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$\pi \leftrightarrow K$ MESON TRANSITIONS (OSCILLATIONS) IN THE MODEL OF DYNAMICAL ANALOGY OF THE CABIBBO-KOBAYASHI-MASKAWA MATRICES

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1 Introduction

The vacuum oscillation of neutral K mesons is well investigated at the present time [1]. This oscillation is the result of d, s quark mixings and is described by Cabibbo-Kobayashi-Maskawa matrices [2]. The angle mixing θ of neutral K mesons is $\theta = 45^O$ since $K^o, \bar{K}^o$ masses are equal (see CPT theorem). Besides, since their masses are equal these oscillations are real ones, i.e. their transitions to each other go without suppression. The case of oscillations of two particles having the masses overlapping their widths was discussed in works [3]. Then we discussed $\pi \leftrightarrow K$ oscillations in work [4] in the framework of the standard weak interaction theory.

This work is devoted to the study of $\pi\pm \leftrightarrow K\pm$ oscillations in the model of dynamical analogy of the Cabibbo-Cobayashi-Maskawa matrices and d, s quark mixings (oscillations).

At first, we will give the general elements of the model of dynamical analogy of the Cabibbo-Cobayashi-Maskawa matrices [5], then $\pi \leftrightarrow K$ transitions (oscillations) are computed in the framework of this model. These transitions are virtual ones since masses of π and K mesons differ considerably. Then we will consider d, s quark mixings (oscillations).

Let us pass to consideration elements of the model of dynamical analogy of Cabibbo-Kobayashi-Maskawa matrices.

2 Elements of the Model of Dynamical Analogy of the Cabibbo-Kobayashi-Maskawa matrices

In the case of three families of quarks the current J^μ has the following form:

$$J^\mu = (\bar{u}\bar{c}\bar{t})_L \gamma^\mu V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \quad (1)$$

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix},$$

where V is Kobayashi-Maskawa matrix [2].

Mixings of the d, s, b quarks are not connected with the weak interaction (i.e., with $W^{\pm}, Z^{o}$ bosons exchanges). From equation (1) it is well seen that mixings of the d, s, b quarks and exchange of $W^{\pm}, Z^{o}$ bosons take place in an independent manner (i.e., if matrix V were diagonal, mixings of the d, s, b quarks would not have taken place).

If the mechanism of this mixings is realized independently of the weak interaction ($W^{\pm}, Z^{o}$ boson exchange) with a probability determined by the mixing angles $\theta, \beta, \gamma, \delta$ (see below), then this violation could be found in the strong and electromagnetic interactions of the quarks as a clear violations of the isospin, strangeness and beauty. But, the available experimental results show that there is no clear violations of the number conservations in strong and electromagnetic interactions of the quarks. Then we must connect the non-conservation of the isospins, strangeness and beauty (or mixings of the d, s, b quarks) with some type of interaction mixings of the quarks. We can do it introducing (together with the $W^{\pm}, Z^{o}$ bosons) the heavier vector bosons $B^{\pm}, C^{\pm}, D^{\pm}, E^{\pm}$ which interact with the d, s, b quarks with violation of isospin, strangeness and beauty.

We shall choose parametrization of matrix V in the form offered by Maiani [6]

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{\gamma} & s_{\gamma} \\ 0 & -s_{\gamma} & c_{\gamma} \end{pmatrix} \begin{pmatrix} c_{\beta} & 0 & s_{\beta}\exp(-i\delta) \\ 0 & 1 & 0 \\ -s_{\beta}\exp(i\delta) & 0 & c_{\beta} \end{pmatrix} \begin{pmatrix} c_{\theta} & s_{\theta} & 0 \\ -s_{\theta} & c_{\theta} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$c_{\theta} = \cos\theta, s_{\theta} = \sin\theta, \exp(i\delta) = \cos\delta + i\sin\delta. \qquad (2)$$

To the nondiagonal terms in (2), which are responsible for mixing of the d, s, b- quarks and CP-violation in the three matrices, we shall make correspond four doublets of vector bosons $B^{\pm}, C^{\pm}, D^{\pm}, E^{\pm}$ whose contributions are parametrized by four angles $\theta, \beta, \gamma, \delta$. It is supposed

that the real part of $Re(s_{\beta}\exp(i\delta)) = s_{\beta}\cos\delta$ corresponds to the vector boson $C^{\pm}$, and the imaginary part of $Im(s_{\beta}\exp(i\delta)) = s_{\beta}\sin\delta$ corresponds to the vector boson $E^{\pm}$ (the couple constant of E is an imaginary value !). Then, when $q^2 << m_W^2$, we get:

$$\tan\theta \cong \frac{m_W^2 g_B^2}{m_B^2 g_W^2},$$
$$\tan\beta \cong \frac{m_W^2 g_C^2}{m_C^2 g_W^2},$$
$$\tan\gamma \cong \frac{m_W^2 g_D^2}{m_D^2 g_W^2},$$
$$\tan\delta \cong \frac{m_W^2 g_E^2}{m_E^2 g_W^2}. \qquad (3)$$

If $g_{B^{\pm}} \cong g_{C^{\pm}} \cong g_{D^{\pm}} \cong g_{E^{\pm}} \cong g_{W^{\pm}}$, then

$$\tan\theta \cong \frac{m_W^2}{m_B^2},$$
$$\tan\beta \cong \frac{m_W^2}{m_C^2},$$
$$\tan\gamma \cong \frac{m_W^2}{m_D^2},$$
$$\tan\delta \cong \frac{m_W^2}{m_E^2}. \qquad (4)$$

Concerning the neutral vector bosons B^0, C^0, D^0, E^0, the neutral scalar bosons $B'^0, C'^0, D'^0, E'^0$and the GIM mechanism [7] we can repeat the same arguments which were given in the previous work [5].

The proposed Lagrangian for expansion of the weak interaction theory (without CP-violation) has the following form:

$$L_{int} = i\sum_i g_i(J^{i,\alpha}A^i_{\alpha} + c.c.), \qquad (5)$$

where $J^{i,\alpha} = \bar{\psi}_{i,L}\gamma^{\alpha}T\varphi_{i,L}$,

$$T = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

$$\begin{array}{ccc} i=1 & i=2 & i=3 \end{array}$$

$$\psi_{i,L} = \begin{pmatrix} u \\ c \end{pmatrix}_L, \begin{pmatrix} u \\ t \end{pmatrix}_L, \begin{pmatrix} c \\ t \end{pmatrix}_L,$$

$$\begin{array}{ccc} i=1 & i=2 & i=3 \end{array}$$

$$\varphi_{i,L} = \begin{pmatrix} d \\ s \end{pmatrix}_L, \begin{pmatrix} d \\ b \end{pmatrix}_L, \begin{pmatrix} s \\ b \end{pmatrix}_L.$$

The weak interaction carriers A^i_α, which are responsible for the weak transitions between different quark families are connected with the B, C, D bosons in the following manner:

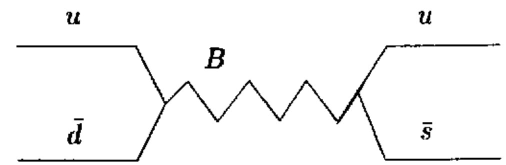
$$A^1_\alpha \to B^\pm_\alpha, A^2_\alpha \to C^\pm_\alpha, A^3_\alpha \to D^\pm_\alpha. \tag{6}$$

Using the data from [5] and equation (4) we have obtained the following masses for $B^\pm, C^\pm, D^\pm, E^\pm$ bosons:

$$\begin{aligned} m_{B^\pm} &\cong 169.5 \div 171.8 \text{ GeV}, \\ m_{C^\pm} &\cong 345.2 \div 448.4 \text{ GeV}, \\ m_{D^\pm} &\cong 958.8 \div 1794 \text{ GeV}, \\ m_{E^\pm} &\cong 4170 \div 4230 \text{ GeV}. \end{aligned} \tag{7}$$

3 The $\pi \overset{B}{\leftrightarrow} K$ Meson Transitions (Oscillations) in the Theory of Dynamical Analogy of Kabibbo-Kobayashi-Maskawa Matrices

The diagram for $\pi \xrightarrow{B} K$ transitions when one takes into account d, s quark mixings and W exchange has the form



It is clear that at d, s mixings the transition from the mass shell of π meson does not take place , i.e. K meson created from π meson remains on the mass shell of π meson.

Repeating the same calculations in [4] for $\pi \overset{B}{\leftrightarrow} K$ transitions for the probability $W(...)$ of $\pi \xrightarrow{B} K$ transitions, one obtains the following expression:

$$W(\pi \xrightarrow{B} K) \cong \frac{G^2 f_\pi^2 \left(\frac{m_W}{m_B}\right)^4 (m_u + m_d)^2 m_\pi}{8\pi} = \tag{8}$$

$$= \left(\frac{g_W^2}{4\sqrt{2} m_W^2}\right)^2 \frac{f_\pi^2 \left(\frac{m_W}{m_B}\right)^4 (m_u + m_d)^2 m_\pi}{8\pi}.$$

Then the time t of $\pi \xrightarrow{B} K$ transition is

$$\tau(\pi \xrightarrow{B} K) = \frac{1}{W(\pi \xrightarrow{B} K)}. \tag{9}$$

The relation between the time $\tau(\pi, \pi)$ of $\pi \overset{W\cos\theta}{\longrightarrow} \pi$ transition and the time $\tau(\pi, K)$ of $\pi \xrightarrow{B} K$ transition is

$$\frac{\tau(\pi \xrightarrow{B} K)}{\tau(\pi \overset{W\cos\theta}{\longrightarrow} \pi)} \cong \frac{1}{\left(\frac{m_W}{m_B}\right)^4}. \tag{10}$$

So, at the $\pi \leftrightarrow K$ transitions, $\bar{s}$ remains on the mass shell of $\bar{d}$ quark (i.e. K is on π mass shell) and then K mesons transit back into π mesons and this process goes on the background of π decays. It is clear that these oscillations (transitions) are virtual ones and can be seen through K meson decays if virtual K mesons transit to their own mass shell. Since K mesons take part in the strong interactions, one can do it through their quasiinelastic strong interactions. This problem will be considered in the next work.

Now we pass to computation of the probability of $\pi \leftrightarrow K$ oscillations.

4 Probability of $\pi \xrightarrow{B} K$ (Virtual) Oscillations

The mass matrix of π and K mesons has the form

$$\begin{pmatrix} m_\pi & 0 \\ 0 & m_K \end{pmatrix}. \tag{11}$$

Due to the presence of strangeness violation in the weak interactions, a nondiagonal term appears in this matrix and then this mass matrix is transformed in the following nondiagonal matrix:

$$\begin{pmatrix} m_\pi & m_{\pi K} \\ m_{\pi K} & m_K \end{pmatrix}, \tag{12}$$

which is diagonalized by turning through the angle β and

$$tg2\beta = \frac{2m_{\pi K}}{\mid m_\pi - m_K \mid},$$

$$sin2\beta = \frac{2m_{\pi K}}{\sqrt{(m_\pi - m_K)^2 + (2m_{\pi K})^2}}. \tag{13}$$

It is interesting to remark that expression (13) can be obtained from the Breit-Wigner distribution [8]

$$P \sim \frac{(\Gamma/2)^2}{(E - E_0)^2 + (\Gamma/2)^2} \tag{14}$$

by using the following substitutions:

$$E = m_K, \; E_0 = m_\pi, \; \Gamma/2 = 2m_{\pi K}, \tag{15}$$

where $\Gamma \equiv W(...)$.

Here take place the two cases of π, K oscillations [4]: real and virtual oscillations.

1. If we consider the real transition of π into K mesons then

$$sin^2 2\beta \cong \frac{4m^2_{\pi K}}{(m_\pi - m_K)^2} \cong 0, \tag{16}$$

i.e. the probability of the real transition of π mesons into K mesons through weak interaction is very small since $m_{\pi K}$ is very small.

How can we understand this real $\pi \to K$ transition?

If $2m_{\pi K} = \frac{\Gamma}{2}$ is not zero, then it means that the mean mass of π meson is m_π and this mass is distributed by $sin^2 2\beta$ (or by the Breit-Wigner formula) and the probability of the $\pi \to K$ transition differs from zero. So, this is a solution of the problem of origin of mixing angle in the theory of vacuum oscillation.

In this case probability of $\pi \to K$ transition (oscillation) is described by the following expression:

$$P(\pi \to K, t) = sin^2 2\beta sin^2 \left[\pi t \frac{m^2_K}{2p}\right],$$

where p is momentum of π meson.

2. If we consider the virtual transition of π into K meson then, since $m_\pi = m_K$,

$$tg2\beta = \infty,$$

i.e. $\beta = \pi/4$, then

$$sin^2 2\beta = 1. \tag{17}$$

In this case probability of $\pi \to K$ transition (oscillation) is described by the following expression:

$$P(\pi \to K, t) = sin^2 \left[\frac{\pi t}{\tau(\pi \xrightarrow{B} K)}\right].$$

Let us pass to consideration of the second case since it is of real interest.

If at $t = 0$ we have the flow $N(\pi, 0)$ of π mesons then at $t \neq 0$ this flow will decrease since π mesons decay and then we have the following flow $N(\pi,t)$ of π mesons:

$$N(\pi,t)=exp(-\frac{t}{\tau_0})N(\pi,0), \qquad (18)$$

where $\tau_0=\tau_0'\frac{E_\pi}{m_\pi}$.

One can express the time $\tau(\pi\leftrightarrow K)$ through the time of τ_0, then

$$\tau(\pi\xrightarrow{B}K)=\tau_0(\frac{m_\mu}{m_u+md})^2\frac{1}{\left(\frac{m_W}{m_B}\right)^4}. \qquad (19)$$

The expression for the flow $N(\pi\to K,t)$, i.e. probability of π to K meson transitions at time t, has the form

$$N(\pi\to K,t)=N(\pi,t)sin^2\left[\frac{\pi t}{\tau(\pi\xrightarrow{B}K)}\right]=$$

$$=N(\pi,0)exp(-\frac{t}{\tau_0})sin^2\left[\frac{\pi t}{\tau_0}\frac{\left(\frac{m_W}{m_B}\right)^4}{(\frac{m_\mu}{m_u+m_{\bar d}})^2}\right]. \qquad (20)$$

Since $\tau(\pi\to K)>>\tau_0$ at $t=\tau(\pi\to K)$ nearly all π mesons will decay, therefore to determine a more effective time (or distance) for observation of $\pi\to K$ transitions it is necessary to find the extremum of $N(\pi\to K,t)$, i.e. Eq. (20):

$$\frac{dN(\pi\to K,t)}{dt}=0. \qquad (21)$$

From Eqs. (20) and (21) one obtains the following equation:

$$\frac{2\pi tg^2\theta}{(\frac{m_\mu}{m_u+m_{\bar d}})^2}=tg\left[\frac{t\pi\left(\frac{m_W}{m_B}\right)^4}{\tau_0(\frac{m_\mu}{m_u+m_d})^2}\right]. \qquad (22)$$

If one takes into account that the argument of the right part of (22) is a very small value, one can rewrite the right part of (22) in the form

$$tg\left[\frac{t\pi\left(\frac{m_W}{m_B}\right)^4}{\tau_0(\frac{m_\mu}{m_u+m_d})^2}\right]\cong\frac{t\pi tg^2\theta}{\tau_0(\frac{m_\mu}{m_u+m_{\bar d}})^2}. \qquad (23)$$

Using (22) and (23) one obtains that the extremum of $N(...)$ takes place at

$$\frac{t}{\tau_0}\cong 2 \quad or \quad t\cong 2\tau_0. \qquad (24)$$

And the extremal distance R for observation of $\pi\to K$ oscillations is

$$R=tv_\pi\cong 2\tau_0 v_\pi, \qquad (25)$$

and the equation for $N(\pi\to K,2\tau_0)$ has the following form:

$$N(\pi\to K,2\tau_0')=N(\pi,0)exp(-2)sin^2\left[2\pi\frac{\left(\frac{m_W}{m_B}\right)^4}{(\frac{m_\mu}{m_u+m_{\bar d}})^2}\right]\cong \qquad (26)$$

$$\cong N(\pi,0)5.1\ 10^{-6},$$

where $m_u+m_{\bar d}\cong 15$ MeV, $\left(\frac{m_W}{m_B}\right)^4\cong 0.048$.

The kinematics of K meson creation processes is given in work [4].

The optimal distances for observation of $\pi\leftrightarrow K$ oscillations can be computed using Eqs.(25) and (27) from [4].

Let us pass to discussion of d,s quarks mixings and oscillations.

5 Mixings and oscillations of d and s quarks

Let us consider mixings of d,s quarks by using nondiagonal mass matrix (without CP violation)

$$\begin{pmatrix} m_d & m_{ds} \\ m_{ds} & m_s \end{pmatrix}, \qquad (27)$$

where m_d,m_s are masses of d,s quarks, m_{ds} is nondiagonal mass term.

Diagonalizing this matrix we get

$$sin2\theta=\frac{2m_{ds}}{\sqrt{(m_d-m_s)^2+4m_{ds}^2}}$$
$$tg2\theta=\frac{2m_{ds}}{|m_d-m_s|} \qquad (28)$$

$$d'=dcos\theta+ssin\theta$$
$$s'=-d\sin\theta+scos\theta. \qquad (29)$$

Experimental value of $sin\theta \equiv \sqrt{0.048}$ [1].

Now for determination of m_{ds} we can use equation (28).

a) In the framework of the considered model [5] the angle mixing θ of d, s quarks is determined by the following equation:

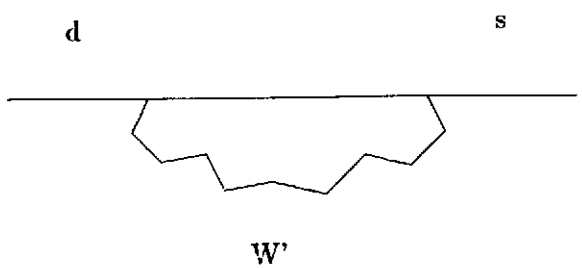
$$\tan\theta \cong \frac{m_W^2 g_B^2}{m_B^2 g_W^2} \cong \frac{2m_{ds}}{\mid m_d - m_s \mid}. \tag{30}$$

This mixing reminds ρ, ω, ϕ mixings in the strong interaction theory [1] where the problem of divergencies does not arise at high energies since at these energies works the chromodynamics but not the strong theory interactions. In our case W, B, C, D, F bosons and quarks can consist of subparticles, then at high energies will interact these subparticles but not bosons and quarks. It is obvious that then in this theory the problem of divergencies also does not arise in full analogy with the strong interactions theory.

b) In the framework of the standard theory using equation (28) we get

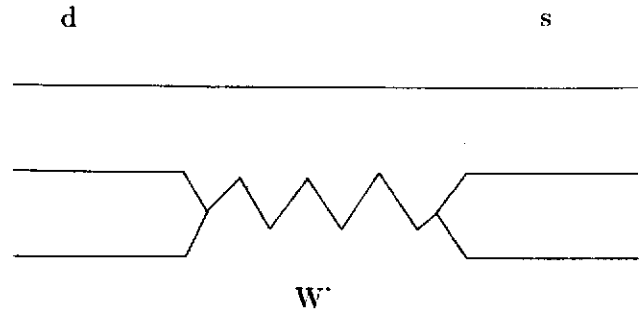
$$m_{ds} \simeq sin\theta m_s. \tag{31}$$

It is interesting to compute this angle mixing in the standard model in the framework of some consistent supposition in analogy to $K^o, \bar{K}^o$ or $\pi\pm, K\pm$ mixings. To do it we suppose that $d \leftrightarrow s$ transitions are generated through exchange of massive boson W' by the following diagram:



It is clear that this diagram is self-energy one, therefore for realization $d \leftrightarrow s$ transitions by using this diagram it is necessary to suppose that d and s quarks consist of three subquarks. Then instead of using of a concrete model we formally will use analogy with a quark model.

Let $d \to s$ transition be described by the following diagram:



then, formally, in analogy with the quark model (see Eq. (8)) we can get that

$$m_{ds} \simeq 2W(d \to s) \simeq$$

$$\simeq (G_F)^2 \frac{f'^2_\pi m_s^3}{8\pi} (\frac{m_W}{m_{W'}})^4 = m_s sin\theta' \tag{32}$$

If we even take $m_{W'} \simeq m_W$ and $f'_\pi \sim$ a few GeV we come to the following result:

$$sin\theta' = (G_F)^2 \frac{f'^2_\pi m_s^2}{8\pi} << sin\theta \simeq \sqrt{0.048} \tag{33}$$

So, we see that the angle mixing $sin\theta'$ obtained in the standard method is very small value and much less than $sin\theta$.

Probability of d to s quark transition (oscillation) is described by the following expression:

$$P(d \to s, t) = sin^2 2\theta sin^2 \left[\pi t \frac{m_s^2}{2p_d}\right].$$

where p_d is momentum of d quark.

6 Conclusion

The elements of the model of dynamical expansion of the theory of weak interaction working on the tree level, i.e. the model of dynamical analogy of Cabibbo-Kobayashi- Maskawa matrices, was given.

In the framework of this model the probability (and time) of $\pi \leftrightarrow K$ transitions (oscillations) were computed. These transitions are virtual ones since masses of π and K mesons differ considerably. These transitions (oscillations) can be registered through K decays after transitions of virtual K mesons to their own mass shell by using their quasielastic strong interactions. But for avoiding the background from inelastic K mesons, the energies E_π of π mesons must be less than the threshold energy of their creation. The optimal distances for observation of these oscillations were computed.

Mixings (oscillations) of d, s quarks were considered. It was shown that mixing angle θ' computed in the framework of the standard method is much less than experimental mixing angle θ.

REFERENCES

1. Review of Particle Prop., Phys. Rev. 1992, D45, N. 11.
2. Cabibbo N., Phys. Rev. Lett., 1963, 10, p.531.
   Kobayashi M. and Maskawa K., Prog. Theor. Phys., 1973, 49, p.652.
3. Beshtoev Kh.M., JINR Commun. E2-93-167, Dubna, 1993;
   JINR Commun. E2-95-326, Dubna, 1995;
   Chinese Journ. of Phys., 1996, 34, p.979.
4. Beshtoev Kh.M., JINR Commun. E2-98-387, Dubna, 1998;
5. Beshtoev Kh.M., JINR Commun. E2-94-293, Dubna, 1994;
   Turkish Journ. of Physics, 1996, 20, p.1245;
   JINR Commun. E2-95-535, Dubna, 1995;
   JINR Commun. P2-96-450, Dubna, 1996.
   JINR Commun. E2-97-210, Dubna, 1997.
6. Maiani L., Proc.Int. Symp. on Lepton-Photon Inter.,Hamburg, DESY, p.867.
7. Glashow S., Iliopoulos J. and Maiani L., Phys. Rev., 1970, D2, p.1285.
8. Blatt J.M., Weisscopf V.F., The Theory of Nuclear Reactions, ONT T. R. 42.

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