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ESTIMATION OF PRIMARY COSMIC RAY CHARACTERISTICS WITH THE HELP OF EAS CERENKOV LIGHT

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Оценка характеристик частиц первичного космического излучения с помощью черенковского света в широком атмосферном ливне

Предлагается новый метод для оценки характеристик частиц и фотонов первичного космического излучения, который основан на регистрации черенковского света в широком атмосферном ливне (ШАЛ). Природа, энергия и направление первичной частицы или фотона получаются как решение нелинейной обратной задачи.

Примерная математическая модель создана на основе анализа наблюдений на широкоугольном черенковском телескопе «Хотово» [1]. Энергетическая зависимость параметров модели изучена с помощью данных, полученных с использованием программы «КОРСИКА» [2] в интервале энергий 30 ГэВ — 3 ТэВ.

Предложенный метод может быть использован для анализа данных ШАЛ, полученных на установках при различном расположении детекторов. Показано, что оценки параметров могут быть получены более эффективно и точно при размещении детекторов по специально выбранной Спирали.

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Estimation of Primary Cosmic Ray Characteristics with the Help of EAS Cerenkov Light

A new method of estimating primary cosmic ray characteristics based on the registration and analysis of EAS Cerenkov light is proposed. The nature, energy and arrival direction of primaries are obtained as a solution of a nonlinear inverse problem.

The applied mathematical model is created by analyzing «Hotovo» telescope experimental data [1]. The behavior of model parameters is studied using CORSIKA code [2] for the primary energy interval 30 GeV — 3 TeV.

This method could be applied successfully for a different kind of detector displacements of EAS arrays. Moreover, it is shown that the shower parameter estimation could be obtained more effectively and precisely in the case of detectors displacement according to a Spiral.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna, 1999

1 Introduction

The gamma ray astronomy based on the Atmospheric Cerenkov Technique (ACT) has achieved remarkable experimental improvement since 1990. The importance of the EAS Cerenkov light increases due to the possibility of registrating gamma quanta initiated events, discriminating the background, generated by primary protons or other cosmic particles.

In this paper, we continue the realization of our program [3] devoted to the development of a new method of estimation of the nature, energy and arrival direction of the primaries initiating EAS Cerenkov Light at the observation level. The Cerenkov light flux is obtained with the help of CORSIKA code in energy interval 30 GeV-3 TeV for primary Gamma quanta and 0.3 - 3 TeV for primary Protons. The flux characteristics are described depending on nature, energy and arrival direction of the initiating primary with the help of code [2], using Cerenkov flux information at the observation level. The characteristics of the primary cosmic ray flux are obtained as a solution of nonlinear inverse with help of code [5, 6], using the information about EAS Cerenkov light flux at the observation level. The influence of the EAS array detector displacement on efficiency and precision of the inverse problem solution is carefully studied.

2 The Method

The lateral distribution function of Atmospheric Cerenkov Light flux (ACL) in EAS depends on the energy and the nature of primary particle(or gamma quantum) and the height of the first interaction [4]. As it is well known, near the shower axis the lateral distribution has more or less plate maximum and after hundred meters it is strongly decreasing. The Mathematical Model of the atmospheric Cerenkov light distribution could be described with a class of functions with such behavior:

$$q\left(X, \overline{x}, \overline{y}, \overline{z}, \theta, \varphi\right), \tag{1}$$

where

$$X = [x_0, y_0, a, b, c, ..., d_N]^T \in R^N \tag{2}$$

is a vector of N real unknowns, x_0 and y_0 are the coordinates of the shower axis in the detector plane,

a,b,c,...,d_N are the $N-2$ parameters of the adopted model function,

$\overline{x}$, $\overline{y}$ and $\overline{z}$ are the detector coordinates

and θ and φ are respectively the zenith and azimuth angles of the EAS.

The number of the Cerenkov photons N_q in the shower is

$$N_q = \int_0^{x_{max}} \int_0^{y_{max}} q\left(X, \overline{x}, \overline{y}, \overline{0}, \theta, \varphi\right) dx dy, \tag{3}$$

where x_{max}, y_{max} are parameters mainly depending on the height of the first interaction of the primary particle and the energy of the initiating primary particle or gamma quantum.

Different triplets $(\overline{x_i}, \overline{y_i}, \overline{z_i})$, $i = 1, ..., M$ respond to different detector coordinates. The unknown components of X are obtained by solution with the code "REGN" [5, 6] of the overdetermined system of nonlinear equations $(M > N)$

$$q\left(X, \overline{x_i}, \overline{y_i}, \overline{z_i}, \theta, \varphi\right) = q_i^{expt}, \tag{4}$$

where q_i^{expt} is the density of ACL flux, measured by the i-th detector.

The system of equations (4) is analyzed by means of the autoregularized Newton Type method [5]. The solution (the values of X) is obtaining by minimization of the functional

$$\chi^2 = \frac{1}{M-N+1} \sum_{i=1}^{M} w_i \left(q\left(X, \overline{x_i}, \overline{y_i}, \overline{z_i}, \theta, \varphi\right) - q_i^{expt} \right)^2, \tag{5}$$

where $w_1, w_2, ..., w_M$ are the weights.

In the REGN computer code, χ^2 is one of the different criteria applied to choose an appropriate mathematical model. The other criteria permits one to chose uniquely between two model functions with the same χ^2 the best function [7, 8, 9].

By examining different $q\left(X, \overline{x}, \overline{y}, \overline{z}, \theta, \varphi\right)$ in the left hand side of (4) for about two thousand real experimental EAS events registered by the Cerenkov light telescope HOTOVO [1] $q_i^{expt}, i = 1, ..7$ (right- hand side of (4)) and solving [7, 8, 9] every time the nonlinear system (4), we arrive at

$$q(X; \overline{x}, \overline{y}, \overline{z}; \theta, \varphi) \equiv q(a, r_0, \gamma, \sigma, R(\overline{x}, \overline{y}, \overline{z}; x_0, y_0, \theta, \varphi)) = \frac{e^a e^{-\frac{\ln(1+R)^2 + [\ln(1+R) - \ln(1+r_0)]^2}{2\sigma^2}}}{\sqrt{2\pi}\frac{\ln(1+R)^2 + [\ln(1+R) - \ln(1+r_0)]^2 + \gamma^2}{2\sigma}}, \tag{6}$$

where the distance R from the detector $(\overline{x}, \overline{y}, \overline{z})$ to the shower axis with the parameters $x_0, y_0, z_0 = 0, \theta$ and φ is given by [10, 11]

$$R = \sqrt{[\xi \cos(\varphi) + \eta \sin(\varphi)]^2 \cos^2\theta + [\xi \sin(\varphi) - \eta \cos(\varphi)]^2},$$
$$\xi = \overline{x} - x_0 - \overline{z} tg(\theta) \cos(\varphi), \tag{7}$$
$$\eta = \overline{y} - y_0 - \overline{z} tg(\theta) \sin(\varphi)$$

and a, r_0, γ, σ are the unknown parameters.

Replacing (6) in the integral (3) with $x_{max} = y_{max} = R_c$, we estimate the number of photons N_q. The energy of the primary particle is

$$E_q = \kappa(E_q) N_q, \tag{8}$$

if we assume a quasi-linear dependence (8) between E_q and N_q, where

$$N_q = 2\pi \cos(\theta) \int_0^{R_c} r q(a, r_0, \gamma, \sigma, r) dr \tag{9}$$

because of the axis symmetry of (8).

3 The functions $\mathbf{q}_{th}^{P,G}(\mathbf{a}_i, \mathbf{r}_{0i}, \gamma_i, \sigma_i, \mathbf{r})$

The energy behavior of the parameters $\mathbf{a}_i, \mathbf{r}_{0i}, \gamma_i, \sigma_i, i = 1, ..., M_E$, where M_E is a number of different energies at which the showers are emulated, will be investigated in coordinate system, in which the shower axes parameters are $x_0 = y_0 = \theta = \varphi = 0$. In this coordinate system the variable

$$r = R(\overline{x}, \overline{y}, \overline{z} = 0, x_0 = 0, y_0 = 0, \theta = 0, \varphi = 0) = \sqrt{\overline{x}^2 + \overline{y}^2}$$

is the distance from the shower axes to the corresponding sensor and the function (6) becomes

$$\mathbf{q}_{th}^{P,G}(\mathbf{a}_i, \mathbf{r}_{0i}, \gamma_i, \sigma_i, \mathbf{r}) = \frac{e^a e^{-\frac{\ln(1+r)^2 + [\ln(1+r) - \ln(1+r_0)]^2}{2\sigma^2}}}{\sqrt{2\pi}\frac{\ln(1+r)^2 + [\ln(1+r) - \ln(1+r_0)]^2 + \gamma^2}{2\sigma}}, i = 1, ..., M_E. \tag{10}$$

The nonlinear overdetermined system (4) becomes

$$\frac{\mathbf{q}_{th}^{P,G}(\mathbf{a}_i, \mathbf{r}_{0i}, \gamma_i, \sigma_i, \mathbf{r}_j) - \mathbf{q}_{em}^{P,G}(i, j)}{\Delta \mathbf{q}_{em}^{P,G}(i, j)} = 0, i = 1, ..., M_E, j = 1, ..., Ns, \tag{11}$$
$$E_i = \kappa_i N_{qi}, i = M_E + 1,$$

where M_E is a number of emulated energies for Gamma or Proton showers, Ns is a number of sensors. The system (12) has dimension $M_E(N_s+1)$.The functions $\mathbf{q}_{th}^{P,G}(\mathbf{a}_i, \mathbf{r}_{0i}, \gamma_i, \sigma_i, \mathbf{r}_j)$ are defined in equations (6,8). The connection between the number of photons and the energy of the primary particle is defined from equations (8,9). The emulated quasiexperimental data for photons distribution $\mathbf{q}_{em}^{P,G}(i,j)$ were obtained with code "CORSIKA" [2] in the energy interval 3 10^{10} - 3 10^{12} eV for primary Gamma and 3 10^{11} - 3 10^{12} eV for primary Proton. The errors (weights) $\Delta\mathbf{q}_{em}^{P,G}(i,j)$ include the shower development fluctuations and constant reception value 30%.

The solution of the system of equations (12) is illustrated in Fig. 3 using the approximation of the emulated data by function (10).

The energy dependence of model function parameters $\mathbf{a}_i, \mathbf{r}_{0i}, \gamma_i, \sigma_i, \kappa_i$ is given in Fig. 3.

For primary Protons the parameter $\mathbf{r}_0 \equiv 0$. This is the reason why its behaviour is not presented in the left picture of Fig. 3.

The energy dependence of the parameters $\mathbf{a}, \mathbf{r}_0, \gamma, \sigma, \kappa$ on energy E is studied with the help of the system (12)

$$\frac{X_{i,j}^{P,G} - F_i^{P,G}(B_i^{P,G}, E_j)}{\Delta F_i^{P,G}(B_i^{P,G}, E_j)} = 0, \qquad (12)$$

where the index i means $\mathbf{a}, \mathbf{r}_0, \gamma, \sigma, \kappa$, j=1, ..., M_E for, respectively Proton and Gamma primaries and the errors $\Delta F_i^{P,G}(B_i^{P,G}, E_j)$ are obtained from the solution of the system (12) (Sea Fig. 3). As a result functions (13) are obtained

$$X_i^{P,G}(E) = \sum_l B_{il}^{P,G} \ln(E)^{(l-1)}, \qquad (13)$$

where the parameters $B_{il}^{P,G}$ are solutions for every ${}_{i,j}^{P,G}$ case of (12).

Inserting results (13) in function (10) one can obtain an analytical function of energy E and distance r from the shower axis which describes the ACL flux distribution in EAS:

$$\mathbf{q}_{th}^{P,G}(E,r) = \mathbf{q}_{th}^{P,G}(\mathbf{a}(E), \mathbf{r}_0(E), \gamma(E), \sigma(E), r). \qquad (14)$$

The connection between the number of photons N_q in the shower and the energy of the primary particle E is

$$E = 2\pi \cos(\theta)\kappa(E) \int_0^{2\sigma(E)} r q_{th}^{P,G}(E,r)dr. \qquad (15)$$

Solving the system (12) with functions (14) for the only one unknown parameter E_{out}, one can test these functions describing the emulated data of shower distribution and their inputted energy spectra E_{in}. Fig. 3 illustrates the results of this test with the behaviour of the function (16)

$$E_{lin} = \frac{E_{in} - E_{out}}{E_{in}}. \qquad (16)$$

Taking into account the comparisons illustrated in the above figures it becomes clear, that the ACL flux can be described with functions (14) depending only on nature

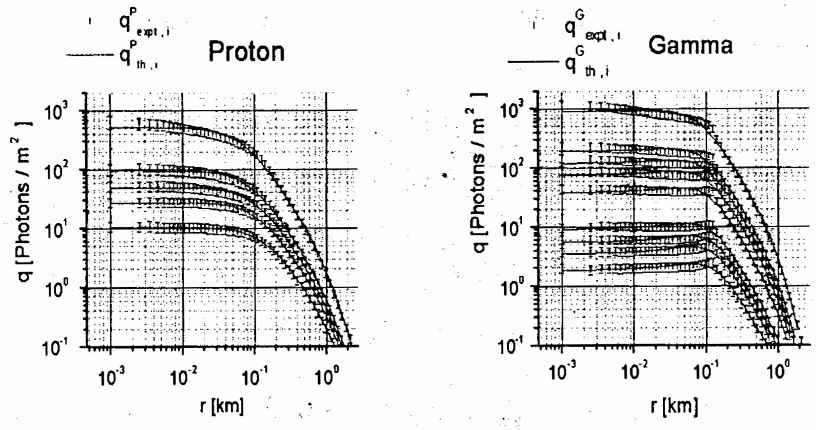

Figure 1: The correspondence between the theoretical model functions and emulated data for primary Gamma and Proton showers

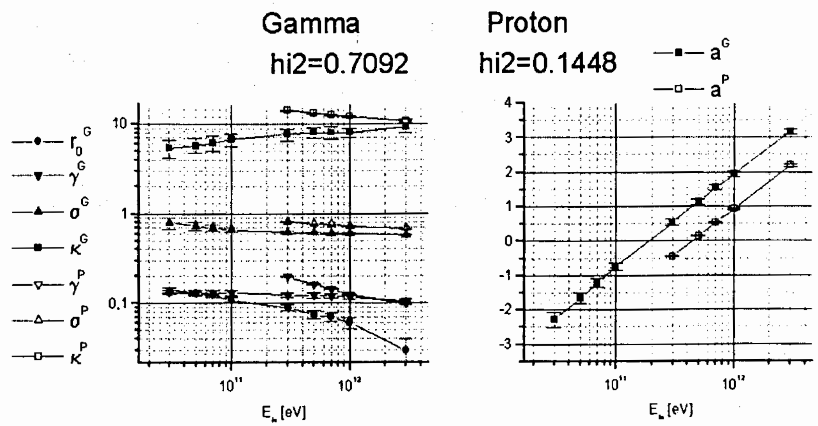

Figure 2: The energy dependence of the model function parameters for primary Gamma and Proton initiated showers

(Gamma or Proton), the energy E of the primary particle and the distance r from the axes shower.The functions (14) describes well the emulated data in the chosen energy interval and differentiates perfectly the showers initiated by Gamma or Proton.

The Fig. 3 illustrates the difference between the behaviour of primary Gamma or Proton initiated showers.

4 The functions $q_{th}^{P,G}(E, R(\overline{x}, \overline{y}, \overline{z}; x_0, y_0, \theta, \varphi))$

The substitution r→ $R(\overline{x}, \overline{y}, \overline{z}; x_0, y_0, \theta, \varphi)$ in functions (14) gives the functions (17)

$$\mathbf{q}_{th}^{P,G}(E, R(\overline{x}, \overline{y}, \overline{z}; x_0, y_0, \theta, \varphi)) \qquad (17)$$

for the Cerenkov light distribution in the coordinate system connected with the given detector displacement of the telescope.

To test the quality of the model, we solve M_E times the system

$$\frac{\mathbf{q}_{th}^{P,G}(E_i, R(\overline{x_j}, \overline{y_j}, \overline{z_j}; x_{0_i}, y_{0_i}, \theta_i, \varphi_i)) - \mathbf{q}_{em}^{P,G}(i,j)}{\Delta \mathbf{q}_{em}^{P,G}(i,j)} = 0, \qquad (18)$$

$$E_i = 2\pi \cos(\theta_i)\kappa(E_i) \int_0^{2\sigma(E_i)} r q_{th}^{P,G}(E_i, r) dr,$$

where i=1,...,M_E, j=1,...,N_s and in the right part of (18) are emulated shower mean distributions in every j'th sensor with energies E_i from the spectra, the variables $x_{0_i}, y_{0_i}, \theta_i, \varphi_i$ are random numbers from the chosen intervals for distance and angles and the errors $\Delta \mathbf{q}_{em}^{P,G}(i,j)$ include the shower development fluctuation values and constant reception value equal to 30%.

Figures 5-7 illustrate the correspondence between the inputted values for the emulation random $E_i, x_{0_i}, y_{0_i}, \theta_i, \varphi_i$ in formula (17) and the values obtained for them from the solution of the system (18) for the Gamma and Proton primeries in the set with sensors displaced at constant distance, $M_E = 520$, Ns = 81. The telescope area is 0.2×0.2 km^2 , the emulated showers parameters are from the energy intervals: $3\ 10^{10} \le E_i \le 3\ 10^{12}$ eV for primary Gamma and $3\ 10^{11} \le E_i \le 3\ 10^{12}$ eV for primary Proton, $|\ x_{0i}\ | \le 0.4$ km, $|\ y_{0i}\ | \le 0.4$ km, $0 \le \theta_i \le 12°$, $|\ \varphi_i\ | \le 180°$, $.i = 1, ..., M_E$.

The next Fig. 4 illustrates the behaviour of the functions $\Delta r_{mean}$and ΔE_{lin} in the case of initiating Gamma primaries. The last point of the graphics are the functions r_{mean} and E_{lin} for all interval of r and E correspondingly.

5 Detector displacement according to a new Spiral set

The quasiexperimental data emulated with 30% reception value of sensors acceptance, which correspond to the real uncertainties in the experiment are emulated like in the previous section, the detectors displacement being chosen to be a new Spiral one.

The Figures 9, 10 illustrate by excellent way the advance of the proposed detector displacement according to a Spiral set.

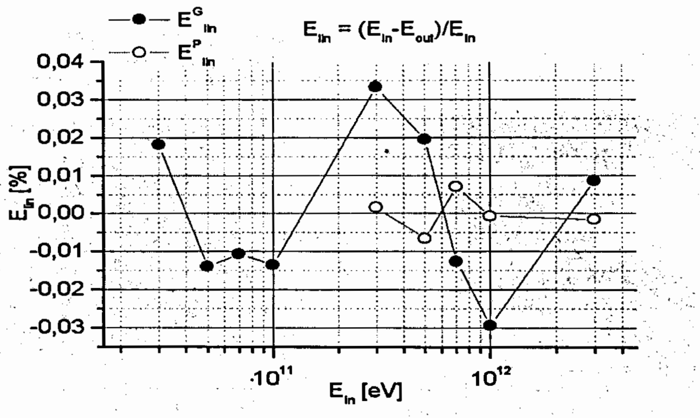

Figure 3: The correspondence between the inputted and estimated energy values for the Gamma and Proton primaries

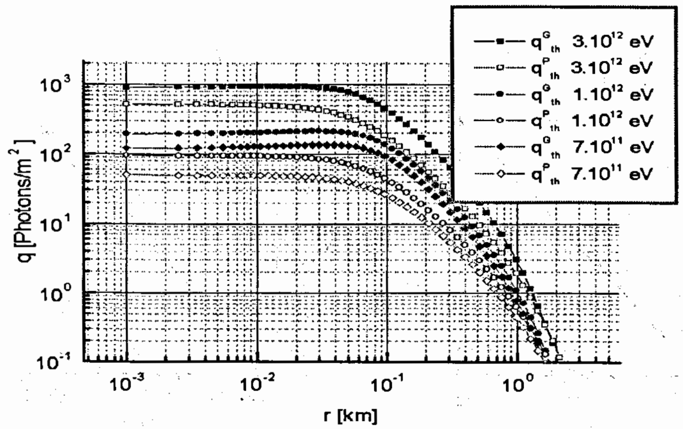

Figure 4: The comparisons of the Gamma and Proton showers behaviour

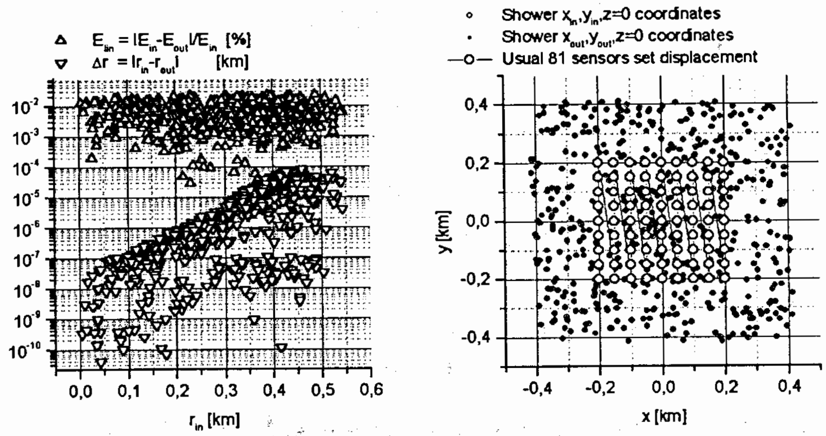

Figure 5: Primary Proton shower array with 81 detectors displaced according uniform grid. The behaviour of $|E_{in} - E_{out}|/E_{in}$, $|r_{in} - r_{out}|$, -in and -out x_0, y_0.

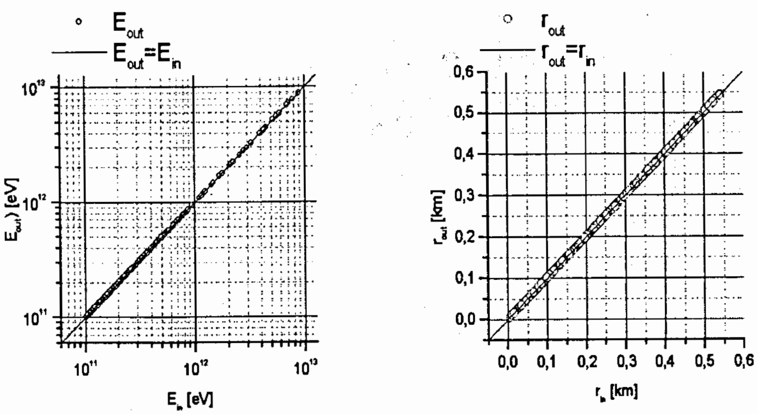

Figure 6: The correspondence E_{in},E_{out} and r_{in},r_{out}. Proton, 81 detectors

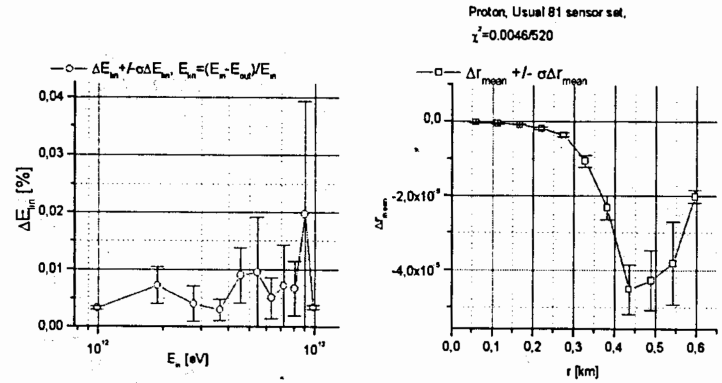

Figure 7: The behaviour of E_{lin} and Δr_{mean},Proton primary, 81 detectors

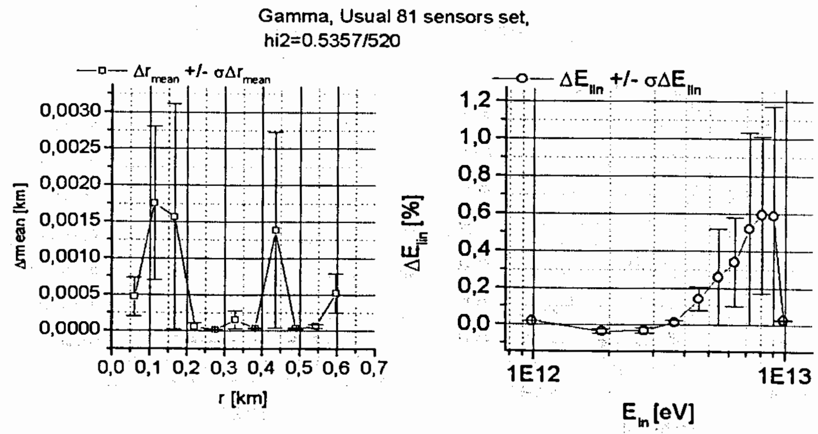

Figure 8: Primary Gamma. Shower array with 81 detectors, displaced according to a uniform grid

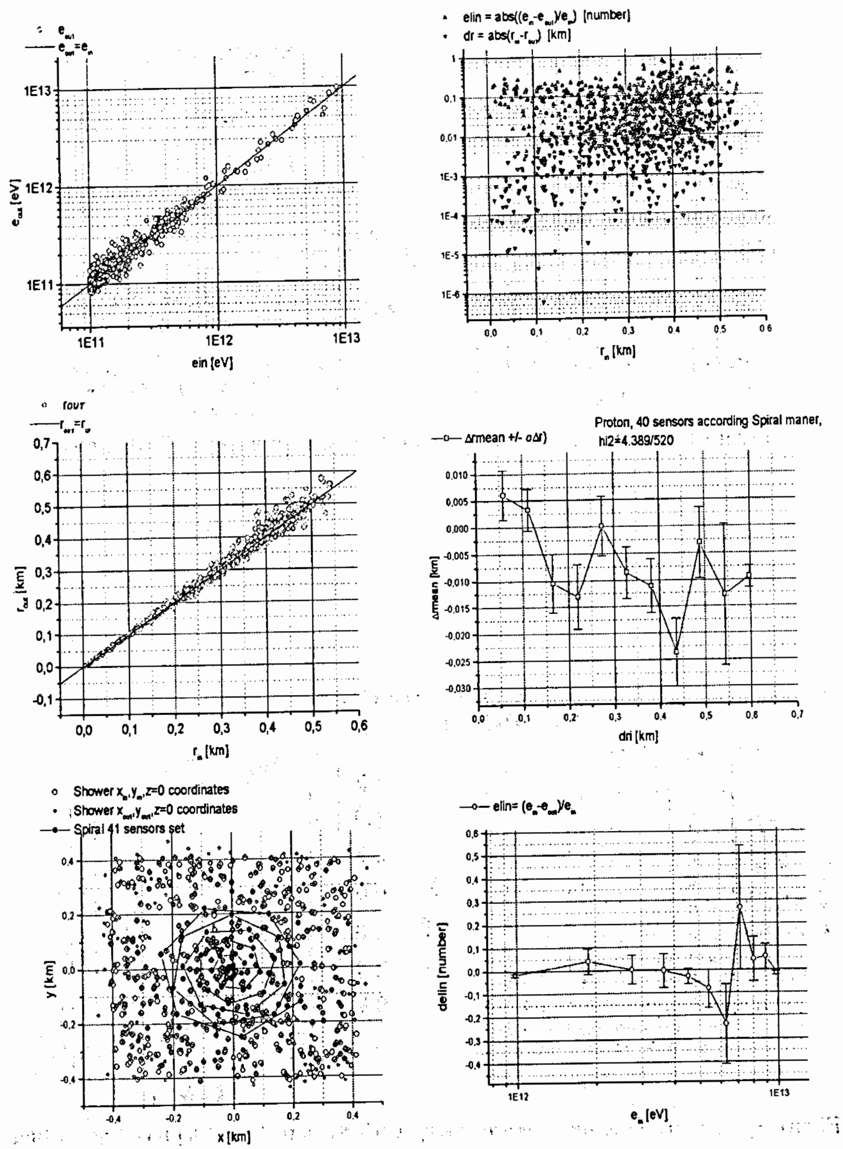

Figure 9: Primary protons, Pseudoexperimental data with constant reception value 30%, EAS array with 40 detectors, displaced according to a Spiral set

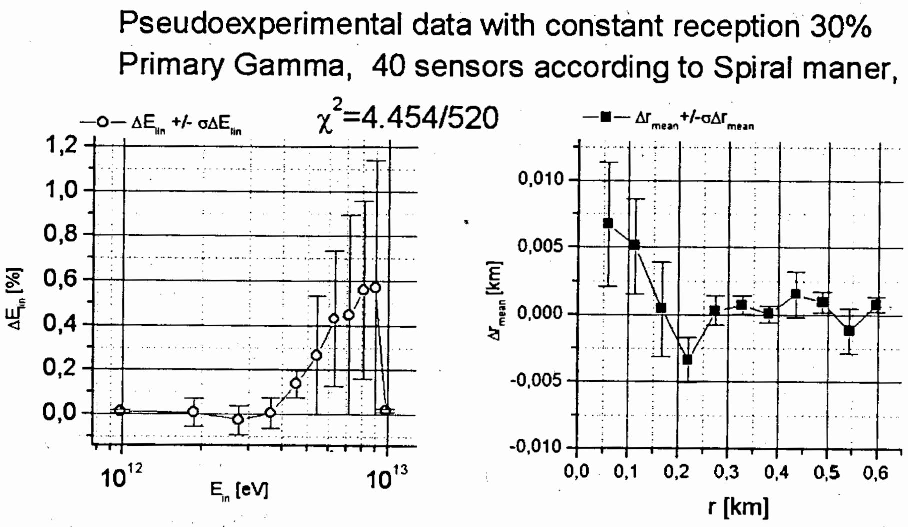

Figure 10: Primary Gamma, EAS array with 40 detectors displaced according to a Spiral set

6 Conclusion

A new method of estimating primary cosmic ray characteristics, based on the registration of Cerenkov light flux in EAS at given observation level is proposed and developed. The flux is described with the model functions defined on the basis of analysis of experimental data, obtained using the wide angle telescope "Hotovo" [1]. The energy dependencies of the function parameters were studied for Proton and Gamma primaries using the emulated data obtained with the code "CORSIKA" for the energy interval 3.10^{10}-3.10^{12} eV.

Cherenkov light lateral distribution for Gamma and Proton showers is obtained like a function only of the measured variables: the nature and energy of the primary particle and the parameters of the shower axis x_0, y_0.

It is shown that the detector displacement according to a new Spiral set permits one to estimate more precisely and effectively the shower parameters in comparison with the usual uniform displacement.

The new method can be applied for different detector arrangements of EAS arrays.

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