tau}(ae), \tag{3.9}$$

corresponding to the transformation of $N(\tau)$ in (2.7).

So, the action (3.4) takes the form

$$S = \int \left\{p_\mu \dot{x}^\mu - e(\tau)\left(\frac{p_\mu^2 + m^2}{2}\right)\right\} d\tau, \tag{3.10}$$

where $\mu = 0, 1, 2, 3$. The action (3.10) describes a massive relativistic particle moving in the four-dimensional space-time. The $e(\tau)$ is an einbein, which plays the role of Lagrange multiplier. Variation of the action (3.10) with respect to $e(\tau)$ leads to the relativistic constraint

$$p_\mu^2 + m^2 = 0, \tag{3.11}$$

which is nothing but the mass-shell condition. When we go over to quantum mechanics, the constraint (3.11) is replaced by the condition on the scalar field ϕ

$$\left(\frac{\partial^2}{\partial x_0^2} - \frac{\partial^2}{\partial x_i^2} + m^2\right)\phi(x_0, x_i) = 0, \tag{3.12}$$

which is the Klein-Gordon equation. Hence, inclusion of an invariant under reparametrization, action leads us to the Schrödinger time-dependent equation for the wave function $\psi(x_i, t)$ in the case of relativistic particle, and at the same time it leads to the Klein-Gordon equation in the case of quantum scalar field $\phi(x_i, t)$.

4 n=2, d= 1 Supersymmetry

In the global $n = 2$ supersymmetric one-dimensional quantum mechanics the simplest action has the form [10, 13, 14]

$$S_{n=2} = \int \left\{ \frac{\dot{x}^2}{2} - i\bar{\chi}\dot{\chi} - 2\Big(\frac{\partial g}{\partial x}\Big)^2 - 2\frac{\partial^2 g}{\partial x^2}\bar{\chi}\chi \right\} dt, \tag{4.1}$$

where the overdote denotes derivatives with respect to t. In the action (4.1) x is an even dynamical variable, unlike χ, which is odd. Note, that the action in (4.1) is the supersymmetric extension of (2.1). The corresponding supersymmetric Hamiltonian is

$$H_0 = \frac{p^2}{2} + 2\Big(\frac{\partial g}{\partial x}\Big)^2 + 2\frac{\partial^2 g}{\partial x^2}\bar{\chi}\chi, \tag{4.2}$$

where $p = \dot{x}$, $\pi_\chi = -i\bar{\chi}$ and $\pi_{\bar{\chi}} = -i\chi$ are the momenta conjugated to x, χ and $\bar{\chi}$, respectively. The Dirac's brackets are defined as

$$\{\chi, \bar{\chi}\}^* = -i, \qquad \{x, p\}^* = 1. \tag{4.3}$$

Applying the Noether theorema to the $n = 2$ supersymmetry invariant action one finds the corresponding conserved supercharges

$$S = \left(ip + 2\frac{\partial g}{\partial x}\right)\chi, \qquad \bar{S} = S^\dagger = \left(-ip + 2\frac{\partial g}{\partial x}\right)\bar{\chi}, \tag{4.4}$$

and F, which is the generator of the $U(1)$ rotation on χ

$$F = \bar{\chi}\chi. \tag{4.5}$$

In terms of the Dirac's brackets (4.3) the quantities H_0, S, $\bar{S}$ and F form a closed super-algebra

$$\begin{aligned} &\{S, \bar{S}\}^* = -2iH_0, \quad \{H_0, S\}^* = \{H_0, \bar{S}\}^* = 0, \\ &\{S, S\}^* = \{\bar{S}, \bar{S}\}^* = 0, \quad \{F, S\}^* = iS, \quad \{F, \bar{S}\}^* = -i\bar{S}. \end{aligned} \tag{4.6}$$

Now, our goal will be to obtain the time-dependent Schrödinger equation for the supersymmetric case. The approach will be similar to that we have followed earlier. Dirac's approach applied to the action (4.1) for the $n = 2$ supersymmetric mechanics in the reparametrization invariant form requires a modification. A direct way to construct such action is a supersymmetric extension of the action (2.5), to the local $n = 2$ supersymmetry (rather $n = 2$ superconformal group in one-dimension) extending simultaneously the time reparametrization (2.6). As a consequence of this extension the new gauge fields $\psi(\tau), \bar{\psi}(\tau)$ and $V(\tau)$ in the action will appear. These gauge fields are the superpartners of the "lapse function" $N(\tau)$.

In order to obtain the supersymmetric extension of the action (2.5) the transformation of the time reparametrization (2.6) must be extended to the $n = 2$ local conformal time supersymmetry $(\tau, \theta, \bar{\theta})$ [15, 16, 17, 18]. The transformations of the supertime $(\tau, \theta, \bar{\theta})$ can be written as

$$\begin{aligned} \delta\tau &= \mathbb{L}(\tau,\theta,\bar{\theta}) + \frac{1}{2}\bar{\theta}D_{\bar{\theta}}\mathbb{L}(\tau,\theta,\bar{\theta}) - \frac{1}{2}\theta D_\theta \mathbb{L}(\tau,\theta,\bar{\theta}), \\ \delta\theta &= \frac{i}{2}D_{\bar{\theta}}\mathbb{L}(\tau,\theta,\bar{\theta}), \qquad \delta\bar{\theta} = -\frac{i}{2}D_\theta\mathbb{L}(\tau,\theta,\bar{\theta}), \end{aligned} \tag{4.7}$$

with the superfunction $\mathbb{L}(\tau, \theta, \bar{\theta})$ defined by

$$\mathbb{L}(\tau,\theta,\bar{\theta}) = a(\tau) + i\theta\bar{\beta}'(\tau) + i\bar{\theta}\beta'(\tau) + b(\tau)\theta\bar{\theta}, \tag{4.8}$$

where $D_\theta = \frac{\partial}{\partial\theta} + i\bar{\theta}\frac{\partial}{\partial\tau}$ and $D_{\bar{\theta}} = -\frac{\partial}{\partial\bar{\theta}} - i\theta\frac{\partial}{\partial\tau}$ are the supercovariant derivatives of the $n = 2$ global supersymmetry, $a(\tau)$ is a local time reparametrization parameter, $\beta'(\tau) = N^{-1/2}\beta$ is the Grassmann complex parameter of the $n = 2$ local conformal supersymmetry transformations and $b(\tau)$ is the parameter of the local $U(1)$ rotations on the complex Grassmann coordinates θ ($\bar{\theta} = \theta^\dagger$). The local supercovariant derivatives have the form $\tilde{D}_\theta = \mathbb{N}^{-\frac{1}{2}}D_\theta$ and $\tilde{D}_{\bar{\theta}} = \mathbb{N}^{-\frac{1}{2}}D_{\bar{\theta}}$.

Then, the superfield generalization of the actions (2.5) and (4.1), which is invariant under the $n=2$ local conformal supersymmetry transformations (4.7), has the form [19, 20]

$$\tilde{S}_{n=2}=\int\left\{\frac{1}{2}I\!N^{-1}D_{\bar{\theta}}\Phi D_{\theta}\Phi-2g(\Phi)\right\}d\theta d\bar{\theta}d\tau, \tag{4.9}$$

where $g(\Phi)$ is the superpotential. In the superfield action (4.9) $I\!N(\tau,\theta,\bar{\theta})$ is absent in the numerator of the second term, this is related to the fact that the superjacobian of the transformations (4.7), as well as the $BerE^A_B$, is equal to one and the quantity $d\theta d\bar{\theta}d\tau$ is an invariant volume. In order to have the component action for (4.9) we must expand in Taylor series the superfields $I\!N$, Φ and the superpotential $g(\Phi)$ with respect to $\theta,\bar{\theta}$.

In the case of the real superfield $I\!N$ ($I\!N^\dagger=I\!N$) we have the following expansion

$$I\!N(\tau,\theta,\bar{\theta})=N(\tau)+i\theta\bar{\psi}'(\tau)+i\bar{\theta}\psi'(\tau)+V'(\tau)\theta\bar{\theta}, \tag{4.10}$$

where $N(\tau)$ is the lapse function, $\psi'=N^{1/2}(\tau)\psi(\tau)$ and $V'(\tau)=NV+\bar{\psi}\psi$. The components $N,\psi,\bar{\psi}$ and V of the superfield $I\!N(\tau,\theta,\bar{\theta})$ are gauge fields of the one-dimensional $n=2$ supergravity. The superfield (4.10) transforms as the one-dimensional vector field under the local supersymmetric transformations (4.7)

$$\delta I\!N=\frac{d}{d\tau}(I\!L I\!N)+\frac{i}{2}D_{\bar{\theta}}I\!L D_{\theta}I\!N+\frac{i}{2}D_{\theta}I\!L D_{\bar{\theta}}I\!N. \tag{4.11}$$

The transformation law for the components $N(\tau),\psi(\tau),\bar{\psi}(\tau)$ and $V(\tau)$ may be obtained from (4.11):

$$\begin{aligned}\delta N&=\frac{d}{d\tau}(aN)+\frac{i}{2}(\beta\bar{\psi}+\bar{\beta}\psi),\qquad \delta V=\frac{d}{d\tau}(aV)+\dot{\hat{b}},\\ \delta\psi&=\frac{d}{d\tau}(a\psi)+D\beta-\frac{i}{2}\hat{b}\psi,\qquad \delta\bar{\psi}=\frac{d}{d\tau}(a\bar{\psi})+D\bar{\beta}+\frac{i}{2}\hat{b}\bar{\psi},\end{aligned} \tag{4.12}$$

where $D\beta=\dot{\beta}+\frac{i}{2}V\beta$ and $D\bar{\beta}=\dot{\bar{\beta}}-\frac{i}{2}V\bar{\beta}$ are the $U(1)$ covariant derivatives and $\hat{b}=b-\frac{1}{2N}(\beta\bar{\psi}-\bar{\beta}\psi)$.

For the real scalar matter superfield $\Phi(\tau,\theta,\bar{\theta})$ we have

$$\Phi(\tau,\theta,\bar{\theta})=x(\tau)+i\theta\bar{\chi}'(\tau)+i\bar{\theta}\chi'(\tau)+F'(\tau)\theta\bar{\theta}, \tag{4.13}$$

where $\chi'=N^{1/2}\chi$ and $F'=NF+\frac{1}{2}(\bar{\psi}\chi-\psi\bar{\chi})$. The transformations law for the superfield $\Phi(\tau,\theta,\bar{\theta})$ is

$$\delta\Phi=I\!L\dot{\Phi}+\frac{i}{2}D_{\bar{\theta}}I\!L D_{\theta}\Phi+\frac{i}{2}D_{\theta}I\!L D_{\bar{\theta}}\Phi. \tag{4.14}$$

The component $F(\tau)$ in (4.13) is an auxiliary degree of freedom (non-dynamical variable), $\chi(\tau)$ and $\bar{\chi}(\tau)$ are the "fermionic" superpartners of the $x(\tau)$. Their transformations laws have the form:

$$\begin{aligned}\delta x&=a\dot{x}+\frac{i}{2}(\beta\bar{\chi}+\bar{\beta}\chi),\qquad \delta F=a\dot{F}+\frac{1}{2N}(\bar{\beta}\tilde{D}\chi-\beta\tilde{D}\bar{\chi}),\\ \delta\chi&=a\dot{\chi}+\frac{\beta}{2}\left(\frac{Dx}{N}+iF\right)-\frac{i}{2}\hat{b}\chi,\qquad \delta\bar{\chi}=a\dot{\bar{\chi}}+\frac{\bar{\beta}}{2}\left(\frac{Dx}{N}-iF\right)+\frac{i}{2}\hat{b}\bar{\chi},\end{aligned} \tag{4.15}$$

where $Dx=\dot{x}-\frac{i}{2}(\psi\bar{\chi}+\bar{\psi}\chi)$, $\tilde{D}\chi=D\chi-\frac{i}{2}(\frac{Dx}{N}+iF)\psi$, $\tilde{D}\bar{\chi}=D\bar{\chi}-\frac{i}{2}(\frac{Dx}{N}-iF)\bar{\psi}$ are the supercovariant derivatives and $D\chi=\dot{\chi}+\frac{i}{2}V\chi$.

It is clear, that the superfield action (4.9) is invariant under the $n=2$ local conformal time supersymmetry. Let us denote the expression under the integral (4.9) by means of certain superfunction $f(I\!N,\Phi)$. Then, the infinitesimal small transformations of the action (4.9) under the superfield transformations (4.11,4.14) have the form

$$\delta\tilde{S}_{n=2}=\frac{i}{2}\int\{D_{\bar{\theta}}(I\!L D_{\theta}f)+D_{\theta}(I\!L D_{\bar{\theta}}f)\}\,d\theta d\bar{\theta}d\tau. \tag{4.16}$$

We can see, that the integrand is a total derivative, *i.e.* the action (4.9) is invariant under the $n=2$ local conformal time supersymmetry.

After integration over the Grassmann complex coordinates θ and $\bar{\theta}$ we find the component action, where $F(\tau)$ is an auxiliary field, and it can be eliminated using its equation of motion. Finally, the action $\tilde{S}_{n=2}$, in terms of the components of the superfields $I\!N$ and Φ, takes the form

$$\tilde{S}_{n=2}=\int\left\{\frac{(Dx)^2}{2N}-i\bar{\chi}D\chi-2N\left(\frac{\partial g}{\partial x}\right)^2-2N\frac{\partial^2 g}{\partial x^2}\bar{\chi}\chi+\frac{\partial g}{\partial x}(\bar{\psi}\chi-\psi\bar{\chi})\right\}d\tau, \tag{4.17}$$

where Dx and $D\chi$ are defined above. The action (4.17) does not include the kinetic terms for $N,\psi,\bar{\psi}$ and V, they are not dynamical. This fact is reflected in the primary constraints

$$\begin{aligned}P_N&=\frac{\partial\tilde{L}_{n=2}}{\partial\dot{N}}=0,\qquad P_\psi=\frac{\partial\tilde{L}_{n=2}}{\partial\dot{\psi}}=0,\qquad P_{\bar{\psi}}=\frac{\partial\tilde{L}_{n=2}}{\partial\dot{\bar{\psi}}}=0,\\ P_V&=\frac{\partial\tilde{L}_{n=2}}{\partial\dot{V}}=0,\end{aligned} \tag{4.18}$$

where $P_N, P_\psi, P_{\bar\psi}$ and P_V are the canonical momenta conjugate to $N, \psi, \bar\psi$ and V, respectively. Then, the canonical Hamiltonian corresponding to the action $\tilde S_{n=2}$ in (4.17) is

$$\tilde H_c = NH_0 + \frac{\bar\psi}{2}S - \frac{\psi}{2}\bar S + \frac{V}{2}F, \tag{4.19}$$

where $H_0, S, \bar S$ and F are defined in (4.2, 4.4, 4.5), and the total Hamiltonian is

$$\tilde H_T = \tilde H_c + u_N P_N + u_\psi P_\psi + u_{\bar\psi} P_{\bar\psi} + u_V P_V. \tag{4.20}$$

The secondary constraints are first-class constraints

$$H_0 = 0, \qquad S = 0, \qquad \bar S = 0, \qquad F = 0, \tag{4.21}$$

which are obtained using the standard Dirac's procedure, *i.e.*, the time derivatives of the primary constraints must be vanishing for all the P_N, P_ψ, $P_{\bar\psi}$, and P_V.

In the quantum theory the first-class constraints (4.21) associated with the invariance of the action (4.17) become conditions on the wave function $\psi = \psi(x, \chi, \bar\chi)$. The quantum constraints are

$$H_0\psi \;=\; 0, \quad S\psi = \bar S\psi = 0, \quad F\psi = 0, \tag{4.22}$$

which are obtained when we change the classical dynamical variables by their corresponding operators. The first equation in (4.22) is the Schrödinger equation, a state with zero energy. Therefore, we have the time-independent Schrödinger equation, this fact is due to the invariance under the reparametrization symmetry of the action (4.17), this problem is well-known as the "problem of time" [1, 2, 3, 4, 5, 6].

So, in order to have a time-dependent Schrödinger equation for the supersymmetric quantum mechanics, we consider the generalization of the reparametrization invariant action S_r in (2.14). In the case of $n = 2$ local supersymmetry it has the superfield form

$$S_{r(n=2)} \;=\; \int \left\{ I\!\!P - \frac{i}{2} I\!\!N^{-1}(D_{\bar\theta}\mathbf{T} D_\theta I\!\!P - D_{\bar\theta} I\!\!P D_\theta \mathbf{T})\right\} d\theta d\bar\theta d\tau. \tag{4.23}$$

The action (4.23) is determined in terms of the new superfields $\mathbf{T}$ and $I\!\!P$. The superfield $\mathbf{T}$ is determined by the odd complex times, $\eta(\tau)$ and $\bar\eta(\tau)$, which are the superpartners of the time $t(\tau)$, and one auxiliary field $m'(\tau)$. Explicitly, we have

$$\mathbf{T}(\tau, \theta, \bar\theta) = t(\tau) + \theta\eta'(\tau) - \bar\theta\bar\eta'(\tau) + m'(\tau)\theta\bar\theta, \tag{4.24}$$

where $\eta' = N^{1/2}\eta$ and $m' = Nm + \frac{i}{2}(\bar\psi\bar\eta + \psi\eta)$. The transformation rule for the superfield $\mathbf{T}(\tau, \theta, \bar\theta)$ under the $n = 2$ local conformal supersymmetry transformations (4.7) is

$$\delta\mathbf{T} = I\!\!L\dot{\mathbf{T}} + \frac{i}{2}D_{\bar\theta} I\!\!L D_\theta \mathbf{T} + \frac{i}{2} D_\theta I\!\!L D_{\bar\theta}\mathbf{T}. \tag{4.25}$$

The superfield $I\!\!P(\tau, \theta, \bar\theta)$ has the form

$$I\!\!P(\tau.\theta, \bar\theta) = \rho(\tau) + i\theta p'_{\bar\eta}(\tau) + i\bar\theta p'_\eta(\tau) + p'_t(\tau)\theta\bar\theta, \tag{4.26}$$

where $p'_\eta = N^{1/2}p_\eta$ and $p'_t = Np_t + \frac{1}{2}(\bar\psi p_\eta - \psi p_{\bar\eta})$. p_η and $p_{\bar\eta}$ are the odd complex momenta, *i.e.* the superpartners of the momentum p_t. The superfield $I\!\!P$ transforms as

$$\delta I\!\!P = I\!\!L\dot{I\!\!P} + \frac{i}{2}D_{\bar\theta} I\!\!L D_\theta I\!\!P + \frac{i}{2} D_\theta I\!\!L D_{\bar\theta} I\!\!P. \tag{4.27}$$

It is easy to show, that the infinitesimal small transformations of the action $S_{r(n=2)}$ under the transformations (4.11, 4.25, 4.27) is a total derivative, then the action $S_{r(n=2)}$ is invariant under the $n = 2$ local supersymmetric transformations (4.7).

After integration over θ and $\bar\theta$ the action (4.23) may be written in its component form

$$\begin{aligned} S_{r(n=2)} \;=\; & \int \left\{ p_t(\dot t + N) + i\dot\eta p_\eta + i\dot{\bar\eta} p_{\bar\eta} + \frac{\bar\psi}{2}(p_\eta + \bar\eta p_t) \right. \\ & - \left. \frac{\psi}{2}(p_{\bar\eta} + \eta p_t) + \frac{V}{2}(\eta p_\eta - \bar\eta p_{\bar\eta}) + p_\rho(\dot\rho - \frac{i}{2}\psi p_{\bar\eta} - \frac{i}{2}\bar\psi p_\eta)\right\} d\tau. \end{aligned} \tag{4.28}$$

As we will see later, the variables ρ and p_ρ are auxiliary, in the sense, that they can be eliminated from the physical variables by some unitary transformation.

Proceeding to Hamiltonian formulation we have the following constraints

$$\begin{aligned} \Pi_3(\eta) &= P_\eta - ip_\eta = 0, \qquad \Pi_4(p_\eta) = P_{p_\eta} = 0, \\ \Pi_5(\bar\eta) &= P_{\bar\eta} - ip_{\bar\eta} = 0, \qquad \Pi_6(p_{\bar\eta}) = P_{p_{\bar\eta}} = 0, \end{aligned} \tag{4.29}$$

where

$$P_\eta = \frac{\partial L_{r(n=2)}}{\partial\dot\eta}, \qquad P_{p_\eta} = \frac{\partial L_{r(n=2)}}{\partial\dot p_\eta} \tag{4.30}$$

are the odd momenta conjugated to η, p_η and their respective complex conjugate. We define the odd canonical Poisson brackets as

$$\{\eta, P_\eta\} = 1, \qquad \{p_\eta, P_{p_\eta}\} = 1. \tag{4.31}$$

So, the constraints (4.29) are of the second-class. Defining the matrix (symmetric for the Grassmann variables) constraint C_{ik} $(i,k = \eta, p_\eta, \bar\eta, p_{\bar\eta})$ as the odd Poisson brackets, we have the following non-zero matrix elements

$$C_{\eta,p_\eta} = C_{p_\eta,\eta} = \{\Pi_3,\Pi_4\} = -i, \qquad C_{\bar\eta,p_{\bar\eta}} = C_{p_{\bar\eta},\bar\eta} = \{\Pi_5,\Pi_6\} = -i \tag{4.32}$$

with their inverse matrix $(C^{-1})^{\eta,p_\eta} = i$ and $(C^{-1})^{\bar\eta,p_{\bar\eta}} = i$. The only non-zero Dirac's brackets are

$$\{\eta, p_\eta\}^* = -i, \qquad \{\bar\eta, p_{\bar\eta}\}^* = -i. \tag{4.33}$$

So, if we take the additional term (4.23) the full action will be

$$\tilde{\tilde{S}} = \tilde{S}_{n=2} + S_{r(n=2)}. \tag{4.34}$$

Then, the canonical Hamiltonian for the action $\tilde{\tilde{S}}$ will have the form

$$\tilde{\tilde{H}}_c = N(p_t + H_0) - \frac{\psi}{2}(-\tilde{S}_{\bar\eta} + \bar{S}) + \frac{\bar\psi}{2}(\tilde{S}_\eta + S) + \frac{V}{2}(F_\eta + F), \tag{4.35}$$

where $\tilde{S}_\eta = (S_\eta - ip_\rho p_\eta)$, $\tilde{S}_{\bar\eta} = (S_{\bar\eta} - ip_\rho p_{\bar\eta})$ and $S_\eta = (p_\eta + \bar\eta p_t)$, $S_{\bar\eta} = (-p_{\bar\eta} - \eta p_t)$, $F_\eta = (\eta p_\eta - \bar\eta p_{\bar\eta})$. As we mentioned earlier the variables ρ and p_ρ can be eliminated. For this goal we make the canonical transformations:

$$\begin{aligned} p_\eta(1 - ip_\rho) &= \tilde{p}_\eta, & \frac{\eta}{1 - ip_\rho} &= \tilde\eta, \\ p_{\bar\eta}(1 + ip_\rho) &= \tilde{p}_{\bar\eta}, & \frac{\bar\eta}{1 + ip_\rho} &= \tilde{\bar\eta}, \end{aligned} \tag{4.36}$$

and after that the unitary transformation is

$$H \to UHU^{-1} \tag{4.37}$$

with $U = \exp(i\eta\bar\eta p_t p_\rho)$. One can check that all p_ρ dependence of $\tilde{\tilde{H}}_c$ disappears. So, we can omit all tildes and write the Hamiltonian in the form

$$H_c = N(p_t + H_0) - \frac{\psi}{2}(-S_{\bar\eta} + \bar{S}) + \frac{\bar\psi}{2}(S_\eta + S) + \frac{V}{2}(F_\eta + F). \tag{4.38}$$

And the total Hamiltonian is

$$H_T = H_c + u_N P_N + u_\psi P_\psi + u_{\bar\psi} P_{\bar\psi} + u_V P_V. \tag{4.39}$$

Due to the conditions

$$\dot{P}_N = \dot{P}_\psi = \dot{P}_{\bar\psi} = \dot{P}_V = 0, \tag{4.40}$$

we now have the first-class constraints

$$H = p_t + H_0, \qquad Q_\eta = S_\eta + S, \qquad Q_{\bar\eta} = -S_{\bar\eta} + \bar{S}, \qquad \mathcal{F} = F_\eta + F. \tag{4.41}$$

The constraints (4.41) form a closed superalgebra with respect to the Dirac's brackets

$$\begin{aligned} \{Q_\eta, Q_{\bar\eta}\}^* &= -2iH, \quad \{H, Q_\eta\}^* = \{H, Q_{\bar\eta}\}^* = 0, \\ \{\mathcal{F}, Q_\eta\}^* &= iQ_\eta, \quad \{\mathcal{F}, Q_{\bar\eta}\}^* = -iQ_{\bar\eta}. \end{aligned} \tag{4.42}$$

After quantization the Dirac's brackets (4.33) become anticomutator for the odd variables

$$\{\eta, p_\eta\} = i\{\eta, p_\eta\}^* = 1, \qquad \{\bar\eta, p_{\bar\eta}\} = i\{\bar\eta, p_{\bar\eta}\}^* = 1, \tag{4.43}$$

with the operator representation $p_\eta = \frac{\partial}{\partial\eta}$ and $p_{\bar\eta} = \frac{\partial}{\partial\bar\eta}$. In order to obtain the quantum expression for $H, Q_\eta, Q_{\bar\eta}$ and $\mathcal{F}$ we use the operator representation $p = -i\frac{d}{dx}$ and χ, $\bar\chi$ as $\{\chi, \bar\chi\} = 1$, $\chi = \sigma_{(-)}$ and $\bar\chi = \sigma_{(+)}$, where $\sigma_\pm = \frac{1}{2}(\sigma_1 \pm i\sigma_2)$. We have then

$$\begin{aligned} H &= -i\frac{d}{dt} + H_0(p, x, \chi, \bar\chi), \qquad Q_\eta = \left(\frac{\partial}{\partial\eta} - i\bar\eta\frac{\partial}{\partial t}\right) + S(p, x, \chi), \\ Q_{\bar\eta} &= -\left(-\frac{\partial}{\partial\bar\eta} + i\eta\frac{\partial}{\partial t}\right) + \bar{S}(p, x, \bar\chi), \qquad \mathcal{F} = \left(\eta\frac{\partial}{\partial\eta} - \bar\eta\frac{\partial}{\partial\bar\eta}\right) + F(\chi, \bar\chi), \end{aligned} \tag{4.44}$$

where

$$H_0 = -\frac{d^2}{dx^2} + 2\left(\frac{\partial g}{\partial x}\right)^2 + \frac{d^2 g}{\partial x^2}[\bar\chi, \chi]$$

and $F = \frac{1}{2}[\bar\chi, \chi] = \frac{1}{2}\sigma_3$. In (4.44) $S_\eta = (\frac{\partial}{\partial\eta} - i\bar\eta\frac{\partial}{\partial t})$ and $S_{\bar\eta} = (-\frac{\partial}{\partial\bar\eta} + i\eta\frac{\partial}{\partial t})$ are the generators of supertranslations on the superspace with coordinates $(t, \eta, \bar\eta)$ and $p_t = -i\frac{\partial}{\partial t}$ is the ordinary time translation operator

$$\{S_\eta, S_{\bar\eta}\} = 2i\frac{\partial}{\partial t}, \tag{4.45}$$

and $F_\eta = (\eta\frac{\partial}{\partial\eta} - \bar\eta\frac{\partial}{\partial\bar\eta})$ is the generator of the $U(1)$ rotation of the complex Grassmann coordinates η $(\bar\eta = \eta^\dagger)$. The algebra of the quantum generators $H_0, S, \bar{S}$ and F is a closed superalgebra

$$\begin{aligned} \{S, \bar{S}\} &= 2H_0, \qquad [S, H_0] = [\bar{S}, H_0] = [F, H_0] = 0, \\ [F, S] &= -S, \qquad [F, \bar{S}] = \bar{S}, \qquad S^2 = \bar{S}^2 = 0, \end{aligned} \tag{4.46}$$

the conserved quantities are $H_0, S, \bar{S}$ and F. We can see, that the generators $H, Q_\eta, Q_{\bar{\eta}}$ and $\mathcal{F}$ satisfy the same superalgebra

$$\begin{aligned} \{Q_\eta, Q_{\bar{\eta}}\} &= 2H, \qquad [Q_\eta, H] = [Q_{\bar{\eta}}, H] = [\mathcal{F}, H] = 0, \\ [\mathcal{F}, Q_\eta] &= -Q_\eta, \qquad [\mathcal{F}, Q_{\bar{\eta}}] = Q_{\bar{\eta}}, \qquad Q_\eta^2 = Q_{\bar{\eta}}^2 = 0. \end{aligned} \tag{4.47}$$

In the quantum theory the first-class constraints (4.44) become conditions on the wave function Ψ. So, we have the supersymmetric quantum constraints

$$H\Psi = 0, \quad Q_\eta \Psi = 0, \quad Q_{\bar{\eta}}\Psi = 0, \quad \mathcal{F}\Psi = 0. \tag{4.48}$$

We will search the wave function in the superfield form

$$\begin{aligned} \Psi(t, \eta, \bar{\eta}, \chi, \bar{\chi}) &= \psi(t, x, \chi, \bar{\chi}) + i\eta\sigma(t, x, \chi, \bar{\chi}) + i\bar{\eta}\phi(t, x, \chi, \bar{\chi}) + \\ &+ \zeta(t, x, \chi, \bar{\chi})\eta\bar{\eta}. \end{aligned} \tag{4.49}$$

This wave function must satisfy the quantum constraints (4.48). In (4.49) ψ, ζ are even components of the wave function, unlike σ, ϕ, which are odd. We take the constraints

$$Q_\eta \Psi = 0, \qquad Q_{\bar{\eta}}\Psi = 0, \tag{4.50}$$

and due to the algebra (4.47) we have

$$\{Q_\eta, Q_{\bar{\eta}}\}\Psi = 2H\Psi = 0. \tag{4.51}$$

This is the time-dependent Schrödinger equation for the supersymmetric quantum mechanics. The conditions (4.50) lead to the following form of the wave function

$$\Psi_* = \psi - \eta(S\psi) - \bar{\eta}(\bar{S}\psi) + \frac{1}{2}(\bar{S}S - S\bar{S})\psi\eta\bar{\eta}, \tag{4.52}$$

where the function $\psi(t, \chi, \bar{\chi})$ satisfies the standard time-dependent Schrödinger equation

$$i\frac{d\psi(t, \chi, \bar{\chi})}{dt} = H_0(p, x, \chi, \bar{\chi})\psi(t, x, \chi, \bar{\chi}). \tag{4.53}$$

If we put in the Schrödinger equation (4.53) the condition of the stationary states given by $\frac{d\psi}{dt} = 0$, we will have $H_0\psi = 0$ and due to the algebra (4.46) we obtain $S\psi = \bar{S}\psi = 0$ and the wave function Ψ_* becomes wave function $\psi(x, \chi, \bar{\chi})$ [10, 11, 19, 20].

5 Conclusions

In this work we have considered systems (including susy), which are not parametrized. Such systems always may be done in a parametrized invariant form. We developed the two-stage procedure of such construction which is applicable in the supersymmetric case as well. First of all, we include in the action the auxiliary gauge field to ensure the reparametrization invariance of the action. Hence, the system of constraints contains generator of reparametrization, which is the Hamiltonian generator. It must annihilate the physical states, this leads to time-independet Schrödinger equation $H_0\psi = 0$. It means, that in the modified system only zero energy states are physical.

In order to have a time-dependent Schrödinger equation, *i.e.* to describe the quantum evolution of the initial system, as we shown in this work, an additional invariant action S_r may be always constructed. The additional action does not change the equations of motion, but the constraint system, one of which becomes time-dependent Schrödinger equation. From our point of view, this fact is very important in those cases, when starting systems are invariant under reparametrization of time, such systems as: general relativity, cosmological models, string theories. These theories contain auxiliary additional gauge degree of freedom (lapse and shift functions) [25]. Such theories have the problem which in literature is known as the "problem of time" [1, 3]. For instance, the Wheeler-DeWitt equation [26].

Naturally, the question arising as a result of this work is: could we construct an additional invariant under general covariant transformations action? If the result of this question is positive, then the additional action will remain without any changes the equations for the physical degree of freedom of the system, but the constraint will be modified leading to time-dependent Schrödinger equation.

Without any difficulties our procedure may be generalized to D-dimensional extended supersymmetry mechanics [14, 21]. This is due to the fact, that the full algebra of the transformations is closed off-shell, and it is $n = 2$ local conformal supersymmetry. So, our procedure represents a direct possibility to apply the Batalin-Vilkovisky formalism [22, 23, 24] to supersymmetric systems.

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