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$\pi\pm \leftrightarrow K\pm$ MESON-VACUUM TRANSITIONS (OSCILLATIONS) IN THE DIAGRAM APPROACH

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1 Introduction

The vacuum oscillations of neutral K mesons are well investigated at the present time [1]. These oscillations are the result of d, s quark mixings described by Cabibbo-Kobayashi-Maskawa matrices [2]. The angle mixing θ of neutral K mesons is $\theta = 45^O$ since $K^o, \bar{K}^o$ masses are equal (see CPT theorem). Besides, since their masses are equal, these oscillations are real, i.e. their transitions to each other go without suppression. Oscillations of two particles having the masses overlapping their widths were discussed in works [3]. Then in previous works [4] we computed probabilities of $\pi \leftrightarrow K$ oscillations in an approach where the phase volume of particles at these transitions is taken into account. The same oscillations arise in the model of dynamical analogy of Cabibbo-Kobayashi-Maskawa matrices [6].

This work is devoted to computation of the same values in a diagram approach which was used at computation of $K^o \leftrightarrow \bar{K}^o$ oscillations [5].

At first, we will consider the general elements of the theory of vacuum oscillations, then come to computation of probabilities of $\pi \leftrightarrow K$ transitions.

It is clear that these transitions are virtual since masses of π and K mesons differ considerably.

Let us pass to consideration of general elements of the theory of vacuum oscillations.

2 Probabilities of $\pi \xrightarrow{W sin\theta} K$ Vacuum Real and Virtual Transitions (Oscillations)

The mass matrix of π and K mesons has a form

$$\begin{pmatrix} m_\pi & 0 \\ 0 & m_K \end{pmatrix}. \tag{1}$$

Due to the presence of strangeness violation in the weak interactions, a nondiagonal term appears in this matrix and then this mass matrix is transformed in the following nondiagonal matrix:

$$\begin{pmatrix} m_\pi & m_{\pi K} \\ m_{\pi K} & m_K \end{pmatrix}, \tag{2}$$

which is diagonalized by turning through the angle β and then

$$\begin{pmatrix} m_\pi & m_{\pi K} \\ m_{\pi K} & m_K \end{pmatrix} \rightarrow \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$$

and

$$tg2\beta = \frac{2m_{\pi K}}{|m_\pi - m_K|},$$

$$sin2\beta = \frac{2m_{\pi K}}{\sqrt{(m_\pi - m_K)^2 + (2m_{\pi K})^2}}, \tag{3}$$

$$m_{1,2} = \frac{1}{2}((m_\pi - m_K) \pm \sqrt{(m_\pi - m_K)^2 + 4(m_{\pi K})^2}),$$

and since π meson without external interactions cannot change its mass shell, the nondiagonal mass term in (1) is equal to the mass difference i.e.,

$$\Delta m_{12} \equiv 2m_{\pi K}. \tag{4}$$

It is necessary to remark that expression (3) can be obtained from the Breit-Wigner distribution [7]

$$P \sim \frac{(\Gamma/2)^2}{(E - E_0)^2 + (\Gamma/2)^2} \tag{5}$$

by using the following substitutions:

$$E = m_K, \ E_0 = m_\pi, \ \Gamma/2 = 2m_{\pi K}, \tag{6}$$

where Γ is width of $\pi \leftrightarrow K$ transitions.

If the mass matrix contains masses in squared form, then oscillations (or mixings) will be described by the expressions (3)-(6) with the following substitutions:

$$m_\pi \rightarrow m_\pi^2, m_K \rightarrow m_K^2, m_{\pi K} \rightarrow m_{\pi K}^2.$$

Here the two cases of π, K oscillations take places [4]: real and virtual oscillations.

1. If we consider the real transition of π into K mesons, then

$$sin^2 2\beta \cong \frac{4m_{\pi K}^2}{(m_\pi - m_K)^2} \cong 0, \tag{7}$$

i.e. the probability of the real transition of π mesons in K mesons through a weak interaction is very small since $m_{\pi K}$ is very small.

How can we understand this real $\pi \rightarrow K$ transition?

If $2m_{\pi K} = \frac{\Gamma}{2}$ is not zero, then it means that the mean mass of π meson is m_π and this mass is distributed by $sin^2 2\beta$ (or by the Breit-Wigner formula) and the probability of the $\pi \rightarrow K$ transition differs from zero. So, this is a solution of the problem of origin of mixing angle in the theory of vacuum oscillation.

In this case the probability of $\pi \rightarrow K$ transition (oscillation) is described by the following expression (see also Exp. (23):

$$P(\pi \rightarrow K, t) = sin^2 2\beta sin^2 \left[\pi t \frac{m_K^2}{2p}\right], \tag{8}$$

where p is momentum of π meson.

2. If we consider the virtual transition of π into K meson then, since $m_\pi = m_K$,

$$tg2\beta = \infty,$$

i.e. $\beta = \pi/4$, then

$$sin^2 2\beta = 1. \tag{9}$$

In this case probability of $\pi \rightarrow K$ transition (oscillation) is described by the following expression:

$$P(\pi \rightarrow K, t) = sin^2 \left[\pi \frac{L}{L_{osc}}\right], \tag{10}$$

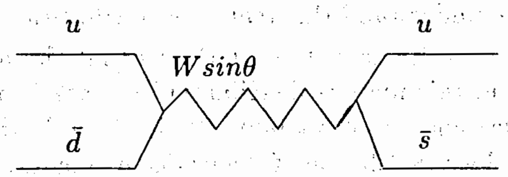
where $L = vt$, v- is velocity of π meson, at $v \cong c$ $L \cong ct$,

$$L_{osc} = \frac{2.48 p_\pi (MeV)}{|m_1^2 - m_2^2| (eV^2)} m.$$

Let us pass to a more detailed consideration of the second case since it is of real interest (i.e. we will compute nondiagonal term of the mass matrix).

3 The $\pi \overset{W sin\theta}{\leftrightarrow} K$ Meson Transitions (Oscillations) in the Standard Theory of Weak Interaction

When d, s quark mixings and W exchange are taken into account the diagram for $\pi \xrightarrow{W \sin\theta} K$ transitions has the form



It is clear that at d, s mixings the transition of π meson mass shell does not take place, i.e. K meson produced from π meson remains on the mass shell of π meson.

The amplitude of this process has the following form (we use Feynman rules):

$$M(\pi \to K) = \frac{G_F}{\sqrt{2}} sin\theta [\bar{d}\gamma_\mu(1-\gamma_5)u][\bar{s}\gamma^\mu(1-\gamma_5)u],$$

or

$$M(\pi \to K) = \frac{G_F}{\sqrt{2}} sin\theta [\bar{d}Q_\mu u][\bar{s}Q^\mu u], \tag{11}$$

where G_F is Fermi constant, $Q_\mu = \gamma_\mu(1-\gamma_5)$ and

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}.$$

The mass Lagrangian L for this diagram in the framework of the standard approach is [5]

$$L = M(\pi \to K). \tag{12}$$

Then the mass differences in square which response for $\pi \to K$ and $K \to \pi$ transitions are

$$m_1^2 - m_2^2 = <\pi \mid L \mid K> + <K \mid L \mid \pi> \tag{13}$$

(we suppose that K meson is on the mass shell of π meson). Therefore

$$m_1^2 - m_2^2 \simeq 2m_\pi \Delta m_{12} \tag{14}$$

or

$$\Delta m_{12} = \frac{1}{2m_\pi}[<\pi \mid L \mid K> + <K \mid L \mid \pi>]. \tag{15}$$

Now we compute mass difference. For this purpose we use the following expressions:

$$\begin{gathered} <0 \mid \bar{d}Q_\mu u \mid \pi> \phi_\pi f_\pi p_\mu, \\ <0 \mid \bar{s}Q^\mu u \mid K> = \phi_K f_K p^\mu, \end{gathered} \tag{16}$$

where ϕ_π, ϕ_K , f_π, f_K, correspondingly are the wave functions and the constant decays of π and K mesons, p_μ is momentum of π meson.

It is necessary to remark that the following relation appears for constant decays on mass shells

$$f_\pi(m_\pi) = f_K(m_\pi). \tag{17}$$

Then from equation (13) using equations (16), (17) we obtain the following expression:

$$\Delta m^2 = m_1^2 - m_2^2 = f_\pi^2 m_\pi^2 \frac{G_F}{\sqrt{2}} sin\theta. \tag{18}$$

or

$$\Delta m_{12} = f_\pi^2 m_\pi \frac{G_F}{\sqrt{2}} sin\theta. \tag{19}$$

4 Probability of $\pi \overset{W sin\theta}{\longleftrightarrow} K$ Virtual Oscillations with account of π decays

If at $t = 0$ we have the flow $N(\pi, 0)$ of π mesons, then at $t \neq 0$ this flow will decrease because of π mesons decay and then we have the following flow $N(\pi, t)$ of π mesons:

$$N(\pi, t) = exp(-\frac{t}{\tau_0'})N(\pi, 0), \tag{20}$$

where $\tau_0' = \tau_0 \frac{E_\pi}{m_\pi}$.

The expression for the flow $N(\pi \leftrightarrow K, t)$, i.e. probability of $\pi \leftrightarrow K$ meson transitions at time t, has the following form

$$N(\pi \to K, t) = N(\pi, t) P(\pi \to K, L) \tag{21}$$

where

$$P(\pi \leftrightarrow K, L) = sin^2\left[\pi \frac{L}{L_{osc}}\right],$$

$$L_{osc} = \frac{2.48 p_\pi (MeV)}{\mid m_1^2 - m_2^2 \mid (eV^2)} m,$$

and

$$m_1^2 - m_2^2 = f_\pi^2 m_\pi^2 \frac{G_F}{\sqrt{2}} sin\theta.$$

In the approach where phase volume is taken into account the expression for the probability of $\pi \to K$ oscillations $P(\pi \to K, t)$ has the following form [4]:

$$N(\pi \leftrightarrow K, t) = N(\pi, t) sin^2\left[\frac{\pi t}{\tau(\pi \xrightarrow{W} K)}\right] =$$

$$= N(\pi, 0) exp(-\frac{t}{\tau_0}) sin^2\left[\frac{\pi t}{\tau_0} \frac{tg^2\theta}{(\frac{m_\mu}{m_u + m_d})^2}\right]. \tag{22}$$

In the case of real oscillations the probability of $\pi \to K$ transitions (oscillations) is described by Exp. (8):

$$P(\pi \leftrightarrow K, t) = sin^2 2\beta sin^2\left[\pi t \frac{m_K^2}{2p}\right],$$

where

$$sin^2 2\beta \cong \frac{\Delta m_{12}^2}{m_K^2} = \frac{\left(f_\pi^2 m_\pi \frac{G_F}{\sqrt{2}} sin\theta\right)^2}{m_K^2} \cong 0. \tag{23}$$

The kinematics of the process of K virtual mesons transition on its mass shell is given in work [4].

5 Conclusion

The elements of the theory of vacuum oscillations were given. Then the probability of real and virtual $\pi \leftrightarrow K$ transitions (oscillations) in diagram approach was calculated. The probability of real vacuum $\pi \leftrightarrow K$ transitions is very small therefore only virtual transitions are the subject of our interest. These transitions (oscillations) can be registered through K decays after transitions of virtual K mesons to their own mass shell by using their quasielastic strong interactions.

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