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PARTIAL SPONTANEOUS BREAKDOWN OF 3-DIMENSIONAL N=2 SUPERSYMMETRY

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1 Introduction

Standard mechanisms of spontaneous breaking of the global D=4, N=1 supersymmetry ($SBGS$) are connected with the constant vacuum solutions for the auxiliary components of chiral and gauge superfields (see reviews [1]-[3]). The constant $SBGS$ solutions are possible in the very restrictive class of interactions of the chiral superfields. In particular, $SBGS$ is not possible for the non-trivial self-interaction of the single chiral superfield. The Fayet-Iliopoulos (FI) mechanism consists in adding the linear term to the action of the N=1 abelian gauge theory, however, this term does not guarantee automatically the appearance of the $SBGS$-solution for any gauge-matter interaction. These standard mechanisms are not very flexible, so the search of new approaches to this problem is desirable, especially for the extended supersymmetry or supersymmetries in low dimensions which have some specific features. The problems of the spontaneous breaking of local supersymmetries will not be discussed in this paper.

The standard linear supermultiplets (standard superfields) are not convenient for the description of the partial spontaneous breaking of the extended global supersymmetries ($PSBGS$) when the invariance with respect to the part of supercharges remains unbroken. In particular, the constant solutions with a degenerate structure of the auxiliary fields are forbidden in many cases. The Goldstone-fermion models with the partial spontaneous breaking of the D=4, N=2 [4] or D=3, N=2 [5] supersymmetries have been constructed using the topologically non-trivial classical solutions preserving the one half of supercharges. These models have been also studied in the method of nonlinear realizations of supersymmetries [6]-[11] using superfields of the unbroken supersymmetry.

Recently the abelian gauge model with two FI-terms has been used to break spontaneously D=4, N=2 supersymmetry to its N=1 subgroup [12]-[14]. This model describes the non-minimal interactions of the complex scalar field with the fermion and $U(1)$-gauge fields. In the D=4, N=2 superspace these interactions correspond to the holomorphic action of the Goldstone-Maxwell (GM) chiral superfield W satisfying the modified superfield 2-nd order constraints. In comparison to the original constraints of the N=2 vector multiplet [15], these constraints contain the constant terms which guarantee the appearance of the unusual constant imaginary part of the isovector auxiliary component and the Goldstone fermion component in the GM-superfield.

The more early example of the Goldstone-type constraint has been considered in the model with the partial breaking of the D=1, N=4 supersymmetry [16]. Thus, these constraints introduce a new type of the supersymmetry representations with the linear Goldstone (LG) fermions. In distinction with the Goldstone fermions of the nonlinear realizations which transforms linearly only in the unbroken supersymmetry, the LG-fermions have their partners in the supermultiplets of the whole supersymmetry. The nonlinear deformation of the standard constraints is also possible [3], however, we shall discuss only constant terms in the modified constraints which are related with the spontaneous breaking of supersymmetries. It will be shown that the models with the LG vector multiplet and the corresponding dual scalar multiplet solve the problem of the partial spontaneous breaking of the D=3, N=2 supersymmetry. Recently these problems have been considered in the framework of the N=1 superspace [11].

The coordinates of the full D=3, N=2 superspace are

$$z = (x^{\alpha\beta}, \theta^\alpha, \bar\theta^\alpha) , \qquad (1.1)$$

where α, β are the spinor indices of the group $SL(2,R)$. The spinor representation of the x-coordinate is connected with the vector representation via the $3D$ γ-matrices $x^{\alpha\beta}$=$(1/2)x^m(\gamma_m)^{\alpha\beta}$. The algebra of spinor derivatives in this superspace has the following form:

$$\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_\beta\} = i\partial_{\alpha\beta} + i\varepsilon_{\alpha\beta}Z , \qquad (1.2)$$

$$\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = 0 , \qquad \{\bar{\mathcal{D}}_\alpha, \bar{\mathcal{D}}_\beta\} = 0 , \qquad (1.3)$$

where Z is the real central charge and

$$\mathcal{D}_\alpha = D_\alpha + \frac{i}{2}\bar\theta_\alpha Z , \qquad D_\alpha = \partial_\alpha + \frac{i}{2}\bar\theta^\beta\partial_{\alpha\beta} ,$$

$$\bar{\mathcal{D}}_\alpha = \bar D_\alpha - \frac{i}{2}\theta_\alpha Z , \qquad \bar D_\alpha = \bar\partial_\alpha + \frac{i}{2}\theta^\beta\partial_{\alpha\beta}. \qquad (1.4)$$

We shall use mainly the spinor derivatives without the central charge D_α and $\bar D_\alpha$.

The corresponding generators of the N=2 supersymmetry are

$$\mathcal{Q}_\alpha = Q_\alpha + \frac{1}{2}\bar\theta_\alpha Z , \qquad \bar{\mathcal{Q}}_\alpha = \bar Q_\alpha - \frac{1}{2}\theta_\alpha Z . \qquad (1.5)$$

The N=2 supersymmetry algebra is covariant with respect to the $U_R(1)$ transformations

$$\theta^\alpha \rightarrow e^{i\rho}\theta^\alpha , \qquad \bar\theta^\alpha \rightarrow e^{-i\rho}\bar\theta^\alpha . \qquad (1.6)$$

We shall consider the following notation for the bilinear combinations of spinor coordinates and differential operators:

$$(\theta)^2 = \frac{1}{2}\theta_\alpha\theta^\alpha , \quad (\bar\theta)^2 = \frac{1}{2}\bar\theta^\alpha\bar\theta_\alpha , \qquad (1.7)$$

$$(\theta\bar\theta) = \frac{1}{2}\theta^\alpha\bar\theta_\alpha , \qquad \Theta^{\alpha\beta} = \frac{1}{2}[\theta^\alpha\bar\theta^\beta + \alpha \leftrightarrow \beta] , \qquad (1.8)$$

$$(D)^2 = \frac{1}{2}D^\alpha D_\alpha , \qquad (\bar D)^2 = \frac{1}{2}\bar D_\alpha \bar D^\alpha , \qquad (1.9)$$

$$(D\bar D) = \frac{1}{2}D^\alpha\bar D_\alpha , \qquad D_{\alpha\beta} = \frac{1}{2}([D_\alpha, \bar D_\beta] + \alpha \leftrightarrow \beta) , \qquad (1.10)$$

and the useful relations

$$D_\alpha\bar D_\beta = \frac{i}{2}\partial_{\alpha\beta} + \varepsilon_{\alpha\beta}(D\bar D) + \frac{1}{2}D_{\alpha\beta} , \qquad (1.11)$$

$$(D\bar D)^2 = \frac{1}{8}\partial^{\alpha\beta}(\partial_{\alpha\beta} - iD_{\alpha\beta}) + \frac{1}{2}(D)^2(\bar D)^2 , \qquad (1.12)$$

$$D_{\alpha\beta}(D)^2 = i\partial_{\alpha\beta}(D)^2 , \qquad (1.13)$$

$$(D)^2(\theta)^2 = 1 , \qquad (\bar D)^2(\bar\theta)^2 = 1 , \qquad (D\bar D)(\theta\bar\theta) = -\frac{1}{2} . \qquad (1.14)$$

The integration measures in the full and chiral superspaces are

$$d^7z = d^3x(D)^2(\bar D)^2 , \qquad d^5\zeta = d^3x_L(D)^2 . \qquad (1.15)$$

They have R-charges 0 and -2, respectively. The complex chiral coordinates can be constructed by the analogy with D=4

$$\zeta = (x_L^{\alpha\beta}, \theta^\alpha) , \qquad x_L^{\alpha\beta} = x^{\alpha\beta} + i\Theta^{\alpha\beta} . \qquad (1.16)$$

It is convenient to use the following rules of conjugation for any operators [3]:

$$(XY)^\dagger = Y^\dagger X^\dagger , \qquad [X,Y\}^\dagger = -(-1)^{p(X)p(Y)}[X^\dagger, Y^\dagger\} , \qquad (1.17)$$

where $[X,Y\}$ is the graded commutator and $p(X) = \pm 1$ is the Z_2-parity. The action of the differential operator X on some function $f(z)$ and the corresponding conjugation are defined as follows:

$$Xf \equiv [X,f\} \Rightarrow (Xf)^\dagger = -(-1)^{p(X)p(f)}X^\dagger f^\dagger . \qquad (1.18)$$

(Remark that the alternative convention of conjugation $(Xf)^\dagger = (-1)^{p(X)p(f)}X^\dagger f^\dagger$ is also possible.)

Consider the conjugation rules for the spinor coordinates and derivatives

$$(\theta^\alpha)^\dagger = \bar\theta^\alpha , \quad [(\theta)^2]^\dagger = (\bar\theta)^2 , \quad (\theta\bar\theta)^\dagger = -(\theta\bar\theta) , \qquad (1.19)$$

$$D_\alpha^\dagger = \bar D_\alpha , \quad [(D)^2]^\dagger = (\bar D)^2 , \quad (D\bar D)^\dagger = -(D\bar D) . \qquad (1.20)$$

It is possible to introduce the real N=2 spinor coordinates $\theta_i^\alpha = (\theta_i^\alpha)^\dagger$

$$\theta^\alpha = \frac{1}{\sqrt2}(\theta_1^\alpha + i\theta_2^\alpha) , \qquad \bar\theta^\alpha = \frac{1}{\sqrt2}(\theta_1^\alpha - i\theta_2^\alpha) , \qquad (1.21)$$

$$(\theta)^2 = \frac{1}{2}[(\theta_2\theta_2) - (\theta_1\theta_1) - 2i(\theta_1\theta_2)] , \qquad (\theta_i\theta_k) \equiv \frac{1}{2}\theta_i^\alpha\theta_{k\alpha} = (\theta_k\theta_i) , \qquad (1.22)$$

$$(\theta\bar\theta) = \frac{1}{2}[(\theta_1\theta_1) + (\theta_2\theta_2)] , \qquad (\theta_i\theta_k)^\dagger = -(\theta_i\theta_k) , \qquad (1.23)$$

$$\Theta^{\alpha\beta} = \frac{i}{2}(\theta_2^\alpha\theta_1^\beta + \alpha \leftrightarrow \beta) , \qquad (\Theta^{\alpha\beta})^\dagger = \Theta^{\alpha\beta} \qquad (1.24)$$

and the corresponding real spinor derivatives

$$\mathcal{D}_\alpha^1 = D_\alpha^1 + \frac{1}{2}\theta_{2\alpha}Z , \qquad \mathcal{D}_\alpha^2 = D_\alpha^2 - \frac{1}{2}\theta_{1\alpha}Z , \qquad (1.25)$$

$$D_\alpha^1 = \frac{1}{\sqrt2}(D_\alpha + \bar D_\alpha) , \qquad D_\alpha^2 = \frac{i}{\sqrt2}(D_\alpha - \bar D_\alpha) , \qquad (1.26)$$

$$\{D_\alpha^1, D_\beta^1\} = \{D_\alpha^2, D_\beta^2\} = i\partial_{\alpha\beta} , \qquad \{D_\alpha^1, D_\beta^2\} = 0 . \qquad (1.27)$$

The D=3, N=2 gauge theories have been considered, for instance, in refs.[17]-[20]. The non-minimal self-interaction of the $U(1)$ gauge supermultiplet in this case is equivalent to the interaction of the $3D$ linear multiplet. We shall analyse the modified LGM-constraints for the $3D$ gauge multiplet. The corresponding real N=2 superfield describes the Goldstone fermions interacting with the scalar and vector fields.

In sect.4 we discuss the prepotential solution for the LGM supermultiplet which contains additional terms manifestly depending on the spinor coordinates and some complex constants playing the role of moduli in the vacuum state of the theory together with the constant of the FI-term. Using this representation in the non-minimal gauge action one can obtain the constant vacuum solutions with the partial spontaneous breaking of the D=3, N=2 supersymmetry. Note that the supersymmetry algebra is modified on the LGM prepotential V by analogy with the similar modified transformations of the $4D$ gauge fields or prepotentials in refs.[13, 14].

The sect.5 is devoted to the description of $PSBGS$ in the interaction of the LG-chiral superfield which is dual to the interaction of the LGM superfield. This manifestly supersymmetric action depends on the sum of the chiral and antichiral superfields and some constant term bilinear in the spinor coordinates. The non-usual transformation of the basic LG-chiral superfield satisfies the supersymmetry algebra with the central-charge term.

The N=1 supermembrane and $D2$-brane actions [11] can be analysed in our approach using the decompositions of N=2 superfields in the 2-nd spinor coordinate θ^α_2. In sect.6, we consider the N=1 components of the extended superfields and the covariant conditions which allow us to express the additional degrees of freedom in terms of the Goldstone superfields.

2 Vector multiplet in D=3, N=2 supersymmetry

The D=3, N=2 gauge theory [17, 18, 19, 20] is analogous to the well-known D=4, N=1 gauge theory, although the three-dimensional case has some interesting peculiarities which are connected with the existence of the topological mass term and duality between the $3D$-vector and chiral multiplets. We shall consider the basic superspace with Z=0.

The abelian $U(1)$-gauge prepotential $V(z)$ possesses the gauge transformation

$$\delta V = \Lambda + \bar{\Lambda} \;, \tag{2.1}$$

where the chiral and anti-chiral parameters are considered

$$\bar{D}_\alpha \Lambda = 0 \;, \qquad D_\alpha \bar{\Lambda} = 0 \;. \tag{2.2}$$

The D=3, N=2 vector multiplet is described by the real linear superfield

$$W(V) = i(D\bar{D})V \tag{2.3}$$

satisfying the basic constraints

$$(D)^2 W = (\bar{D})^2 W = 0 \;. \tag{2.4}$$

The additional useful relations for this superfield have the following form:

$$D_\alpha(D\bar{D})W = -\frac{i}{2}\partial_{\alpha\beta}D^\beta W \;, \tag{2.5}$$

$$(D\bar{D})^2 W = \frac{1}{8}\partial^{\alpha\beta}(\partial_{\alpha\beta} - iD_{\alpha\beta})W \;. \tag{2.6}$$

The components of the vector multiplet can be calculated as the θ=0 parts of basic superfields and their spinor derivatives

$$\varphi(x) = W|_0 = i(D\bar{D})V|_0 \;, \quad \lambda_\alpha(x) = (D_\alpha W)|_0 \;, \tag{2.7}$$

$$\bar{\lambda}_\alpha(x) = -\bar{D}_\alpha W|_0 \;, \qquad A_{\alpha\beta}(x) = D_{\alpha\beta}V)|_0 \;, \tag{2.8}$$

$$F_{\alpha\beta}(x) = D_{\alpha\beta}W|_0 \;, \qquad G(x) = i(D\bar{D})W|_0 \;, \tag{2.9}$$

where $A_{\alpha\beta}$ and $F_{\alpha\beta}$ are the $3D$-vector field and its field-strength , G is the real auxiliary component and φ, λ and $\bar{\lambda}$ are the physical scalar and spinor fields. The scalar field appears as the $3D$ analog of the 3-rd component of the $4D$ gauge field.

The low-energy effective action of the $3D$ vector multiplet describes a non-minimal interaction of the real scalar field with the fermion and gauge fields. For the $U(1)$ gauge superfield V this action has the following general form:

$$S(W) = -\frac{1}{2}\int d^7z H(W) \;, \qquad \tau(W) = H''(W) > 0 \;, \tag{2.10}$$

where $H(W)$ is the real convex function of W. Note that the action $S(W)$ conserves the $U_R(1)$ invariance.

The interesting feature of the $3D$ gauge theory is the existence of the Chern-Simons term [17]

$$S_{CS} = \frac{ik}{4}\int d^7z V(D\bar{D})V \;, \tag{2.11}$$

where k is some constant. The component form of this action contains the topological gauge term $\int d^3x A_{\alpha\beta}\partial^\alpha_\gamma A^{\gamma\beta}$. Note that the non-abelian generalization of this term has been constructed in ref.[18].

The $3D$ linear multiplet is dual to the chiral multiplet ϕ. The Legendre transform describing this duality is

$$S[B, \Phi] = -\frac{1}{2}\int d^7z[H(B) - \Phi B] \;, \tag{2.12}$$

where B is the real unconstrained superfield and Φ=$\phi + \bar{\phi}$. Varying the Lagrange multipliers ϕ and $\bar{\phi}$ one can obtain the constraints (2.4).

Using the solution of the algebraic B-equation

$$H'(B) \equiv f(B) = \Phi \tag{2.13}$$

one can pass to the self-interaction of the chiral superfields

$$B \;\Rightarrow\; B(\Phi) = f^{-1}(\Phi) \;, \tag{2.14}$$

$$S(\Phi) = -\frac{1}{2}\int d^7z\hat{H}(\Phi)\ , \tag{2.15}$$

$$\hat{H}(\Phi) = H[B(\Phi)] - \Phi B(\Phi)\ , \tag{2.16}$$

$$\frac{\partial\Phi}{\partial B} = \tau(B)\ , \qquad \frac{\partial^2\hat{H}}{\partial\Phi^2} \equiv \hat{\tau} = -\frac{1}{\tau}\ . \tag{2.17}$$

The corresponding superfield equation of motion is

$$(\bar{D})^2\hat{H}'(\Phi) = \hat{\tau}(\Phi)(\bar{D})^2\bar{\phi} + \frac{1}{2}\hat{\tau}'(\Phi)\bar{D}_\alpha\bar{\phi}\bar{D}^\alpha\bar{\phi} = 0\ . \tag{2.18}$$

This chiral model describes the special case of the Kähler supersymmetric σ-model which is completely determined by the real function H and possesses by construction the additional abelian isometry

$$\phi \rightarrow \phi + i\beta\ , \tag{2.19}$$

where β is some real parameter.

3 Difficulties with spontaneous breaking of supersymmetry

Let us consider the spontaneous breaking of supersymmetry in the non-minimal gauge model (2.10) with the additional linear FI-term

$$S_{FI} = \frac{1}{2}\xi\int d^7zV\ , \tag{3.1}$$

where ξ is a constant of the dimension -1. Varying the superfield V one can derive the corresponding superfield equation of motion

$$-i(D\bar{D})H'(W) + \xi = -i\tau(W)(D\bar{D})W - \frac{i}{2}\tau'(W)D^\alpha W\bar{D}_\alpha W + \xi = 0\ , \tag{3.2}$$

where

$$\tau(W) = H''(W)\ , \qquad \tau'(W) = H'''(W)\ . \tag{3.3}$$

The spinor derivatives of this superfield equation generate the component equations of motion of different dimension

$$D_\alpha(D\bar{D})H' = -\frac{i}{2}\tau\partial_{\alpha\beta}D^\beta W + \frac{1}{2}\tau' D_\alpha W(D\bar{D})W$$
$$+\frac{1}{2}\tau' D^\beta W(D_{\alpha\beta} + \frac{i}{2}\partial_{\alpha\beta})W - \frac{1}{4}\tau''\bar{D}_\alpha W D^\beta W D_\beta W - \frac{1}{2}\tau'\bar{D}_\alpha W(D)^2W = 0\ , \tag{3.4}$$

where the last term vanishes due to the constraint (2.4). The vacuum solutions can be analysed with the help of the equation

$$(D\bar{D})^2H'(W) = 0\ . \tag{3.5}$$

We shall study the constant solutions of the equation of motion using the following vacuum ansatz:

$$V_0 = 2i(\theta\bar{\theta})a - 2(\theta)^2(\bar{\theta})^2G\ , \qquad W_0 = a + 2i(\theta\bar{\theta})G\ , \tag{3.6}$$

where a and G are constants. The lowest vacuum components of the equations (3.2) and (3.5) read

$$G\tau(a) - \xi = 0\ , \tag{3.7}$$

$$G^2\tau'(a) = 0\ . \tag{3.8}$$

The non-trivial solution $G_0 \neq 0$ is possible for the quadratic function H only.

It is useful to consider the real $3D$ spinors $\lambda_i^\alpha = (\lambda_i^\alpha)^\dagger$

$$\lambda^\alpha = \frac{1}{\sqrt{2}}(\lambda_1^\alpha + i\lambda_2^\alpha) \tag{3.9}$$

and the corresponding real spinor parameters of the N=2 supersymmetry

$$\epsilon^\alpha = \frac{1}{\sqrt{2}}(\epsilon_1^\alpha + i\epsilon_2^\alpha)\ . \tag{3.10}$$

It is clear that the constant solution (3.6) can only break spontaneously *both* supersymmetries

$$\delta_\epsilon\lambda^\alpha = iG_0\bar{\epsilon}^\alpha\ , \qquad \delta_\epsilon\bar{\lambda}^\alpha = -iG_0\epsilon^\alpha \tag{3.11}$$

since it generates two real Goldstone fermions.

Thus, the full spontaneous breakdown of the N=2 supersymmetry is possible only for the free theory with the $W^2(V)$-interaction and the FI term. The partial spontaneous breaking is forbidden if one uses the vector multiplet satisfying the standard constraint (2.4).

Let us estimate the role of the Chern-Simons term (2.11) in the vacuum equations. Varying the action $S(W) + S_{CS} + S_{FI}$ one can obtain the modified equation of motion

$$-i(D\bar{D})H' + kW + \xi = 0\ . \tag{3.12}$$

This superfield equation produces the following modified vacuum equations:

$$G\tau(a) - ka - \xi = 0\ , \tag{3.13}$$

$$G^2\tau'(a) - kG = 0\ . \tag{3.14}$$

The scalar potential of this model is

$$\mathcal{V}_k(a) = \frac{1}{2\tau(a)}(\xi + ka)^2\ . \tag{3.15}$$

For the arbitrary function $\infty > \tau(a) > 0$ this potential has the unique manifestly supersymmetric minimum $a = -\xi k^{-1}$. Thus, even the free $SBGS$ solution $\tau = const$ disappears in the presence of the CS-term.

4 Partial spontaneous breakdown of supersymmetry

We shall define the modified Goldstone-type constraints for the $3D$ vector multiplet by the analogy with refs.[16, 14] and show that the partial spontaneous breaking of the $D=3$, $N=2$ supersymmetry is possible for the non-trivial gauge interaction in the framework of this approach.

Consider the following deformation of the constraints (2.4):

$$(D)^2\hat{W} = C\,, \qquad (\bar{D})^2\hat{W} = \bar{C}\,, \tag{4.1}$$

where C and $\bar{C}$ are some constants. These relations break manifestly the $U_R(1)$ invariance.

The solution of these constraints can be constructed by analogy with Eq.(2.3)

$$\hat{W} = i(D\bar{D})V + (\theta)^2 C + (\bar{\theta})^2\bar{C}\,. \tag{4.2}$$

This LGM superfield contains new constant auxiliary structures which change radically the matrix of the vacuum fermion transformations

$$\delta_\epsilon\lambda^\alpha = -C\epsilon^\alpha + iG_0\bar{\epsilon}^\alpha\,, \tag{4.3}$$
$$\delta_\epsilon\bar{\lambda}^\alpha = -iG_0\epsilon^\alpha - \bar{C}\bar{\epsilon}^\alpha\,. \tag{4.4}$$

It is evident that the $PSBGS$-condition corresponds to the degeneracy of these transformations

$$C\bar{C} - G_0^2 = 0\,. \tag{4.5}$$

In this case one can choose the single real Goldstone spinor field as some linear combination of λ_i^α. For the case of the pure imaginary constant C the LG fermion can be identified with λ_2^α.

It should be stressed that the shifted quantity $W(V) = i(D\bar{D})V$ in Eq.(4.2) is not a standard superfield

$$\delta_\epsilon\hat{W} = i\epsilon_k^\alpha Q_\alpha^k\hat{W} \;\rightarrow\; \delta_\epsilon W(V) = C\epsilon^\alpha\theta_\alpha - \bar{C}\bar{\epsilon}^\alpha\bar{\theta}_\alpha + i\epsilon_k^\alpha Q_\alpha^k W(V)\,, \tag{4.6}$$

The algebra of these transformations is not changed on the gauge-invariant superfield

$$[\delta_\eta,\delta_\epsilon]W(V) = \epsilon_k^\alpha\eta_l^\beta\{Q_\alpha^k, Q_\beta^l\}W(V)\,. \tag{4.7}$$

The transformation of the LGM-prepotential V will be considered in the end of this section.

The action of the LGM-superfield (4.2) has the following form:

$$\hat{S}(V) = -\frac{1}{2}\int d^7z[H(\hat{W}) - \xi V] \tag{4.8}$$

and depends on three constants ξ, C and $\bar{C}$.

The non-derivative terms in the component Lagrangian

$$\frac{1}{2}(G^2 - |C|^2)\tau(\varphi) - \xi G\,, \tag{4.9}$$

produce the following scalar potential

$$\mathcal{V}(\varphi) = \frac{1}{2}[|C|^2\tau(\varphi) + \xi^2\tau^{-1}(\varphi)]\,. \tag{4.10}$$

The vacuum equations of this model are

$$G\tau(a) - \xi = 0\,, \tag{4.11}$$
$$(G^2 - |C|^2)\tau'(a) = 0\,. \tag{4.12}$$

The $PSBGS$ solution (4.5) arises for the non-trivial interaction $\tau'(a)\neq 0$. This solution determines the minimum point a_0 of the model

$$\tau(a_0) = \frac{|\xi|}{|C|}\,. \tag{4.13}$$

The vacuum auxiliary field can be calculated in the point a_0

$$G_0 = \frac{\xi}{\tau(a_0)} = \pm|C|\,. \tag{4.14}$$

Using the $U_R(1)$ transformation one can choose the pure imaginary constant $C \rightarrow c = i|c|$ (without the loss of generality) then

$$G_0 = -ic = |c|\,. \tag{4.15}$$

This choice corresponds to the following decomposition of the LGM-superfield (4.2)

$$\hat{W} = W_s(V_s) + 2i|c|(\theta_2\theta_2)\,, \tag{4.16}$$
$$W_s(V_s) = \frac{i}{4}(D^{1\alpha}D_\alpha^1 + D^{2\alpha}D_\alpha^2)V_s\,. \tag{4.17}$$

where V_s is the shifted LGM-prepotential which has the vanishing vacuum solution for the auxiliary component. It is evident that this representation breaks spontaneously the 2-nd supersymmetry only.

Let us consider now the supersymmetry transformations of W_s and V_s

$$\delta_\epsilon W_s = i\epsilon_k^\alpha Q_\alpha^k\hat{W} = -2i|c|\epsilon_2^\alpha\theta_{2\alpha} + i\epsilon_k^\alpha Q_\alpha^k W_s\,, \tag{4.18}$$
$$\delta_\epsilon V_s = \Delta(\epsilon,\theta) + i\epsilon_k^\alpha Q_\alpha^k V_s\,, \tag{4.19}$$
$$\Delta(\epsilon,\theta) = 2|c|\epsilon_2^\alpha\theta_{2\alpha}(\theta_1^\beta\theta_{1\beta}) = -2\sqrt{2}|c|i\epsilon_2^\alpha[\bar{\theta}_\alpha(\theta)^2 + \theta_\alpha(\bar{\theta})^2]\,. \tag{4.20}$$

The supersymmetry algebra of the V_s-transformations is essentially modified by the analogy with the transformations of the prepotentials in refs.[13, 14]

$$\begin{aligned}[\delta_\eta,\delta_\epsilon]V_s &\equiv \epsilon_k^\alpha\eta_l^\beta\{\tilde{Q}_\alpha^k, \tilde{Q}_\beta^l\}V_s \\ &= 4|c|(\epsilon_2^\alpha\eta_1^\beta - \eta_2^\alpha\epsilon_1^\beta)\theta_{1\beta}\theta_{2\alpha} + \epsilon_k^\alpha\eta_l^\beta\{Q_\alpha^k, Q_\beta^l\}V_s\,,\end{aligned} \tag{4.21}$$

where $\tilde{Q}_\alpha^k$ are the generators of the modified transformations.

The modified part of the supersymmetry algebra of transformations has the following form:

$$\{\tilde{Q}^1_\alpha, \tilde{Q}^2_\beta\}_{mod} V_s = 4|c|\theta_{1\alpha}\theta_{2\beta} = 4i|c|\Theta_{\alpha\beta} + 2i|c|\varepsilon_{\alpha\beta}[(\theta)^2 + (\bar{\theta})^2] \,. \tag{4.22}$$

It should be stressed that both terms in this anticommutator can be decomposed as a sum of chiral and anti-chiral functions and do not contribute to the Lie bracket on the superfield W_s

$$\Theta^{\alpha\beta} = -\frac{i}{2}(x^{\alpha\beta}_L - x^{\alpha\beta}_R) \,, \qquad x^{\alpha\beta}_R \equiv (x^{\alpha\beta}_L)^\dagger \,. \tag{4.23}$$

The modified anticommutator contains the additional vector and scalar generators $T_{\alpha\beta}, T$ and $\bar{T}$

$$\{\tilde{Q}^1_\alpha, \tilde{Q}^2_\beta\} = \varepsilon_{\alpha\beta}(T + \bar{T}) + T_{\alpha\beta} \,. \tag{4.24}$$

$$T_{\alpha\beta} V_s = 4i|c|\Theta_{\alpha\beta} \,, \quad T V_s = 2i|c|(\theta)^2 \,. \tag{4.25}$$

The additional generators belong to the infinite Lie algebra of the $U(1)$-gauge transformations which arises in the (x, θ)-decomposition of the chiral gauge parameters Λ (2.1). These generators vanish on the gauge invariant quantity W_s. One should also include in the modified N=2 supersymmetry algebra all nontrivial commutators of the T generators with the spinor generators $\tilde{Q}^k_\alpha$.

Consider the spinor gauge connection

$$A_\alpha(z) = D_\alpha V_s \,, \qquad \delta_\Lambda A_\alpha = D_\alpha \Lambda \tag{4.26}$$

in the chiral representation ($\bar{A}_\alpha$=0). The inhomogeneous term in the modified supersymmetry transformation of this gauge superfield has the following form:

$$\delta A_\alpha = -2\sqrt{2}i|c|[\epsilon_{2\alpha}(\bar{\theta})^2 - \theta_\alpha \epsilon^\beta_2 \bar{\theta}_\beta] + i\epsilon^\alpha_k Q^k_\alpha A_\alpha \,. \tag{4.27}$$

It should be remarked that the minimal interaction of the charged chiral superfields with the LGM-prepotential V_s breaks the supersymmetry. The analogous problem of the LGM interaction with the charged matter appears also in the $PSBGS$ model with D=4, N=2 supersymmetry [14].

5 The $3D$ chiral interaction with the partial breaking

The general effective action of the chiral superfields ϕ_i (i is some internal index) can be written as follows:

$$\int d^4x d^4\theta K(\phi_k, \bar{\phi}_k) + [\int d^4x d^2\theta P(\phi_i) + \text{c.c.}] \,, \tag{5.1}$$

where K is the Kähler potential and P is the chiral superfield potential.

The existence of the non-trivial $SBGS$ solution implies the degeneracy of the matrix $\partial_i\partial_k P$. The vacuum equation for the single chiral superfield ϕ may have the non-vanishing $SBGS$ solution only in the trivial case of the linear function $P(\phi)$ and the free Kähler potential $K = \phi\bar{\phi}$.

We shall show that the spontaneous breaking of supersymmetry is possible for the non-trivial interaction of the LG chiral superfield which possesses the inhomogeneous supersymmetry transformation. Let us consider the dual picture for the $PSBGS$ gauge model with the FI-term (4.8)

$$\hat{S}(B, \phi, \bar{\phi}) = -\frac{1}{2}\int d^7z[H(B) - B\hat{\Phi}] - \frac{1}{2}[\bar{C}\int d^5\zeta\phi + \text{c.c.}] \,, \tag{5.2}$$

where the modified constrained LG superfield is introduced

$$\hat{\Phi} \equiv \phi + \bar{\phi} + 2i\xi(\theta\bar{\theta}) \,, \qquad (D\bar{D})\hat{\Phi} = -i\xi \,. \tag{5.3}$$

Varying the chiral and antichiral Lagrange multipliers ϕ and $\bar{\phi}$ one can obtain the LGM-constraints (4.2) on the superfield B and then pass to the gauge phase $B \to W(V)$ where the $(\theta\bar{\theta})$-term in $\hat{\Phi}$ transforms to the FI-term.

The algebraic B-equation

$$H'(B) \equiv f(B) = \hat{\Phi} \,, \tag{5.4}$$

provides the transform to the 'chiral' phase

$$B \;\rightarrow\; f^{-1}(\hat{\Phi}) \equiv \hat{B}(\hat{\Phi}) \,. \tag{5.5}$$

The transformed chiral action is

$$\hat{S}(\hat{\Phi}) = -\frac{1}{2}\int d^7z\{\hat{H}(\hat{\Phi}) + [\bar{C}(\theta)^2 + \text{c.c.}]\hat{\Phi}\} \,, \tag{5.6}$$

$$\hat{H}(\hat{\Phi}) = H[\hat{B}(\hat{\Phi})] - \hat{\Phi}\hat{B}(\hat{\Phi}) \tag{5.7}$$

The linear terms with C and $\bar{C}$ break the $U_R(1)$-symmetry (1.6), however, this action is invariant with respect to the isometry transformation (2.19).

It should be underlined that the LG-superfield $\hat{\Phi}$ transforms homogeneously, while the supersymmetry transformation of the LG-chiral Lagrange multiplier ϕ contains the inhomogeneous term

$$\delta_\epsilon\phi = -i\xi(\theta^\alpha\bar{\epsilon}_\alpha) + i\epsilon^\alpha_k Q^k_\alpha\phi \,. \tag{5.8}$$

The action $\hat{S}$ is invariant with respect to the LG representation of the N=2 supersymmetry, since the 1-st term of this action depends manifestly on the covariant superfield $\hat{\Phi}$, and the 2-nd one is invariant due to the linear θ-dependence of the inhomogeneous part of $\delta_\epsilon\phi$.

Consider the θ-decomposition of the LG-chiral superfield

$$\phi = A(x_L) + \theta^\alpha\psi_\alpha(x_L) + (\theta)^2 F(x_L) \,, \tag{5.9}$$

where x_L is the complex coordinate of the chiral basis.

The Lie bracket of the modified supersymmetry transformation (5.8)

$$[\delta_\eta, \delta_\epsilon]\phi = i\xi(\epsilon^\alpha\bar{\eta}_\alpha - \eta^\alpha\bar{\epsilon}_\alpha) + \epsilon^\alpha_k\eta^\beta_l\{Q^k_\alpha, Q^l_\beta\}\phi \tag{5.10}$$

contains the composite central charge parameter corresponding to the following action of the generator Z on the chiral superfield:

$$Z\phi = \xi \ , \qquad (Z\bar{\phi} = -\xi) \ . \tag{5.11}$$

Thus, the Goldstone boson field $\mathrm{Im}\, A(x)$ for the central-charge transformation appears in this model. It should be remarked that the isometry transformation (2.19) in the chiral model without $PSBGS$ cannot be identified with the central-charge transformation.

It is interesting that we can define the deformed chiral superfield

$$\phi_\xi = \phi + i\xi(\theta\bar{\theta}) = e^{i(\theta\bar{\theta})Z}\phi \ , \qquad Z^2\phi = 0 \ ,$$
$$\bar{D}_\alpha \phi_\xi = (\bar{D}_\alpha - \frac{i}{2}\theta_\alpha Z)\phi_\xi = 0 \tag{5.12}$$

satisfying the unusual covariant condition.

The superfield equation of motion for the action (5.6)

$$(\bar{D})^2 \hat{H}'(\hat{\Phi}) + \bar{C} = 0 \tag{5.13}$$

generates the vacuum component equations

$$\bar{F}\hat{\tau}(b) + \bar{C} = 0 \ , \qquad b = A + \bar{A} \tag{5.14}$$
$$(|F|^2 - \xi^2)\hat{\tau}'(b) = 0 \ , \tag{5.15}$$
$$\hat{\tau} = \hat{H}'' = -\tau^{-1} \ . \tag{5.16}$$

The scalar potential of this model depends on the one real scalar component only

$$\mathcal{V}(b) = \frac{1}{2}[\xi^2 \hat{\tau}(b) + |C|^2 \hat{\tau}^{-1}(b)] \ . \tag{5.17}$$

The minimum point b_0 of this potential can be defined by the equation

$$\mathcal{V}' = \frac{1}{2}\hat{\tau}'(b)[\xi^2 - |C|^2\hat{\tau}^{-2}(b)] = 0 \ , \tag{5.18}$$
$$\tau^2(b_0) = \hat{\tau}^{-2} = \xi^2|C|^2 \tag{5.19}$$

using the condition $\tau'(b)\neq 0$.

The vacuum transformations of the spinor components of the LG superfields ϕ and $\bar{\phi}$ have the following form:

$$\delta_\epsilon \psi^\alpha = F_0\epsilon^\alpha - i\xi\bar{\epsilon}^\alpha \ , \tag{5.20}$$
$$\delta_\epsilon \bar{\psi}^\alpha = i\xi\epsilon^\alpha + \bar{F}_0\bar{\epsilon}^\alpha \ . \tag{5.21}$$

The vacuum solution $|F_0|^2 = \xi^2$ corresponds to the degeneracy condition for these transformations. The choice $F_0 = i\xi$ breaks the 2-nd supersymmetry.

Thus, the non-trivial interaction of the LG-chiral superfield ϕ provides the partial spontaneous breaking of the D=3, N=2 supersymmetry. This phenomenon has been analysed also in the formalism of the D=3, N=1 Goldstone-type superfields [11].

6 Passing to N=1 superfields

Let us assume that the spinor coordinates θ_1^α parameterize N=1 superspace, and the generators Q_α^1 form the corresponding subalgebra of the N=2 supersymmetry. The complex chiral coordinates ζ (1.16) can be written via the real spinor coordinates

$$x_L^{\alpha\beta} = x^{\alpha\beta} + \frac{1}{2}(\theta_1^\alpha\theta_2^\beta + \alpha \leftrightarrow \beta) \ , \tag{6.1}$$
$$\theta^\alpha = \frac{1}{\sqrt{2}}(\theta_1^\alpha + i\theta_2^\alpha) \ . \tag{6.2}$$

We shall use the relations

$$(D)^2 = \frac{1}{2}(D^1D^1) - \frac{1}{2}(D^2D^2) - i(D^1D^2) \ , \qquad (D^iD^k) \equiv \frac{1}{2}D^{i\alpha}D^k_\alpha \ , \tag{6.3}$$
$$D^1_\alpha D^1_\beta = \frac{i}{2}\partial_{\alpha\beta} + \varepsilon_{\alpha\beta}(D^1D^1) \ , \qquad \{D^1_\alpha, (D^1D^1)\} = 0 \ , \tag{6.4}$$
$$[D^1_\alpha, (D^1D^1)] = -i\partial_{\alpha\beta}D^{1\beta} \ , \qquad (D^1D^1)^2 = \frac{1}{8}\Box_3 \ , \qquad \Box_3 = \partial^{\alpha\beta}\partial_{\alpha\beta} \ . \tag{6.5}$$

The chirality condition in the real basis

$$\bar{D}_\alpha\phi = \frac{1}{\sqrt{2}}(D^1_\alpha + iD^2_\alpha)\phi = 0 \tag{6.6}$$

can be solved via the complex unrestricted N=1 superfield χ

$$\phi = \chi(x,\theta_1) + i\theta_2^\alpha D^1_\alpha\chi(x,\theta_1) + (\theta_2\theta_2)(D^1D^1)\chi(x,\theta_1) \ . \tag{6.7}$$

To prove the chirality in the N=1 representation one should use Eqs.(6.4,6.5) and the relation

$$D^2_\alpha\chi(x,\theta_1) = \frac{i}{2}\theta_2^\beta\partial_{\alpha\beta}\chi(x,\theta_1) \ . \tag{6.8}$$

Using Eq.(6.3) one can readily obtain the relation between the chiral and N=1 integrals

$$\int d^3x(D)^2\phi = \int d^3x(D^1D^1)\chi(x,\theta_1) \ , \tag{6.9}$$

where $d^2\theta_1$=(D^1D^1) is the imaginary spinor measure of the N=1 superspace.

The transformation (5.8) has the following N=1 decomposition:

$$\delta\phi = -\frac{1}{2}\xi\theta_1^\alpha(\epsilon_{2\alpha} + i\epsilon_{1\alpha}) + \frac{1}{2}\xi\theta_2^\alpha(\epsilon_{1\alpha} - i\epsilon_{2\alpha}) + \epsilon_2^\alpha(-\partial^2_\alpha + \frac{i}{2}\theta_2^\beta\partial_{\alpha\beta})\phi + i\epsilon_1^\alpha Q^1_\alpha\phi \tag{6.10}$$

and generates the corresponding transformation of the complex N=1 superfield:

$$\delta\chi = -\frac{1}{2}\xi\theta_1^\alpha(\epsilon_{2\alpha} + i\epsilon_{1\alpha}) - i\epsilon_2^\alpha D^1_\alpha\chi + i\epsilon_1^\alpha Q^1_\alpha\chi \ . \tag{6.11}$$

Consider the θ_2-decomposition of the basic superfield (5.3) of the chiral $PSBGS$ model

$$\hat{\Phi} = \chi + \bar{\chi} + i\theta_2^\alpha D^1_\alpha(\chi - \bar{\chi}) + (\theta_2\theta_2)(D^1D^1)(\chi + \bar{\chi}) + i\xi[(\theta_1\theta_1) + (\theta_2\theta_2)]$$
$$= \Sigma + \theta_2^\alpha D^1_\alpha\rho + (\theta_2\theta_2)[(D^1D^1)\Sigma + 2i\xi] \ , \tag{6.12}$$
$$\Sigma(x,\theta_1) = \chi + \bar{\chi} + i\xi(\theta_1\theta_1) \ , \qquad \rho = i\chi - i\bar{\chi} \tag{6.13}$$

where Σ is the massive real N=1 superfield and ρ is the real Goldstone superfield for the 2-nd supersymmetry

$$\delta\Sigma = -\epsilon_2^\alpha D^1_\alpha \rho + i\epsilon_1^\alpha Q^1_\alpha \Sigma \,. \tag{6.14}$$

$$\delta\rho = -i\xi\epsilon_2^\alpha \theta_{1\alpha} + \epsilon_2^\alpha D^1_\alpha \Sigma + i\epsilon_1^\alpha Q^1_\alpha \rho \,. \tag{6.15}$$

The analogous transformations of N=1 superfields have been proposed in ref.[11]. The authors of this work have shown that the additional superfield can be constructed in terms of the spinor derivative of the Goldstone superfield ρ in order to built the supermembrane action. The massive degrees of freedom in our approach can be removed using the covariant condition

$$\hat{\Phi}^2 = 0 \,, \tag{6.16}$$

which allows us to construct Σ via $D^1_\alpha \rho$ by analogy with the similar construction in the D=4, N=2 theory [10].

The superfield ρ possesses also the central-charge transformation induced by the corresponding transformation of the chiral superfield (5.11).

Our N=2 action (5.6) can be rewritten via the both N=1 components of $\hat{\Phi}$

$$-\frac{1}{2}\bar{C}\int d^3x(D)^2\phi + \text{c.c.} = \frac{1}{2}\int d^3x d^2\theta_1[(C-\bar{C})\Sigma + i(C+\bar{C})\rho] + \text{const} \,, \tag{6.17}$$

$$-\frac{1}{2}\int d^7z \hat{H}(\hat{\Phi}) = -\frac{1}{2}\int d^3x(D^1D^1)(D^2D^2)\hat{H}(\hat{\Phi})$$

$$= \frac{1}{2}\int d^3x d^2\theta_1\{[2i\xi + (D^1D^1)\Sigma]\hat{H}'(\Sigma) + \frac{1}{2}\hat{\tau}(\Sigma)D^{1\alpha}\rho D^1_\alpha\rho\} \,, \tag{6.18}$$

Note that these integrals, including the linear in ρ term, are invariant with respect to the N=2 supersymmetry transformations (6.14,6.15).

Let us analyse the N=1 decomposition of the gauge prepotential

$$V_s(x,\theta_1,\theta_2) = \kappa(x,\theta_1) + i\theta_2^\alpha V_\alpha(x,\theta_1) + i(\theta_2\theta_2)M(x,\theta_1) \tag{6.19}$$

and the chiral gauge parameter

$$\Lambda = [1 + i\theta_2^\alpha D^1_\alpha + (\theta_2\theta_2)(D^1D^1)]\lambda(x,\theta_1) \,. \tag{6.20}$$

The gauge transformations of the N=1 components are

$$\delta_\lambda\kappa = \lambda + \bar{\lambda} \,, \tag{6.21}$$

$$\delta_\lambda V_\alpha = D^1_\alpha(\lambda - \bar{\lambda}) \,, \tag{6.22}$$

$$\delta_\lambda M = -i(D^1D^1)(\lambda + \bar{\lambda}) \,. \tag{6.23}$$

Thus, κ is a pure gauge degree of freedom, V_α is the N=1 gauge superfield, and M is the scalar N=1 component of the N=2 supermultiplet.

The 2-nd supersymmetry transformations of the N=1 superfields have the following form:

$$\delta_2\kappa = -i\epsilon_2^\alpha V_\alpha \,, \tag{6.24}$$

$$\delta_2 V_\alpha = -\epsilon_{2\alpha}[M + 4i|c|(\theta_1\theta_1)] - \frac{1}{2}\epsilon_2^\beta\partial_{\alpha\beta}\kappa \,, \tag{6.25}$$

$$\tilde{\delta}_2 M = \frac{1}{2}\epsilon_2^\alpha\partial_{\alpha\beta}V^\beta \,. \tag{6.26}$$

The deformation of the supersymmetry algebra (4.22) can be studied also in this representation.

Consider the N=1 decomposition of the linear superfield (4.17)

$$W_s(V_s) = \frac{i}{2}[(D^1D^1) + (D^2D^2)]V_s = w + i\theta_2^\alpha F_\alpha(V) - (\theta_2\theta_2)(D^1D^1)w \,, \tag{6.27}$$

$$[(D^1D^1) - (D^2D^2)]W_s = 0 \,, \qquad (D^1D^2)W_s = 0 \,. \tag{6.28}$$

where the gauge-invariant scalar and spinor superfields are defined

$$w = \frac{1}{2}[M + i(D^1D^1)\kappa] \,, \tag{6.29}$$

$$F_\alpha(V) = \frac{i}{2}(D^1D^1)V_\alpha + \frac{1}{4}\partial_{\alpha\beta}V^\beta \,, \qquad D^\alpha F_\alpha = 0 \,. \tag{6.30}$$

The Goldstone transformation of W_s (4.18) produces the following ϵ_2-transformations of the N=1 superfields:

$$\delta w = -i\epsilon_2^\alpha F_\alpha \,, \tag{6.31}$$

$$\delta F_\alpha = -\epsilon_{2\alpha}[2|c| + i(D^1D^1)w] + \frac{i}{2}\epsilon_2^\alpha\partial_{\alpha\beta}w \,. \tag{6.32}$$

The spinor superfield strength F_α is analogous to the Goldstone spinor superfield of ref.[11]. It describes the Goldstone degree of freedom of the $D2$-brane, and the superfield w corresponds to the massive degrees of freedom. Our construction introduces the N=1 gauge superfield V_α as the basic object of this model and allows us to study the modification of the supersymmetry algebra on the gauge fields of the $D2$-brane. It is not difficult to rewrite the N=2 action (4.8) in terms of the N=1 superfields.

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