

# 0БЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ 

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# RHO-MESON SELF-ENERGY 

IN ROTATING MATTER

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[^0]Рассматривается пионная среда с нулевым средним значением углового момента. Сделанное приближение позволяет вычислить собственно энергетическую функцию. $\rho$-мезона при нулевом пространственном импульсе. Найдено, что присутствие среднего углового момента приводит к определенному поляризационному эффекту - неоднородному распределению диэлектронов с равным нулю полным импульсом, что является следствием различия в средних числах заполнения для векторных частиц с разными проекциями спина.

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We consider pion matter at finite temperature with nonzero net value of angular momentum. Our approach allows us to evaluate $\rho$-meson self-energy and distribution function for zero value of the external spatial momentum. We found a specific polarization effect - inhomogeneous angular distributions with zero total spatial momentum near the two pion threshold due to the difference of mean numbers of $\rho$-mesons with different spin orientation.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Introduction. The question of how the hadron properties are modified in hot and dense nuclear matter still attracts close attention of both experimenters and theoretists. The $\rho$ dynamics is crucially important here because it may be related to observables. In particular, possible modifications of the in-medium $\rho$-meson properties are believed to be seen in the spectra of lepton pairs produced in heavy ion collisions. The commonly expected phenomena are the modification of the effective mass and decay width [1-12]. Another observable is the polarization effect caused by the fact that vector particle could exhibit a different inmedium behavior being in different states of polarization [6,8,13]. The physical reason is that the matter has its own four-velocity so that one can distinguish between states polarized longitudinally or transversally with respect to direction of matter motion (equivalently, one can consider the motion of a vector particle while the matter remainss at rest).

From statistical physics we know that an equilibrated system may have (apart from the motion as a whole) a rotation. As well as in the case of motion, there is a preferable direction - the one of the angular velocity, so one could distinguish between longitudinal and transverse polarizations with respect to this direction. The basic question is, whether this state of matter could be realized experimentally, for example, in heavy ion collisions? A possible answer is that peripheral collisions may provide a large momentum transfer whose mean value could be described via introducing the rotation with some value of angular velocity [14].

In this paper, we investigate this case within the canonical approach. We consider a rotating pion matter with a nonzero value of net angular momentum projection. We expect some fraction of momentum to be carried by massive vector mesons produced in $\pi-\pi$ interactions. This would mean, in particular, that there must be some difference in numbers of vector mesons with a different value of the total angular momentum projection. Furthermore, while exists, this difference must also holds for vector mesons at rest - the mean numbers of vector mesons with a different spin orientation must also differ. As a result, one could expect an interesting in-medium effect - inhomogeneous angular distribution of lepton pairs with vanishing total spatial momenta.

Our starting point is the Hibbs statistical operator for the rotating system

$$
\begin{equation*}
\hat{\rho}=Z^{-1} e^{-\beta(\hat{H}-\mathbf{\Omega} \hat{\mathbf{J}})} \tag{1}
\end{equation*}
$$

where $\hat{H}$ is the Hamiltonian of the system, $\hat{\mathbf{J}}$ is the angular momentum operator and $\Omega$ is the angular velocity for which we take $\Omega=(0,0, \Omega)$. For a highly correlated system, the condition $R \Omega<1$ ( $R$ is the radius of the system) has to be satisfied. This condition is just a criterion for the validity of Eq. (1). It is our assumption that this statistical operator can also be applied to the system described above, at least for the small values of $\Omega$, which now plays the role of the Lagrange multiplier for the corresponding conserved charge (the angular momentum).

The model. As it has been mentioned in Introduction, we are interested in the evaluation of the mean numbers of vector mesons with a zero spatial momentum in different polarization states. These quantities are directly related to the observable back to back lepton rate. The quantity we are going to calculate is self-energy which vector particle develops while propagating in the heated matter admitting, in our case, the non-vanishing value of net angular momentum. Self-energy function contains all the information we need to estimate both mean number of vector mesons and dilepton production rate in matter. The model we use for calculations is based on the Lagrangian

$$
\begin{equation*}
\mathcal{L}=\frac{1}{2}\left(D^{\nu} \varphi\right)^{2}-\frac{1}{2} m_{\pi}^{2} \varphi^{2}-\frac{1}{4} \rho_{\mu \nu} \rho^{\mu \nu}+\frac{1}{2} m_{\rho}^{2} \rho^{2} \tag{2}
\end{equation*}
$$

where $\varphi$ is the complex charged scalar field, $\rho$ stands for the vector field with strength $\rho_{\mu \nu}=\partial_{\mu} \rho_{\nu}-\partial_{\nu} \rho_{\mu}$, and $D_{\nu}=\partial_{\nu}-i g_{\rho} \rho_{\nu}$ is the covariant derivative. To calculate thermal quantities, one can use direct way of construction, as a first step, the statistical sum of the system in the form of functional integral. All the quantities of interest may then be obtained by functional differentiation [16]. In particular, $p$-self-energy is given by the functional derivatives of the logarithm of statistical sum with respect to a bare $\rho$ propagator. However, this is rather a long program and inconvenient for the first estimation. We are going to evaluate $\rho$ selfenergy in the first non-vanishing order in the coupling constant $g_{\rho}$. All we need for our aim is the thermal pion propagator, for which we can use expressions (19) - (20), given in Appendix. Although these expressions are obtained for the neutral scalar field, we would have the same for charged pion field as long as we do not consider isotopically asymmetric matter.

Results and discussion. The calculation of $\rho$-meson self-energy is given in Appendix. The results for particles with spin projection $\lambda=$ $\pm 1,0$ are
where $\omega_{ \pm}=\omega-\Omega m, m= \pm 1$ is the energy of a particle with angular momenta projection $m$ in the frame rotating with angular velocity $\Omega$.

$$
\begin{align*}
& \Pi_{ \pm 1}\left(p_{0}\right)= g_{\rho}^{2} \frac{1}{2 \pi^{2}} \int d \omega\left(\omega^{2}-m_{\pi}^{2}\right)^{1 / 2}\left\{1+2 n_{B}(\omega)\right\}+ \\
& g_{\rho}^{2} \frac{1}{6 \pi^{2}} \int \frac{d \omega}{\omega}\left(\omega^{2}-m_{\pi}^{2}\right)^{3 / 2}\left\{\frac{\left(1+n_{B}\left(\omega_{ \pm}\right)\right)\left(1+n_{B}^{\prime}(\omega)\right)-n_{B}\left(\omega_{ \pm}\right) n_{B}(\omega)}{p_{0}-\omega_{ \pm}-\omega}-\right. \\
& \frac{\left(1+n_{B}\left(\omega_{ \pm}\right)\right)\left(1+n_{B}(\omega)\right)-n_{B}\left(\omega_{ \pm}\right) n_{B}(\omega)}{p_{0}+\omega_{ \pm}+\omega}+ \\
& \frac{n_{B}(\omega)\left(1+n\left(\omega_{ \pm}\right)\right)-n_{B}\left(\omega_{ \pm}\right)(1+n(\omega))}{p_{0}-\left(\omega_{ \pm}-\omega\right)} \\
&\left.+\frac{n_{B}\left(\omega_{\mp}\right)(1+n(\omega))-n_{B}(\omega)\left(1+n_{B}\left(\omega_{\mp}\right)\right)}{p_{0}-\left(\omega-\omega_{\mp}\right)}\right\} \\
& \Pi_{0}\left(p_{0}\right)=\Pi_{ \pm}\left(p_{0}\right)_{\Omega=0} \tag{3}
\end{align*}
$$

Having these expressions, one can obtain the quantities of interest, in particular, distribution functions for vector mesons in different polarization states and the back to back leptons rate.
a) Vector meson distribution.Let us discuss the physical meaning of expression (3). The quantity $p_{0}$, after being continued from the discrete Matsubara frequencies into the Minkowski space, is the energy of a vector particle in the rotating frame. Four terms in (3) represent the four different forward $\rho \pi \rightarrow \rho \pi$ processes, with statistical factors in numerators corresponding to different time orderings. As it has been shown by Weldon [17], their imaginary parts define a difference between probabilities of the decay $\rho \rightarrow \pi \pi(\rho \pi \rightarrow \pi)$ and backward formation process $\pi \pi \rightarrow \rho(\pi \rightarrow \rho \pi)$. To write the explicit expressions for the imaginary parts of $\Pi_{ \pm 1}$ in the laboratory frame, one should recall that energy in the laboratory system $E_{L}$ is related to the one in the rotating system $E_{R}$ by $E_{R}=E_{L}-\Omega \mathrm{J}$, so that for $\lambda= \pm 1$ one should make a replacement $p_{0} \rightarrow M \mp \Omega$, respectively, where $M$ is the experimentally measured invariant mass of the vector meson. After this remark, one can write

$$
\begin{aligned}
& I m \Pi_{ \pm}(M \mp \Omega) \equiv \operatorname{Im} \Pi_{ \pm}^{L}(M)=-g_{\rho}^{2} \frac{1}{48 \pi} M^{2}\left(1-\frac{4 m_{\pi}^{2}}{M^{2}}\right)^{\frac{3}{2}} \\
& \left\{\left[1+n_{B}(M / 2 \mp \Omega)\right]\left[\left(1+n_{B}(M / 2)\right]-n_{B}(M / 2 \mp \Omega) n_{B}(M / 2)\right\}\right.
\end{aligned}
$$

$$
\begin{align*}
& \operatorname{Im} \Pi_{0}^{L}(M)=-g_{\rho}^{2} \frac{1}{48 \pi} M^{2}\left(1-\frac{4 m_{\pi}^{2}}{M^{2}}\right)^{\frac{3}{2}} \\
& \left\{\left[1+n_{B}(M / 2)\right]\left[\left(1+n_{B}(M / 2)\right]-n_{B}(M / 2) n_{B}(M / 2)\right\}\right. \tag{4}
\end{align*}
$$

Note that the last two terms in (3), whose imaginary parts would correspond to the Landau damping in rotating frame, give no contribution at finite values of $M$. Expressions (4) can be rewritten as $[8,17]$

$$
\begin{equation*}
\operatorname{Im} \Pi_{\lambda}^{L}(M)=-M\left(\Gamma_{d}^{\lambda}-\Gamma_{f}^{\lambda}\right)=-M\left(e^{\beta(M-\lambda \Omega)}-1\right) \Gamma_{f}^{\lambda}, \tag{5}
\end{equation*}
$$

where $\Gamma_{f}^{\lambda}=\frac{g_{\rho}^{2}}{48 \pi} M\left(1-\frac{4 m_{\pi}^{2}}{M^{2}}\right)^{\frac{3}{2}} n_{B}(M / 2-\lambda \Omega) n_{B}(M / 2)$ is the thermal $\pi \pi \rightarrow \rho$ formation rate. Following Weldon [17], we can calculate the time evolution of the distribution function for vector particles at rest. The result is

$$
\begin{equation*}
n_{\lambda}(M, t)=n_{B}(M-\lambda \Omega)+C(M) e^{-\left(\Gamma_{d}^{\lambda}-\Gamma_{j}^{\hat{j}}\right) t}, \tag{6}
\end{equation*}
$$

where $C(M)$ is the time-independent constant. Equation (6) shows that distribution of the vector particles at rest with energy $M$ and spin projection $\lambda$ (with respect to the rotation axis) reaches the equilibrium distribution $n_{B}(M-\lambda \Omega)$ during the time defined by the inverse time scale $t^{-1} \sim\left(\Gamma_{d}^{\lambda}-\Gamma_{f}^{\lambda}\right) / 2$.
b)Dilepton production rate. In-medium $\rho$-meson self-energy is directly related to the observable dilepton production rate. At finite temperature the explicit expression for the differential rate of lepton pairs reads $[8,18]$ :

$$
\begin{equation*}
E_{+} E_{-} \frac{d R}{d^{3} p_{+} d^{3} p_{-}}=F(M) L_{\mu \nu} \operatorname{Im} D^{\mu \nu} n_{B}(E) \tag{7}
\end{equation*}
$$

where $p_{+}, p_{-}$are the four momenta of leptons, $E=E_{+}+E_{-}$is the total energy of the pair, $M$ is their invariant mass, $M^{2}=\left(p_{+}+p_{-}\right)^{2}$. Factor $F(M)$ is a numerical constant and $L_{\mu \nu}, I m D^{\mu \nu}$ are the leptonic tensor and imaginary part of the $\rho$ propagator,

$$
\begin{align*}
& L^{\mu \nu}=p_{+}^{\mu} p_{-}^{\nu}+p_{+}^{\nu} p_{-}^{\mu}-p_{+} p_{-} g^{\mu \nu} \\
& D^{\mu \nu}=-\frac{P_{L}^{\mu \nu}}{M^{2}-m_{\rho}^{2}-F}-\frac{P_{T}^{\mu \nu}}{M^{2}-m_{\rho}^{2}-G} . \tag{8}
\end{align*}
$$

In the above expressions, $F$ and $G$ are the scalar functions parameterizing in-medium $\rho$-meson self-energy, whose general form at finite temperature is given by:

$$
\begin{equation*}
\Pi^{\mu \nu}=F \cdot P_{L}^{\mu \nu}+G \cdot P_{T}^{\mu \nu} \tag{9}
\end{equation*}
$$

Projection tensors $P_{L, T}^{\mu \nu}$ correspond to the vector particle longitudinally and transversally polarized with respect to its spatial momentum [16].

In our case, however, these expressions must be modified. The decomposition (9) is impossible for vector meson at rest. Instead, one can decompose the spatial components of self-energy (other components vanish in the case of zero spatial momentum due to the four-transversality of self-energy) as follows

$$
\begin{equation*}
\Pi^{a b}(M)=\sum_{\lambda} e_{\lambda}^{* a} e_{\lambda}^{\dot{b}} \Pi_{\lambda}^{L}(M), \quad \Pi_{\lambda}^{L}(M) \equiv \Pi_{\lambda}(M-\lambda \Omega) \tag{10}
\end{equation*}
$$

Each term in the sum (10) corresponds to the proper spin orientation of the vector meson with respect to $\Omega=(0,0, \Omega)$.

The same decomposition is also valid for the propagator.
Another ingredient to be modified in (7) is an overall factor $n_{B}(E)=$ $[\exp (\beta E)-1]^{-1}$. The presence of this factor in (7) is necessary in order to pick up the contribution of only formation rate (see Eq. (5)). In our case this component cannot be factored out as being different for different $\lambda$.

After these remarks, the expression for dilepton production rate at zero total spatial momentum can be written as

$$
\begin{equation*}
E_{+} E_{-} \frac{d r}{d^{3} p_{+} d^{3} p_{-}}=F(M) \sum_{\lambda} L_{a b} I m D_{\lambda}^{a b} n_{B}(M-\lambda \Omega) \tag{11}
\end{equation*}
$$

where $E_{+}=E_{-}=M / 2$ and $I m D_{\lambda}^{a b}$ is an imaginary part of the propagator for the vector meson with the zero spatial momentum and spin projection $\lambda$,

$$
\begin{equation*}
\operatorname{Im} D_{\lambda}^{a b}=-e_{\lambda}^{* a} e_{\lambda}^{b} \frac{\operatorname{Im} \Pi_{\lambda}^{L}(M)}{\left(M^{2}-m_{\rho}^{2}-\operatorname{Re} \Pi_{\lambda}(M)\right)^{2}+\left(\operatorname{Im} \Pi_{\lambda}(M)\right)^{2}} \tag{12}
\end{equation*}
$$

The medium affected quantities enter both the numerator and the denominator of (12). However, the presence of $\Pi$ in the denominator produces a relatively small effect of positive pole position slift. The most valuable phenomenon changing an effective mass of the $\rho$-meson in matter
is expected to come from the change of vacuum properties, in particular, the decreasing of chiral condensate [3,4,19], which is not included in our model. Furthermore, the real part of self-energy is divergent and requires renormalization, which could produce an ambiguous result in the phenomenological model. For these reasons, we neglect any medium effects in the denominators of (12). The final expression we use for the numerical estimations reads:

$$
\begin{align*}
& E_{+} E_{-} \frac{d r}{d^{3} p_{+} d^{3} p_{-}}= \\
& \tilde{F}(M) \Gamma(M) n_{B}(M / 2)\left\{\bar{n}_{B}^{ \pm}(M / 2)\left(1+\cos ^{2} \theta\right)+n_{B}(M / 2)\left(1-\cos ^{2} \theta\right)\right\} \\
& \tilde{F}(M)=\frac{e^{4} m_{\rho}^{4}}{(2 \pi)^{6} g_{\rho}^{2} M^{3}}\left\{\left(M^{2}-m_{\rho}^{2}-\operatorname{Re} \Pi_{v a c}(M)\right)^{2}+\operatorname{Im}_{v a c}(M)^{2}\right\}^{-1} \\
& \Gamma(M)=-\frac{1}{M} \operatorname{Im} \Pi_{v a c}(M)=\frac{g_{\rho}^{2}}{48 \pi} M\left(1-\frac{4 m_{\pi}^{2}}{M^{2}}\right)^{\frac{3}{2}} \\
& \bar{n}_{B}^{ \pm}(M / 2)=\frac{1}{2}\left\{n_{B}(M / 2-\Omega)+n_{B}(M / 2+\Omega)\right\} \tag{13}
\end{align*}
$$

where $\operatorname{Re} \Pi_{v a c}(M)$ denotes the renormalized real vacuum part of selfenergy $[8,9]$ and $e$ is the electromagnetic coupling constant.

As one can see from expression (13), in the case of nonzero value of $\Omega$ the angular dependence survives. This is just a manifestation of the fact that mean numbers of vector mesons produced in matter with the nonzero total angular momentum depend on their spin orientation, so that some fraction of the total angular momentum is carried by vector mesons at rest.

To quantify this effect, it seems to be useful to estimate the asymmetry $A(\theta)$ defined as

$$
\begin{equation*}
A(\theta)=\frac{d R(\theta)-d R(0)}{d R(\pi / 2)} \tag{14}
\end{equation*}
$$

This quantity was investigated in details in [13], were it was shown that this specific in-medium effect, inhomogeneous angular distribution, may occur for the dileptons having nonzero spatial momentum. In the case under consideration, the asymmetry survives even if the total angular momentum of the pair is equal to zero. For $A(\theta=\pi / 2)$ we have

$$
\begin{equation*}
A(\pi / 2)=\frac{n_{B}(M / 2)-\bar{n}_{B}^{ \pm}(M / 2)}{n_{B}(M / 2)+\bar{n}_{B}^{ \pm}(M / 2)} \tag{15}
\end{equation*}
$$

This quantity is shown in Fig.2, where asymmetry $A(\pi / 2)$ is estimated for different values of $\Omega$ at temperature $T=100 \mathrm{MeV}$. As one can see, the asymmetry is rather small for $\Omega=10 \mathrm{MeV}$ but reaches the value of almost $15 \%$ at $\Omega=50 \mathrm{MeV}$. In all cases this effect decreases with increasing invariant mass $M$ from its maximum value near the pion threshold.

Conclusion. The angular asymmetry in the spectra of dielectrons with the vanishing total spatial momentum would be a manifestation of the presence of the net angular momentum in the hadronic matter created in heavy ion collisions. However, the important question is which values of the parameter $\Omega$ could be considered realistically. This question is currently beyond the scope of the present work and probably requires a macroscopic model of non-central collisions. At present, we concentrate on the qualitative aspect of the problem and consider the effect which might be experimentally observed. Our consideration is restricted by taking into account only the $\pi \pi$-annihilation process mostly contributing to the imaginary part of $\rho$ self-energy above the pion threshold. However, there might be other reactions producing vector mesons with masses less a $2 m_{\pi}$, for example, bremsstrahlung-like processes. They would lead to the same distribution of vector mesons as (6), which is required by both energy conservation and detailed equilibrium balance [17]. For small values of $M$ the mean numbers of transversally polarized vector mesons could differ significantly from the longitudinally polarized ones even at $\Omega$ of order of a few $M e V$. So we hope that the total dielectron rate at small $M$ might exhibit the angular asymmetry large enough to be experimentally observed.

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Appendix. Let us first obtain the expression for the free scalar field propagator in the presence of rotation. We write down the formal expression for the free scalar field operator of our system in the sphericalwave basis,

$$
\begin{align*}
& \phi\left(x_{0}, \mathbf{x}\right)=e^{i t \hat{H}^{\prime}} \phi(\mathbf{x}) e^{-i t \hat{H}^{\prime}}=\int \frac{4 \pi k^{2} d k}{(2 \pi)^{3 / 2} \sqrt{2 \omega_{k}}} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \\
& \left\{e^{i t\left(\omega_{k}-\Omega m\right)} a_{l m}^{\dagger}(k) \psi_{l m}^{*}(\mathbf{x})+e^{-i t\left(\omega_{k}-\Omega m\right)} a_{l m}(k) \psi_{l m}(\mathbf{x})\right\} \tag{16}
\end{align*}
$$

where the time evolution operator $\hat{H}^{\prime}=\hat{H}-\Omega \hat{J}_{z} ; \psi_{l m}(\mathrm{x})=$ ${ }^{(i)}{ }^{l} j_{l}(k x) Y_{l m}(\theta, \phi)$ are the spherical wave functions and we have introduced the proper creation and annihilation operators $a_{l m}^{\dagger}(k), a_{l m}(k)$, which act on the states with angular momentum $l$, its projection $m$ and energy $\omega_{k}=\sqrt{k^{2}+m_{\pi}^{2}}$. These operators are related to those for planewave states,

$$
a(\mathbf{k})=\sum_{l, m} a_{l m}(k) Y_{l m}\left(\theta_{k}, \phi_{k}\right), \quad a^{\dagger}(\mathbf{k})=\sum_{l, m} a_{l m}^{\dagger}(k) Y_{l m}^{*}\left(\theta_{k}, \phi_{k}\right),
$$

and satisfy the commutation relation:

$$
\begin{equation*}
\left[a_{l m}(k), a_{l^{\prime} m^{\prime}}^{\dagger}\left(k^{\prime}\right)\right]=\delta_{l l^{\prime}} \delta_{m m^{\prime}} \frac{1}{k^{2}} \delta\left(k-k^{\prime}\right) \tag{17}
\end{equation*}
$$

The thermal averages of their products can be evaluated in the usual manner (see e.g. [15]) by using the cyclicity of the trace operation and the commutation relation $\left[\hat{H}^{\prime}, a_{l m}(k)\right]=-\left(\omega_{k}-\Omega m\right) a_{l m}(k)$. Taking the average of (17), one obtains

$$
\begin{align*}
& <a_{l m}(k) a_{l^{\prime} m^{\prime}}^{\dagger}\left(k^{\prime}\right)>_{\beta}=n_{B}\left(\omega_{k}-\Omega m\right) \delta_{l l^{\prime}} \delta_{m m^{\prime}} \frac{1}{k^{2}} \delta\left(k-k^{\prime}\right) \\
& <a_{l^{\prime} m^{\prime}}^{\dagger}\left(k^{\prime}\right) a_{l m}(k)>_{\beta}=\left(1+n_{B}\left(\omega_{k}-\Omega m\right)\right) \delta_{l l^{\prime}} \delta_{m m^{\prime}} \frac{1}{k^{2}} \delta\left(k-k^{\prime}\right) \tag{18}
\end{align*}
$$

where $n_{B}(x)$ is Bose-Einstein statistical weight.
Now we are able to write the expression for the propagator in the imaginary time formulation in the time-momentum representation

$$
\begin{aligned}
& G\left(\mathbf{k}_{1}, \mathbf{k}_{2}, \tau\right)= \\
& \frac{1}{2 \omega_{k}} \sum_{l m}\left\{n_{B}\left(\omega_{k}-\Omega m\right) e^{|\tau|\left(\omega_{k}-\Omega m\right)} Y_{l m}^{*}\left(\theta_{k 1}, \phi_{k 1}\right) Y_{l m}\left(\theta_{k 2}, \phi_{k 2}\right)+\right. \\
& \left.\left[1+n_{B}\left(\omega_{k}-\Omega m\right)\right] e^{-\mid \tau\left(\omega_{k}-\Omega m\right)} Y_{l m}^{*}\left(\theta_{k 2}, \phi_{k 2}\right) Y_{l m}\left(\theta_{k 1}, \phi_{k 1}\right)\right\} \\
& (2 \pi)^{3} \frac{1}{k_{1}^{2}} \delta\left(k_{1}-k_{2}\right) .
\end{aligned}
$$

This expression is not quite well defined. One can see that at some values of $\omega_{k}$ and $m$ the difference $\omega_{k}-\Omega m$ could become negative producing the divergence of the sum. This is a consequence of the infinite volume limit, when the angular momentum and energy are independent quantum
numbers. In the real system of finite size such a discrepancy would never appear, so we should modify our propagator. We make this by assuming the sum to be convergent which enables us to make the replacement $\omega_{k}-\Omega m \rightarrow \omega_{k}-\Omega \partial / i \partial \phi_{k}$ and factor out $m$-containing functions. Then we have

$$
\begin{align*}
& G\left(\mathbf{k}_{1}, \mathbf{k}_{2}, \tau\right)=\frac{1}{2 \omega_{k}}\left\{n_{B}\left(\omega_{k}-\Omega \hat{l}_{z 1}\right) e^{|\tau|\left(\omega_{k}-\Omega \hat{l}_{z 1}\right)}\right. \\
& \left.+\left[1+n_{B}\left(\omega_{k}-\Omega \hat{l}_{z 2}\right)\right] e^{-|\tau|\left(\omega_{k}-\Omega \hat{l}_{z 2}\right)}\right\}(2 \pi)^{3} \delta^{3}\left(\mathbf{k}_{1}-\mathbf{k}_{2}\right), \tag{19}
\end{align*}
$$

where $k=\left|\mathbf{k}_{1,2}\right|, \hat{l}_{z 1,2}=\partial /\left(i \partial \phi_{\mathbf{k}_{1}, 2}\right)$ and we have made use of the relation $\sum_{l m} Y_{l m}\left(\theta_{k 1}, \phi_{k 1}\right) Y_{l m}^{*}\left(\theta_{k 2}, \phi_{k 2}\right)=\sin \left(\theta_{k 1}\right)^{-1} \delta\left(\theta_{k 1}-\theta_{k 2}\right) \delta\left(\phi_{k 1}-\phi_{k 2}\right)$. We also give the expression for Euclidean propagator

$$
\begin{align*}
& G\left(\mathbf{k}_{1}, \mathbf{k}_{2}, i \omega_{n}\right)=\int_{0}^{\beta} d \tau e^{i \omega_{n} \tau} G\left(\mathbf{k}_{1}, \mathbf{k}_{2}, \tau\right)= \\
& \frac{-1}{\left(i \omega_{n}-\Omega \hat{l}_{z 1}\right)^{2}-\omega_{k 1}^{2}}(2 \pi)^{3} \delta^{3}\left(\mathbf{k}_{1}-\mathbf{k}_{2}\right), \tag{20}
\end{align*}
$$

where $\omega_{n}=2 \pi n T$ are the Matsubara frequencies. Using expression for propagator (19) we can evaluate vector meson self-energy. In the lowest order, it is given by the diagrams shown in Fig.1. In our approach the first, contact diagram is not modified by the presence of nonzero $\Omega$. For the second diagram one obtains, after some algebraic manipulations, the following expression:

$$
\begin{align*}
& \Pi_{2}^{a b}\left(p_{0}, \Omega\right)=-g_{\rho}^{2} \int_{0}^{\beta} d \tau e^{-p_{0} \tau} \int \frac{d^{3} \mathbf{k}}{(2 \pi)^{3}} \frac{1}{\omega^{2}} k^{a} f(\hat{l}, \tau) k^{b}, \\
& f(\hat{l}, \tau)=N(\omega-\Omega \hat{l}) N(\omega) e^{-\tau(2 \omega-\Omega \hat{l})}+n_{B}(\omega-\Omega \hat{l}) n(\omega) e^{\tau(2 \omega-\Omega \hat{l})}+ \\
& n_{B}(\omega-\Omega \hat{l}) N(\omega) e^{-\tau \Omega \hat{l}}+N(\omega+\Omega \hat{l}) n_{B}(\omega) e^{-\tau \Omega \hat{l}} \tag{21}
\end{align*}
$$

where $\omega=\sqrt{\mathbf{k}^{2}+m_{\pi}^{2}}, \hat{l}=\partial / i \partial \phi_{k}$ and $N(\ldots)=1+n_{B}(\ldots)$.
Using the spin 1 polarization vectors $\mathbf{e}_{ \pm 1}=\mp \frac{i}{\sqrt{2}}(1, \pm i, 0)$, $\mathrm{e}_{0}=i(0,0,1)$, we can rewrite $\Pi_{2}^{a b}\left(p_{0}, \Omega\right)$ as follows:

$$
\begin{aligned}
& \Pi_{2}^{a b}\left(p_{0}, \Omega\right)=-g_{\rho}^{2} \int_{0}^{\beta} d \tau e^{-p_{0} \tau} \sum_{\lambda, \lambda^{\prime}} e_{\lambda}^{* a} e_{\lambda^{\prime}}^{b} \int \frac{d^{3} \mathbf{k}}{(2 \pi)^{3}} \frac{1}{\omega^{2}}\left(\mathbf{k e}_{\lambda}\right) f(\hat{l}, \tau)\left(\mathbf{k e}_{\lambda^{\prime}}^{*}\right)= \\
& -g_{\rho}^{2} \int_{0}^{\beta} d \tau e^{-p_{0} \tau} \sum_{\lambda} e_{\lambda}^{* a} e_{\lambda}^{b} \int \frac{d^{3} \mathbf{k}}{(2 \pi)^{3}} \frac{\mathbf{k}^{2}}{\omega^{2}} \frac{4 \pi}{3} Y_{1 \neq 1}\left(\theta_{k}, \phi_{k}\right) f(\hat{l}, \tau) Y_{1 \pm 1}\left(\theta_{k}, \phi_{k}\right)
\end{aligned}
$$

where we have used the relations $\mathbf{e}^{* \pm} \mathbf{k}=-|\mathbf{k}| \sqrt{4 \pi / 3} Y_{1 \pm 1}\left(\theta_{k}, \phi_{k}\right)$. Replacing $\hat{l}$ by its eigenvalues, integrating over angles and $\tau$ and adding the contribution from the contact diagram, one finally arrives at expression (3).


Fig.1. The Feynman diagrams that contribute to the $\rho$ self-energy in the lowest order.


Fig.2. Asymmetry $A(\pi / 2)$ as a function of the invariant mass at temperature $T=100 \mathrm{MeV}$. Solid, long-dashed and dashed lines correspond to $\Omega=50,30$ and 10 MeV , respectivly.

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