ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E2-99-113

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A-DEPENDENCE OF Z-SCALING

Submitted to «Journal of Modern Physics A»

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1999

1 Introduction

A study of the A-dependence of particle production in lA and hA collisions is traditionally connected with phenomena of nuclear matter influence on particle formation. The difference between the cross sections of particle production on free and bound nucleons is normally considered as an indication of new physics phenomena (EMC-effect [1, 2], J/ψ-suppression [3, 4], enhanced A-dependence in hadron production [5]). It is expected that the commissioning of such large accelerators of heavy ions as the Relativistic Heavy Ion Collider (RHIC) at Brookhaven or the Large Hadron Collider (LHC) at CERN [6, 7, 8] gives a new possibility of searching for quark-gluon plasma (QGP), exotic phase of nuclear matter [9]-[17]. The A-dependence of the cross section is usually presented in the form $\sigma = \sigma_0 \cdot A^{\alpha(x,p_\perp)}$. Here σ_0 is the cross section on free nucleon, the factor $\alpha(x,p_\perp)$ characterizes the influence of nuclear matter on the mechanism of particle formation.

Numerous experimental results on particle spectra measured in pA, and AA collisions at BNL, CERN, and Fermilab [18, 19, 20] over a wide energy and transverse momentum range show that the shapes of the distributions are not simple exponential in any representation ($p_\perp, m_\perp, m_\perp - m_0$ etc.). The deviations from a pure exponential in $m_\perp$-representation are discussed in Refs.[21, 22, 23]. The slope constants that are used to characterize the spectra depend on particle type, rapidity, centrality and the energy of collisions, too.

The A-dependence of hadron production over a high and low transverse momentum range was investigated in [5, 24] and [25, 26, 27, 28], and a nontrivial α-dependence on $x = p/p_{max}$ and $p_\perp$ was found. The A-dependence of cumulative particle production in pA collisions on H, D, Be, Al, Cu, Ta, Pb nuclei was studied in Refs.[31, 32].

As shown in Ref.[31], the ratio of the inclusive cross section for π^+-meson production $R^{A/D}(X, A) = A^{-1}\rho^\pi_A/\rho^\pi_D$ as a function of cumulative number X [26] demonstrates a strong A-dependence. The ratio was found to be practically constant over the range $X < 1$ and to reveal an exponential growth at $X > 1$. The A-dependence of the ratio $R^{A/D}$ shows an asymptotic behaviour with increasing atomic number. A similar exponential behaviour for the ratio $R^{C/D}(x) = F^C_2/F^D_2$ of deep-inelastic structure functions at $x > 1$ was found in [33].

A detailed review of the A-dependence of particle production in hA and AA collisions can be found in [24, 28, 32],[34]-[36] (see also references therein).

One of the methods to study the properties of nuclear matter is to search for the violation of scaling laws established in elementary collisions (pp, $\bar{p}p$, lp etc.). In this paper, we study the A-dependence of z-scaling for hadron ($\pi^\pm, K^\pm, \bar{p}$) production over a high $p_\perp$ range in pA collisions. The scaling was proposed in [37] to describe the feature of hadron production in pp and $\bar{p}p$ collisions. The idea of the scaling was developed for the analysis of direct photon production in pp [38], $\bar{p}p$ [39] and pA [40] collisions, too. The general concept of the scaling is based on such fundamental principles as self-similarity, locality, fractality and scale-relativity. The first one reflects the dropping of certain dimensional quantities or parameters out of a physical picture of interactions. The second principle concludes that the momentum-energy conservation law is locally valid for interacting constituents. The third fractality principle says that both the structure of interacting particles and their formation mechanism are self-similar over a kinematic range. The forth one, a scale relativity principle, states that the structures of interaction and interacting objects reveal self-similarity and fractality on any scale [41, 42].

As shown in Refs.[37, 46, 42], the experimental observables, inclusive cross section $Ed^3\sigma/dq^3$ and the multiplicity density of charged particles $dN/d\eta|_{\eta=0} = \rho(s)$, can be used to obtain a new presentation (z-presentation) of data. The scaling function $H(z)$ is found to be independent of center-of-mass energy $\sqrt{s}$ and the angle of produced particle θ over a wide kinematic range. The properties of the scaling are assumed to reflect the fundamental properties of particle structure, interaction and production. The scaling function $H(z)$ describes the probability to form the hadron with formation length z. The existence of the scaling means that the hadronization mechanism of particle production reveals such fundamental properties as self-similarity, locality, fractality and scale-relativity.

The A-dependence of z-scaling for hadron $(\pi^\pm, K^\pm, \bar{p})$ production in pA collisions is studied in this present paper. The symmetry property of the scaling function under the scale transformation $z \to \alpha z$, $\psi \to \alpha^{-1}\psi$ is used to determine the A-dependence of transformation parameter α. It is shown that α depends on the atomic number only. The properties of z-scaling for particle production in pA collisions are used to predict the particle yields at RHIC energies.

The paper is organized as follows. A general concept of z-scaling and the method of constructing the scaling function for the $p + A \to h + X$ process is described in Section 2. New results on the A-dependence of z-scaling for $\pi^\pm, K^\pm, \bar{p}$ particle production on nuclei from D up to Pb based on the analysis of the experimental data, discussion of the obtained results, physical interpretation of the scaling function and the variable z are presented in Section 3. Conclusions are summarized in Section 4.

2 Z-scaling

In this section, we would like to remember the basic ideas of z-scaling dealing with the investigation of the inclusive process

$$P_1 + P_2 \to q + X. \tag{1}$$

The momenta and masses of the colliding nuclei and the inclusive particle are denoted by P_1, P_2, q and M_1, M_2, m_1, respectively. In accordance with Stavinsky's ideas [26], the gross features of the inclusive particle distributions for reaction (1) at high energies can be described in terms of the corresponding kinematic characteristics of the exclusive subprocess written in the symbolic form

$$(x_1M_1) + (x_2M_2) \to m_1 + (x_1M_1 + x_2M_2 + m_2). \tag{2}$$

The parameter m_2 is introduced in connection with internal conservation laws (for isospin, baryon number, and strangeness). The x_1 and x_2 are the scale-invariant fractions of the incoming four-momenta P_1 and P_2 of the colliding objects. The energy of the parton subprocess defined as

$$\hat{s}_x^{1/2} = \sqrt{(x_1P_1 + x_2P_2)^2} \tag{3}$$

represents the center-of-mass energy of the constituents involved in the collision. In accordance with a space-time picture of hadron interactions at the parton level, the cross section for the production of the inclusive particle is governed by a minimum energy of colliding partons

$$d\sigma/dt \sim 1/\hat{s}^2_{min}(x_1, x_2). \tag{4}$$

The corresponding energy $\hat{s}^{1/2}_{min}$ is fixed as a minimum of Eq. (3) which is necessary for the production of the secondary particle with mass m_1 and four-momentum q. Below we present a scheme from which a more general structure of the variables x_1 and x_2 follows. We would like to emphasize two main points of this approach. The first one is a fractal character of the parton content of the involved composite structures. The second one is based on the self-similarity of the mechanism underlying the particle production at the level of elementary constituent interactions. Both points will be discussed in the other sections.

2.1 Momentum fractions x_1 and x_2

The elementary parton-parton collision is considered as a binary subprocess which is satisfied to the condition

$$(x_1P_1 + x_2P_2 - q)^2 = (x_1M_1 + x_2M_2 + m_2)^2. \tag{5}$$

The equation expresses the 4-momentum conservation law for an elementary subprocess. Relationship between x_1 and x_2 is written in the form

$$x_1x_2 - x_1\lambda_2 - x_2\lambda_1 = \lambda_0, \tag{6}$$

where

$$\lambda_1 = \frac{(P_2q) + M_2m_2}{(P_1P_2) - M_1M_2}, \qquad \lambda_2 = \frac{(P_1q) + M_1m_2}{(P_1P_2) - M_1M_2}, \qquad \lambda_0 = \frac{0.5(m_2^2 - m_1^2)}{(P_1P_2) - M_1M_2}. \tag{7}$$

Considering process (2) as a parton-parton collision, we introduce the coefficient Ω which connects kinematic and dynamic characteristics of the interaction. The coefficient is chosen in the form

$$\Omega(x_1, x_2) = m(1 - x_1)^{\delta_1}(1 - x_2)^{\delta_2}, \tag{8}$$

where m is the mass constant and δ_1 and δ_2 are the factors relating the fractal structure of the colliding objects [42]. We define the fractions x_1 and x_2 to maximize the value of $\Omega(x_1, x_2)$, simultaneously fulfilling condition (6)

$$\frac{d\Omega(x_1, x_2)}{dx_1} = 0. \tag{9}$$

The prominent expression for $x_{1,2} = x_{1,2}(\lambda_0, \lambda_1, \lambda_2, \delta_1, \delta_2)$ was obtained in Ref.[42]. A physical interpretation of the coefficient Ω and the factors δ_2 and δ_1 of is given in Sec. IV.

Equation (7) satisfies the 4-momentum conservation law in the whole phase space. The variables $x_{1,2}$ are equal to unity along the phase space limit and cover the full phase space accessible at any energy. The restriction $\lambda_1+\lambda_2+\lambda_0 \le 1$ can be obtained from the condition $x_i \le 1$. The inequality corresponds to the threshold condition

$$(M_1+M_2+m_2)^2+E^2-m_1^2 \le (\sqrt{s_A}-E)^2. \tag{10}$$

Here $\sqrt{s_A}$ and E are the energy in the center-of-mass of the reaction and the energy of produced inclusive particle. The inequality (10) bounds kinematically the maximum energy E of the inclusive particle m_1 in the c.m.s. of reaction (1).

2.2 Scaling variable z and scaling function $\psi(z)$

In accordance with the self-similarity principle, we search for the solution depending on a single scaling variable z in the form

$$\psi(z) \equiv \frac{1}{<N>\sigma_{inel}}\frac{d\sigma}{dz}. \tag{11}$$

Here σ_{inel} is the inelastic cross section, $<N>$ is the average multiplicity. The function $\psi(z)$ has to be dependent on the scaling variable z. All the quantities refer to pA interactions. The function ψ is expressed via the invariant differential cross section for the production of the inclusive particle m_1 and introduced as follows (see Ref.[42])

$$\psi(z) = -\frac{\pi s A}{\rho_A \sigma_{inel}} J^{-1} E\frac{d\sigma}{dq^3}, \tag{12}$$

where, the factor J is given by

$$J = \frac{\partial y}{\partial \lambda_1}\frac{\partial z}{\partial \lambda_2} - \frac{\partial y}{\partial \lambda_2}\frac{\partial z}{\partial \lambda_1}. \tag{13}$$

Here s is the square of the center-of-mass energy of the corresponding NN-system and A is the atomic number. Expression (12) relates the inclusive differential cross section and the average multiplicity density $\rho_A(s,\eta) = d<N>/d\eta$ to the scaling function $\psi(z)$. As usual, the combination $y = 0.5\ln(\lambda_2/\lambda_1)$ is approximated to (pseudo)rapidity η at high energies.

We choose z as a physically meaningful variable which could reflect self-similarity (scale invariance) as a general pattern of hadron production in accordance with the ansatz suggested in Ref.[42]

$$z = \frac{\sqrt{\hat{s}_\perp}}{\Omega \cdot \rho(s)}, \tag{14}$$

where $\hat{s}_\perp^{1/2}$ is the transverse kinetic energy of subprocess (2), defined by the expression $\hat{s}_\perp^{1/2} = \hat{s}_\lambda^{1/2} + \hat{s}_\chi^{1/2} - m_1 - (M_1x_1+M_2x_2+m_2)$; Ω is the fractal measure given by Eq.(8) and $\rho(s) = dN/d\eta|_{\eta=0}$ is the average multiplicity density of charged particles produced in the central region of the corresponding nucleon-nucleon interaction.

The transverse energy consists of two parts

$$\hat{s}_\lambda^{1/2} = \sqrt{(\lambda_1P_1+\lambda_2P_2)^2}, \qquad \hat{s}_\chi^{1/2} = \sqrt{(\chi_1P_1+\chi_2P_2)^2}, \tag{15}$$

which represent the transverse energy of the inclusive particle and its recoil, respectively. We would like to note that the form of z, as defined by Eqs. (14) and (8),

determines its variation range. The boundaries of the range are 0 and ∞. These values are scale independent and kinematically accessible at any energy.

2.3 Fractality and scale-relativity

Let us focus our attention on some symmetry properties of z-scaling construction. As noted above, Eq.(6) describes the 4-momentum conservation law for the elementary subprocess. The equation is a covariant one under the scale transformation

$$\lambda_{1,2} \to \rho_{1,2}\cdot\lambda_{1,2}, \qquad x_{1,2} \to \rho_{1,2}\cdot x_{1,2}, \qquad \lambda_0 \to \rho_1\cdot\rho_2\cdot\lambda_0. \tag{16}$$

The scale parameters, $\rho_{1,2}$, are chosen in accordance with the type of the collisions. This is reasonable for the description of the pp, pA and AA interaction to use $\rho_1 = 1, \rho_2 = 1$ and $\rho_1 = 1, \rho_2 = A_2$ and $\rho_1 = A_1, \rho_2 = A_2$, respectively. Here A_1 and A_2 are the corresponding atomic numbers. The scale transformation allows us to consider the collision of complex objects in terms of a suitable subprocess of interacting elementary constituents. The choice of the elementary subprocess suitable to the problem is important for the development of a microscopic scenario of the collision.

The coefficient Ω connects the kinematic with dynamic characteristics of the interaction. The factors δ_1 and δ_2 are related to the fractal structure of the colliding objects. (The fractal dimension of the nucleus was found to be expressed via the fractal dimension of the nucleon, $\delta_A = A\cdot\delta_N$ [42].) The fractal structure itself is defined by the structure of interacting constituents which is not an elementary one either. In this scheme, hadron-hadron, hadron-nucleus and nucleus-nucleus collisions are considered as an interaction of two fractals. The measure of the interaction is written as

$$\Omega = V^\delta, \tag{17}$$

where δ is the coefficient (fractal dimension) describing the fractal structure of the elementary collision. The factor V is part of the full phase-space of fractions $\{x_1, x_2\}$ corresponding to such parton-parton collisions in which the inclusive particle can be produced. The fractal property of the collision reveals itself so that only the part of all multiscatterings corresponding to the phase space V^δ produces the inclusive particle.

The full phase-space is determined by the condition $\{0 \le x_1, x_2 \le 1.\}$ Equation (7) represents the kinematic restriction on binary parton-parton collisions in which the inclusive particle with momentum q can be produced. The volume V of the full phase-space, accessible for the production of the inclusive particle given by the values of kinematic variables P_1, P_2, and q, is approximately written as $V = (1-x_1)(1-x_2)$.

In the case of collisions of asymmetric objects, the approximation for the measure Ω is written as follows

$$\Omega = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2} = (1-\bar{x}_1)^{\bar{\delta}_1}(1-\bar{x}_2)^{\bar{\delta}_2}. \tag{18}$$

The equation shows the correlation between the fraction x_i and fractal dimension δ_i. (The scale transformation can be chosen so that $\bar{\delta}_1 = \bar{\delta}_2$.) The measure is the invariant under the simultaneity of the scale transformation of Lorenz invariants x_i and multiplicative transformation of δ_i.

Physically, the asymmetry $\delta_1 \neq \delta_2$ is due to a richer structure content of the studied objects (hadron, nucleus etc.) in comparison with the structure of probe (lepton, hadron etc.).

The other method to describe the asymmetry of incoming objects is to use the parameter which is the ratio of the corresponding fractal dimensions $\alpha = \delta_2/\delta_1$ [42]. The parameter describes a relative resolution of one fractal structure with respect to the other one and plays the role of a parameter which labels single scale-inertial frames [41, 42]. Constructing a self-consistent and self-similar solution of the problem, we have to rely on special relativity which yields the limitation of any velocity, $v/c < 1$. It gives the restriction on any scales $\alpha > 0$ [42]. Besides the laws of motion, the relativity principle applies to the laws of scale [41], as well. It states that the laws of Nature are identical in all scale-inertial systems. In the considered perspective, the principle of scale relativity states that the Einstein-Lorenz composition law of velocities applies to the systems of reference whatever their state of scale [41].

3 A-dependence of z-scaling in pA-collisions

Two sets of experimental data for hadron production in pA collisions obtained at Fermilab [5] and Protvino [24] were analyzed. The first set includes the results of measurements of the invariant cross section $Ed^3\sigma/dq^3$ for the production of hadrons $(\pi^\pm, K^\pm, \bar{p})$ at a large transverse momentum $q_\perp$ of 200, 300 and 400 GeV protons incident on the H, D, Be, Ti and W targets. The measurements were made at a laboratory angle of 77 mrad, which corresponds to angles near 90^0 in the c.m. system of the incident proton and a single nucleon at rest. The $q_\perp$ range for the data is $0.77 < q_\perp < 6.91$ GeV/c. The second one was obtained at the accelerator U70 using a proton beam with an incident momentum of 70 GeV/c. The D, C, Al, Cu, Sn and Pb nuclear targets were used. The produced hadrons were registered in the transverse momentum range $q_\perp = 0.99 - 4.65$ GeV/c and at an angle θ_{NN} of 90^0.

Here, we describe the procedure of z-analysis of the data [5],[24]. The function ψ is calculated for every nucleus using Eq.(12). The normalization factor $\sigma^{pA}_{inel}/\sigma^{pp}_{inel}$ [43] was used instead of σ_{inel} in the expression for the inclusive cross section. The factor σ^{pA}_{in} is the total inelastic cross section for pA interactions. To construct the function $\psi(z)$ for particle production in pA interactions, it is necessary to know the values of the average multiplicity density $\rho_A(s,\eta)$ of secondaries produced in pA collisions. The relevant multiplicity densities of charged particles were obtained by the Monte Carlo generator HIJING [44, 45] for different nuclei ($A = 27 - 197$) and parametrized [46] by the formula

$$\rho_A(s) \simeq 0.67 \cdot A^{0.18} \cdot s^{0.105}, \qquad A \geq 2. \tag{19}$$

The obtained results for $\rho_A(s)$ are in good agreement with the experimental data [47]. The scaling functions for different nuclei obtained in this manner reveal an energy and angular independence. The symmetry transformation

$$z \to \alpha \cdot z, \qquad \psi \to \alpha^{-1} \cdot \psi \tag{20}$$

of the function $\psi(z)$ and the argument z was used to compare the functions ψ for different nuclei. Figure 1(a) shows the dependence of the function ψ on z for

π^+-meson produced in pA collisions at an incident proton momentum p_{lab} of 70 and 400 GeV/c and an angle of $\simeq 90^0$. The dependence of the inclusive cross section $Ed^3\sigma/dq^3$ on the transverse momentum $q_\perp$ of the same experimental data is shown in Figure 1(b). The solid and dashed lines restrict the regions corresponding to the sets of data, [5] and [24], respectively. The results presented in Figure 1(a) confirm the validity of z-scaling for the lightest (D) and heaviest (Pb) nuclei. The property of the function $\psi(z)$ allows us to say that the mechanism of hadron formation demonstrates a feature depending on general properties of nuclear matter. The function characterizing the influence of matter on hadron formation is found to be $\alpha = \alpha(A)$. The dependence of the function $\alpha(A)$ on atomic number A is shown in Figure 2. One can see that the function is independent of momentum p_{lab}. In contrast to the z-presentation of experimental data, the $q_\perp$-presentation of the same data demonstrates a strong energy dependence [5]. The values of the cross section of the same nucleus (D) at $p_{lab} = 70$ and 400 GeV/c differ more than by two orders at $q_\perp > 4$ GeV/c. Thus, the results presented in Figures 1(a) and 2 illustrate the existence of the A-dependence of z-scaling for π^+-meson production in pA collisions at a high colliding energy $\sqrt{s}$ over a high transverse momentum range and at a produced angle of near 90^0.

Figures 3(a) and 3(b) present the dependences of ψ on z and $Ed^3\sigma/dq^3$ on $q_\perp$ for π^--meson production, respectively. The $\alpha(A)$ dependence obtained for π^+-meson production (Figure 2) was used to calculate the scaling function of different nuclei for π^--meson production, too. Thus, the z-scaling of π^--meson produced in pA collisions over the range of incident proton momentum $p_{lab} = 70 - 400$ GeV/c is confirmed.

Figures 4,5 and 6 show the dependence of the function ψ on z and inclusive cross section $Ed^3\sigma/dq^3$ on transverse momentum $q_\perp$ for K^+, K^- and $\bar{p}$ hadrons produced in pA collisions at a proton momentum p_{lab} of 200,300 and 400 GeV/c and an angle of 90^0, respectively. During the calculation the same $\alpha(A)$ dependence was used. The solid lines are obtained by fitting the experimental data corresponding to different p_{lab} and the same nuclear target, W. As we see from Figures 4,5 and 6, the difference between the cross sections increases with p_{lab} and $q_\perp$. Here, we would like to note that the function ψ, corresponding to the second data set [24], reveals a deviation from the first one [5]. The result indicates the correlation between colliding energy $\sqrt{s}$ and the mass of produced particle. We found that the violation of z-scaling increases with decreasing p_{lab} and increasing hadron mass m_1. The restoration of z-scaling is observed at large enough $\sqrt{s}$. We would like to remember that the scaling in the proposed form is valid for pp collisions in the energy range $\sqrt{s} > 20$ GeV [37]. It is reasonable to assume that a similar restriction exists for proton-nucleus interactions and that the full asymptotic regime is achieved for corresponding center-of-mass energies.

We use the properties of z-scaling to calculate the cross section of particle production in pA collisions at RHIC energies. Figures 7-11 show the dependence of the inclusive differential cross section of π^+, π^-, K^+, K^- and $\bar{p}$ particles produced in $p-Be$ (a) and $p-W$ (b) collisions on transverse momentum $q_\perp$ at different colliding energy $\sqrt{s}$ and $\theta_{NN} \simeq 90^0$. The points and solid line are the results obtained at

[5] The other presentations of experimental data are discussed in [42].

different colliding energy ($\diamond - 63\ GeV, \circ - 200\ GeV, + - 500\ GeV$). The experimental data, $\triangle - 27\ GeV$, are taken from [5]. It should be noted that the obtained $q_\perp$-dependence of the cross section demonstrates a non-exponential behaviour in a high transverse momentum region ($q_\perp$ up to 10 or more GeV/c) for all types of produced particles and nuclear targets.

4 Discussion

In this section, we qualitatively discuss the obtained results in the framework of a fractal picture of constituent interaction. Its substantial element is the idea of fractality of constituents (hadrons, nuclei etc.) and their interactions. We would like to emphasize that z-scaling was observed in the production of particles with high $q_\perp$ at high energies. This means that the scaling function describes the fragmentation process of point-like produced partons into observable hadrons. The cross section of hadron interactions at the level of point-like parton-parton scattering (4) is governed by the minimum energy $\hat{s}^{1/2}_{min}$ of colliding constituents. Thus, the point-likeness of the constituents is defined by colliding energy $\sqrt{s}$ and the transverse momentum of registered hadron.

A fractal character in the initial state regards the parton compositeness of hadrons and nuclei and reveals itself with a larger resolution at high energies. Led by these principles, the variable z is constructed according to Eq.(14). The fractal objects are usually characterized by a power law dependence of their fractal measures [41]. The fractal measure, considered in our case, is given by all possible configurations of elementary interactions that lead to the production of the inclusive particle. It takes the following form

$$\Omega(x_1, x_2) \sim (1 - x_1)^{\delta_1}(1 - x_2)^{\delta_2}.$$

The formula expresses the factorization of the fractal measure with respect to the fractal measures of colliding objects. Both are described by power law dependence in the space of fractions $\{x_1, x_2\}$. A single measure reflects the number of constituent configurations in the colliding object involved in the production of the inclusive particle. The measure is characterized by the fractal dimension δ. Fractal dimensions can be different for various colliding objects. The results of the analysis [42] show that the fractal dimension of nucleus δ_A is related to the nucleon fractal dimension δ_N for different types of produced hadrons ($\pi^\pm, K^\pm, \bar{p}$) by the following simple form

$$\delta_A = A \cdot \delta_N. \tag{21}$$

The relation describes the additivity of fractal dimension of nucleus.[6] In the framework of the fractal picture, the number of initial configurations is maximized according to Eq.(8), and the variable z describes the energy of elementary constituent collision per initial configuration and per produced particle.

The existence of z-scaling itself is the confirmation of hadron interaction self-similarity at the constituent level. The property reflects the simplest property of

[6]The breaking of the additivity of fractal dimension was discussed in Ref.[42].

the corresponding equation, scale invariance. According to the principle of scale relativity, the basic equations should be covariant under scale transformation. This means that the objects or phenomena described by the equations should demonstrate the properties of fractals.

The measure Ω, introduced above, describes the structure of colliding objects by the power law. Moreover, equation (9) determining the function ψ reveals the property of scale covariance under scale transformation (16). We consider the results as an indication of the fractal structure of interacting objects.

Now, we would like to discuss the physical interpretation of the function ψ. As one can see from Figure 1(a), the z-dependence of ψ demonstrates a different behaviour over a low and high z range. The ranges correspond to low and high $q_\perp$ ranges, respectively. According to the general idea that the function ψ describes the hadronization mechanism, we interpret a different behaviour of the function as a transition from the soft to hard regime of hadron formation in the surrounding matter. The hard regime is characterized by the non-exponential $q_\perp$-dependence of inclusive cross section in contrast to the soft one.

Here, we suggest a scenario of hadron formation. The decrease of formation length is assumed to be due to the influence of the *hadronic phase* of nuclear matter. The presence of nuclear matter in this form intensifies the "dressing" of a bare parent parton and accelerates its transformation into a real hadron. We assume that the influence of nuclear matter on hadron formation depends on the transverse momentum characterizing the compositeness of colliding objects and the elementary subprocess. The hadronization process in which the produced bare quark dresses itself dragging out some matter of the vacuum forming the dressed quark is a self-similar one. The derivative $d\psi/dz$ characterizing the process decreases with z. The hard regime is preferable to study the space-time evolution of hadron formation.

The results presented in Figures 1(a),3(a)-6(a) give some indications of asymptotic regime, $\psi \sim z^{-\rho}$, over a high z range. The slope parameter $\rho = -d\log(\psi)/d\log(z)$ tends to a constant value. It means that hadron formation as a fractal process is characterized by the power law with the fractal dimension ρ. We assume that the change of the fractal dimension of particle formation is a signature of new physics phenomena (quark compositeness, new type of interaction, phase transition etc.). In the framework of the proposed scenario, the mechanism of hadron formation is considered as a process of construction of complex fractal (hadron) from elementary fractal blocks. The size and structure of blocks depends on the colliding energy and transverse momentum of the produced hadron. The value of z is proportional to the number of blocks and can be considered as a relative formation length.

In addition, we would like to discuss the problem of rescattering and multiscattering mechanisms in the framework of the scenario of fractal particle interaction. The values of colliding energy $\sqrt{s}$ and transverse momentum $q_\perp$ define the resolution of interacting constituents. The point-like probe resolutes the constituents of the hadron or nucleus and the rescattering of the probe off the hadron or nuclear constituents take place.

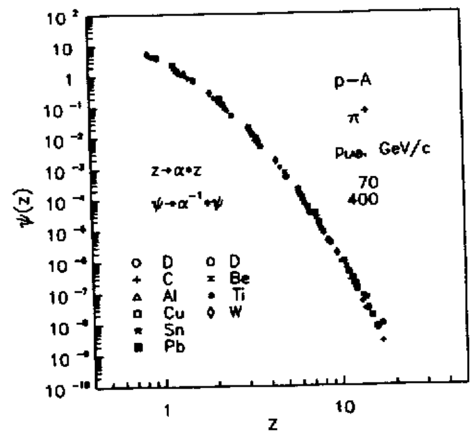
The probe itself is not a point-like one in hh, hA and AA collisions. Therefore the resolution of colliding objects is defined by the transverse momentum of the produced hadron and also the probe size. The size and number of hadron and nuclear constituents involved in the interactions are governed by initial conditions of collisions. In the framework of fractal structure of colliding objects, it is practically impossible to distinguish between the rescattering and multiscattering mechanisms. The rescattering of constituent at a structure level reveals itself as multiscattering in the other one.

The A-dependence of z-scaling obtained in our analysis does not violate the general features of z-scaling construction [37, 42]. Moreover, the observed form of the function ψ for pD and pPb collisions is the same. Thus, the influence of surrounding matter on the mechanism of particle formation is described by a smooth function $\alpha(A)$ depending on the atomic number, only. The obtained results show that the fractal character of the hadronization mechanism is not changed by nuclear medium. The situation may change if the nuclear medium has inhomogeneous domains (for example, DCC [48]). The search for scaling violations in pp, pA and AA collisions at high energies, especially in the region of high transverse momenta, could be very interesting for our understanding the structure of nuclear matter.

One of the manifestations of the transition of nuclear matter to parton phase corresponding to the joining of partons from different nucleons of nuclei is known as a cumulative process [25, 26, 27]. The corresponding regime of particle production is kinematically forbidden in nucleon-nucleon collisions. A higher stage of cumulation corresponds the larger values of the variable z and can manifest itself more prominent just in a high momentum tail of the spectrum. The cumulation of the nucleus can be responsible for the creation of the exotic physical state of nuclear matter which can reveals itself in the violation of z-scaling.

5 Conclusions

The A-dependence of inclusive particle production in pA collisions at high energies in terms of z-scaling is considered. The experimental data on the inclusive cross sections for $\pi^{\pm}, K^{\pm}, \bar{p}$ particles produced on different nuclei ($A = D,Be,C,Al,Cu,Ti,Sn,W,Pb$) over a high transverse momentum range ($q_{\perp} = 1 - 7$ GeV/c) obtained at U70 and Tevatron were used for the analysis. The momentum of incident proton p_{lab} changes from 70 to 400 GeV/c. The function $\psi(z)$, presenting a new presentation of experimental data, is expressed via the invariant inclusive cross section $Ed^3\sigma/dq^3$ and normalized to the multiplicity density of particles produced in pA collisions. The symmetry transformations of the function ψ and its argument, $\psi \to \alpha^{-1}\psi$ and $z \to \alpha z$, are used to establish the A-dependence of the transformation parameter α. The function $\alpha = \alpha(A)$ is found to be the same one for $\pi^{\pm}$ and $K^{\pm}, \bar{p}$ hadron produced for $p_{lab} = 70 - 400$ GeV/c and $p_{lab} = 200 - 400$ GeV/c, respectively. The fractal dimensions of nuclei are expressed via the fractal dimension of the nucleon in the form $\delta_A = A \cdot \delta_N$ for different types of produced hadrons. The function $\psi(z)$ demonstrates main features of the hadronization process in terms of relative formation length z. The obtained results show that the general properties of the particle production mechanism such as self-similarity, locality, scale-relativity and fractality reveal themselves in pA collisions as well as in pp and $\bar{p}p$. Using the properties of z-scaling, the dependence of the cross sections of $\pi^{\pm}, K^{\pm}, \bar{p}$ hadron produced in pA collisions on transverse momentum over the central range at RHIC energies were



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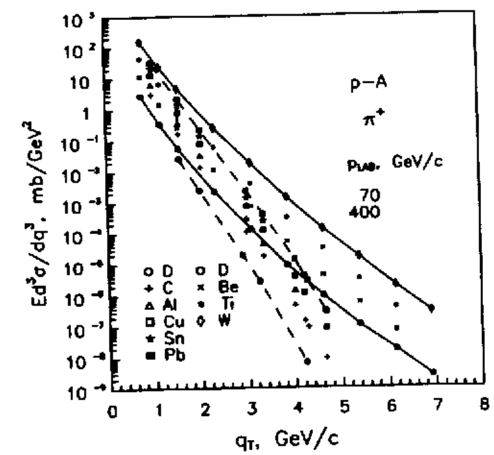

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Figure 1. (a) Scaling function $\psi(z)$ for the π^+-meson produced in $p - A$ interactions at $p_{lab} = 70, 400$ GeV/c and in the central region, $\theta_{NN} \simeq 90^0$ as a function of z. (b) The corresponding inclusive differential cross sections as functions of the transverse momentum $q_{\perp}$. Solid and dashed lines are obtained by fitting of the data for D, W and D, Pb, respectively. Experimental data are taken from Refs. [5, 24].

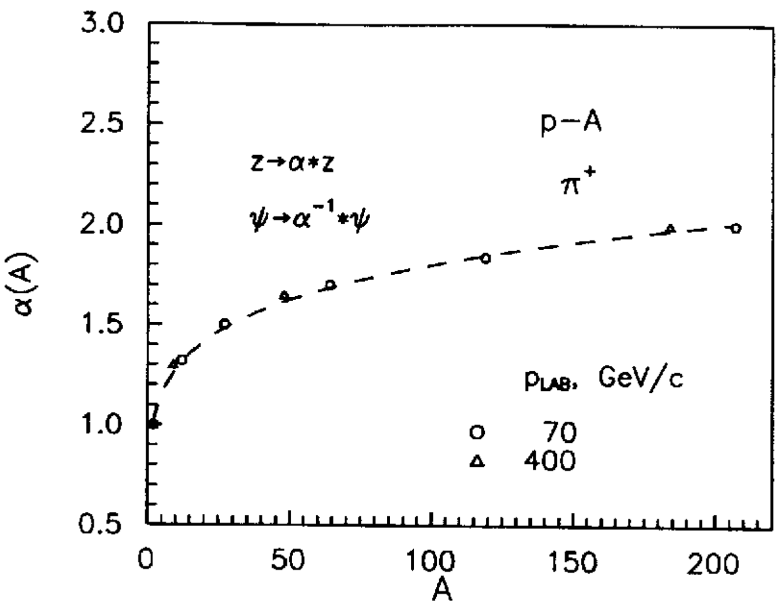
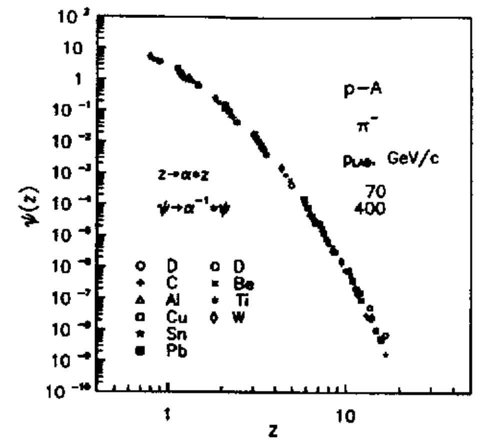


Figure 2. A-dependence of transformation parameter α. The line is obtained by the fitting of the points.



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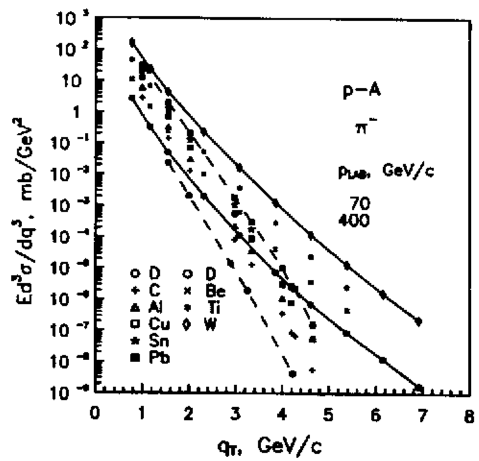
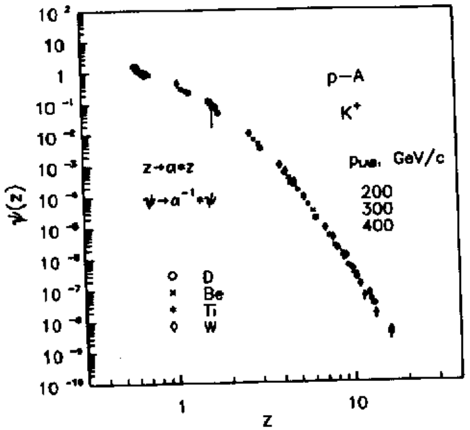


b)

Figure 3. (a) Scaling function $\psi(z)$ for the π^--meson produced in $p-A$ interactions at $p_{lab} = 70, 400\ GeV/c$ and in the central region, $\theta_{NN} \simeq 90^0$ as a function of z. (b) The corresponding inclusive differential cross sections as functions of the transverse momentum $q_\perp$. Solid and dashed lines are obtained by fitting of the data for D, W and D, Pb, respectively. Experimental data are taken from Refs. [5, 24].



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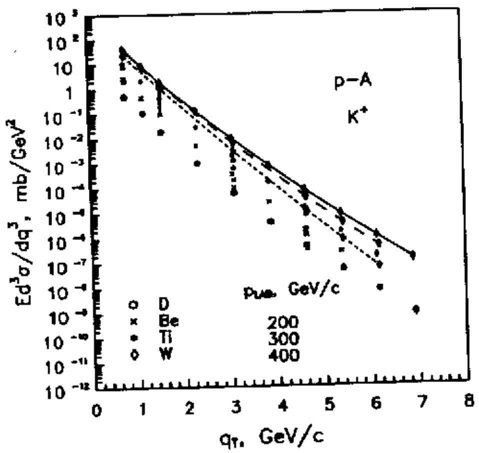

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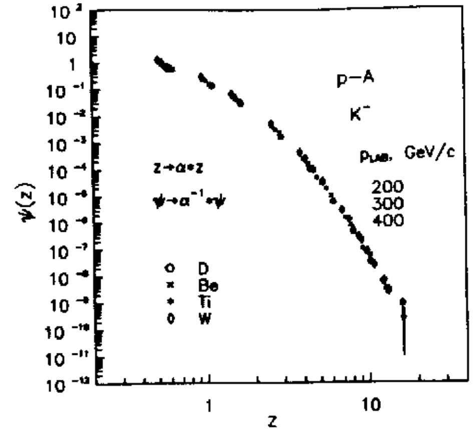
Figure 4. (a) Scaling function $\psi(z)$ for the K^+-meson produced in $p-A$ interactions at $p_{lab} = 200, 300$ and $400\ GeV/c$ and in the central region, $\theta_{NN} \simeq 90^0$ as a function of z. (b) The corresponding inclusive differential cross sections as functions of the transverse momentum $q_\perp$. Dotted, dashed and solid lines are obtained by fitting of the data for W target at $p_{lab} = 200, 300$ and $400\ GeV/c$, respectively. Experimental data are taken from Ref.[5].



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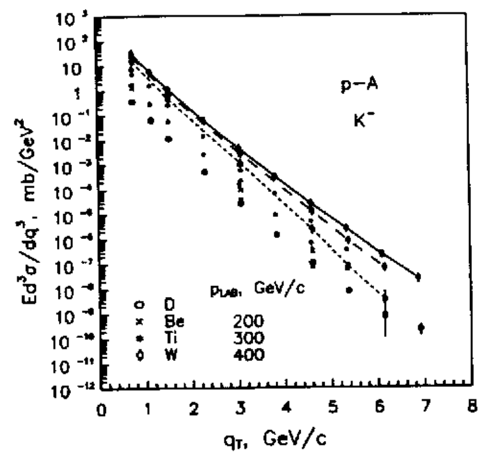

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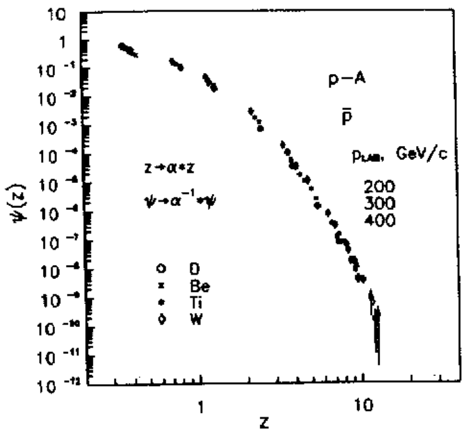
Figure 5. (a) Scaling function $\psi(z)$ for the K^--meson produced in $p-A$ interactions at $p_{lab} = 200, 300$ and $400\ GeV/c$ and in the central region, $\theta_{NN} \simeq 90^0$ as a function of z. (b) The corresponding inclusive differential cross sections as functions of the transverse momentum $q_\perp$. Dotted, dashed and solid lines are obtained by fitting of the data for W target at $p_{lab} = 200, 300$ and $400\ GeV/c$, respectively. Experimental data are taken from Ref. [5].



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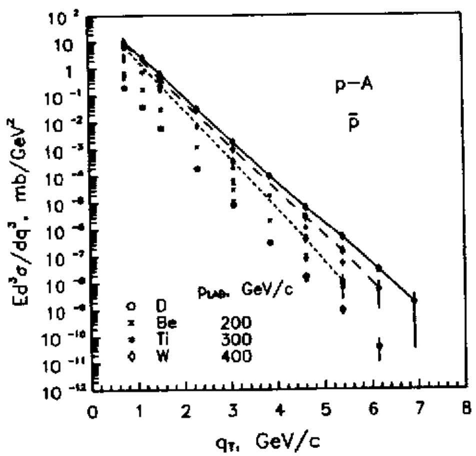

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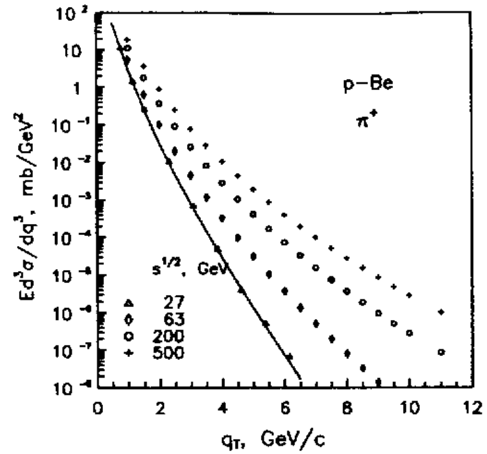
Figure 6. (a) Scaling function $\psi(z)$ for the $\bar{p}$ produced in $p-A$ interactions at $p_{lab} = 200, 300$ and $400\ GeV/c$ and in the central region, $\theta_{NN} \simeq 90^0$ as a function of z. (b) The corresponding inclusive differential cross sections as functions of the transverse momentum $q_\perp$. Dotted, dashed and solid lines are obtained by fitting of the data for W target at $p_{lab} = 200, 300$ and $400\ GeV/c$, respectively. Experimental data are taken from Ref. [5].



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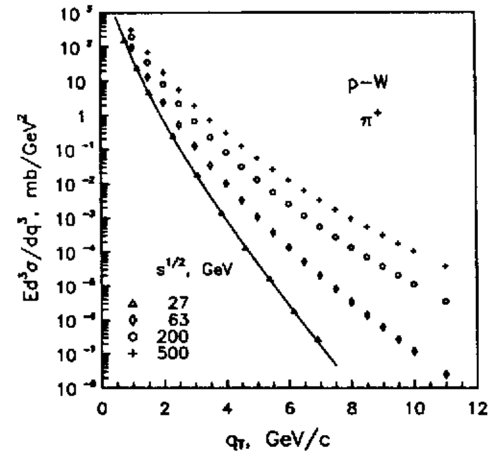

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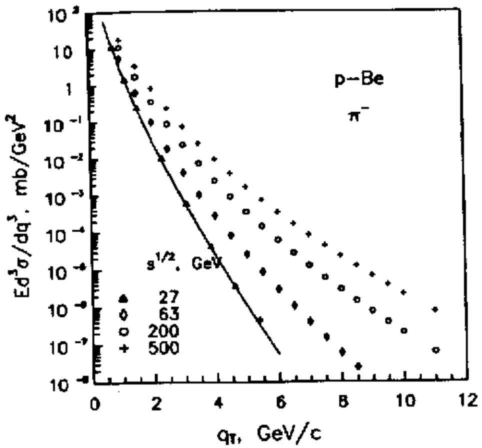
Figure 7. The dependence of inclusive differential cross section of π^+-meson production in $p-Be$ (a) and $p-W$ (b) collisions on the transverse momentum $q_\perp$ at different colliding energy $\sqrt{s}$ and $\theta_{NN} \simeq 90^0$. Points, $\diamond - 63\ GeV, \circ - 200\ GeV, + - 500\ GeV$, and solid line are calculation results. Experimental data, $\Delta - 27\ GeV$, are taken from [5].



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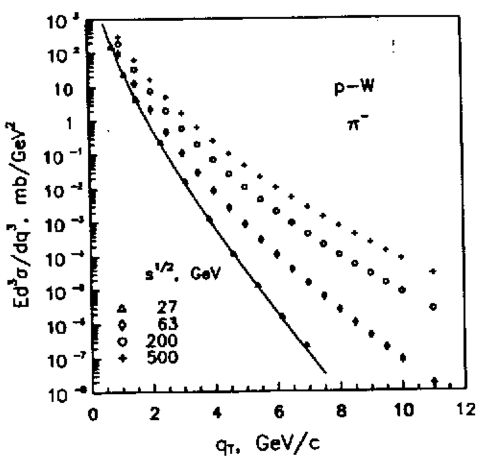

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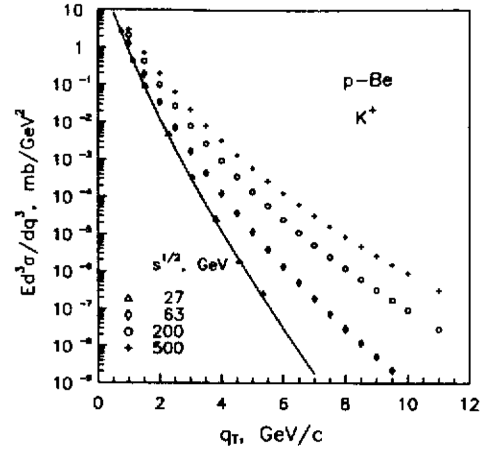
Figure 8. The dependence of inclusive differential cross section of π^--meson production in $p-Be$ (a) and $p-W$ (b) collisions on the transverse momentum $q_\perp$ at different colliding energy $\sqrt{s}$ and $\theta_{NN} \simeq 90^0$. Points, $\diamond - 63\ GeV, \circ - 200\ GeV, + - 500\ GeV$, and solid line are calculation results. Experimental data, $\Delta - 27\ GeV$, are taken from [5].



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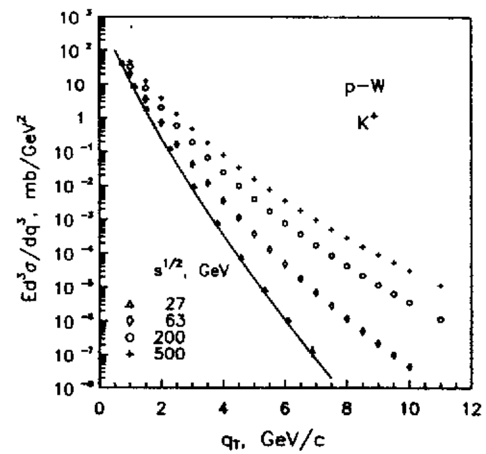

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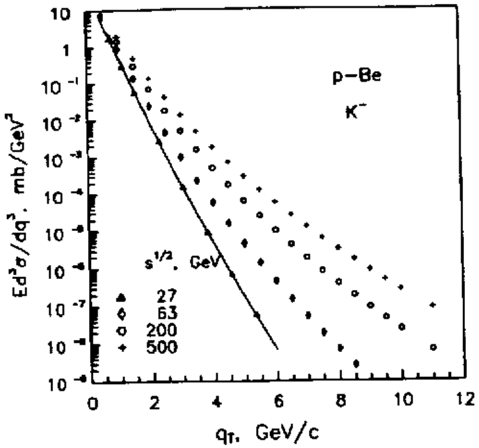
Figure 9. The dependence of inclusive differential cross section of K^+-meson production in $p-Be$ (a) and $p-W$ (b) collisions on the transverse momentum $q_\perp$ at different colliding energy $\sqrt{s}$ and $\theta_{NN} \simeq 90^0$. Points, $\diamond - 63\ GeV, \circ - 200\ GeV, + - 500\ GeV$, and solid line are calculation results. Experimental data, $\Delta - 27\ GeV$, are taken from [5].



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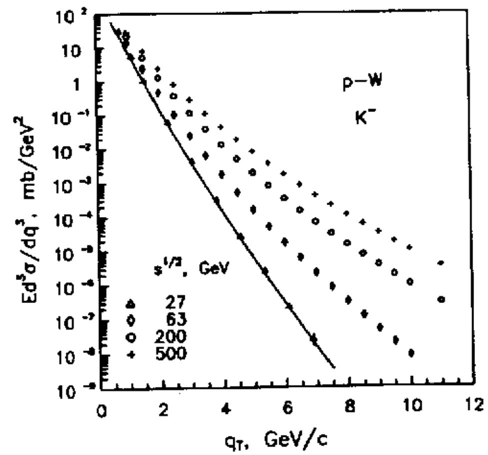
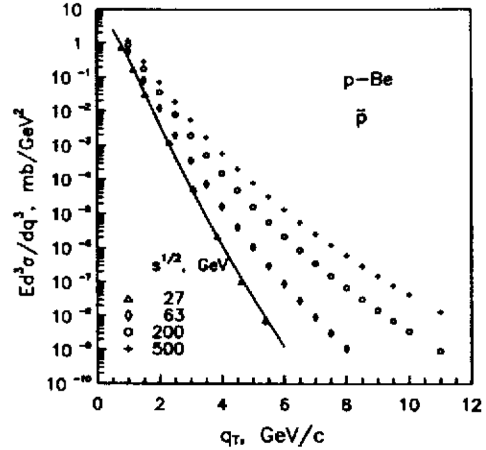


b)

Figure 10. The dependence of inclusive differential cross section of K^--meson production in $p-Be$ (a) and $p-W$ (b) collisions on the transverse momentum $q_\perp$ at different colliding energy $\sqrt{s}$ and $\theta_{NN} \simeq 90^0$. Points, $\diamond - 63$ GeV, $\circ - 200$ GeV, $+ - 500$ GeV, and solid line are calculation results. Experimental data, $\Delta - 27$ GeV, are taken from [5].



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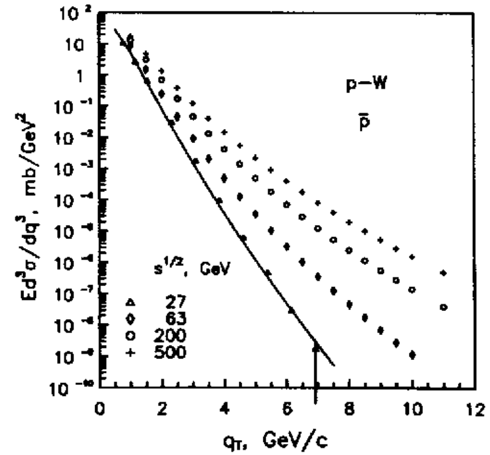

b)

Figure 11. The dependence of inclusive differential cross section of $\bar{p}$ production in $p-Be$ (a) and $p-W$ (b) collisions on the transverse momentum $q_\perp$ at different colliding energy $\sqrt{s}$ and $\theta_{NN} \simeq 90^0$. Points, $\diamond - 63$ GeV, $\circ - 200$ GeV, $+ - 500$ GeV, and solid line are calculation results. Experimental data, $\Delta - 27$ GeV, are taken from [5].

predicted. The verification of the predictions and the search for the violation of z-scaling in pp, pA and AA collisions in future experiments planned at RHIC (BNL) and LHC (CERN) were suggested.

Acknowledgments

This work has been partially supported by Grant of the Czech Academy of Sciences No. 1048703.

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Received by Publishing Department
on April 19, 1999.