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О некоторых свойствах представлений конформной супералгебры

В работе рассматривается вопрос о приводимости ранее найденных представлений конформной супералгебры с произвольной лоренцовской структурой. Показано, что неприводимые подпространства возникают при определенных соотношениях между параметрами, характеризующими данное представление. Найдены глобальные преобразования, соответствующие генераторам рассматриваемой супералгебры.

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On Some Properties of the Conformal Superalgebra Representations

Reducibility of the previously found representations of the conformal superalgebra for arbitrary Lorentz structure is examined. It is shown that irreducible subspaces arise provided certain relations between the parameters characterizing the given representation take place. The global transformations generated by the considered superalgebra are found.

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I. In the present paper we examine some properties of the conformal superalgebra representations proposed in /1/. We recall that the generators of the conformal superalgebra are realized as differential operators in a space of functions of the variables x_μ, θ^+_a, ξ^-_a*. The explicit form of the generators is the following:

$$\hat{P}_\mu = -i\frac{\partial}{\partial x_\mu},\quad \hat{S}^+_a = i\left(\gamma^\circ\frac{\partial}{\partial\theta^+}\right)_a,\quad \hat{T}^-_a = i\left(\gamma^\circ\frac{\partial}{\partial\xi^-}\right)_a + i\left(\hat{x}\gamma^\circ\frac{\partial}{\partial\theta^+}\right)_a,$$

$$\hat{M}_{\mu\nu} = \Sigma_{\mu\nu} + i\left(x_\mu\frac{\partial}{\partial x^\nu} - x_\nu\frac{\partial}{\partial x^\mu} - \theta^+\gamma^\circ\sigma_{\mu\nu}\gamma^\circ\frac{\partial}{\partial\theta^+} - \xi^-\gamma^\circ\sigma_{\mu\nu}\gamma^\circ\frac{\partial}{\partial\xi^-}\right),$$

$$\hat{D} = -i\left\{d + x^\mu\frac{\partial}{\partial x^\mu} + \frac{1}{2}\left[\theta^+\frac{\partial}{\partial\theta^+} - \xi^-\frac{\partial}{\partial\xi^-}\right]\right\},$$

$$\hat{\Pi} = -z + \theta^+\frac{\partial}{\partial\theta^+} + \xi^-\frac{\partial}{\partial\xi^-}, \tag{1.1}$$

$$\hat{K}_\mu = 2x^\nu\Sigma_{\mu\nu} + i\left\{(2x_\nu x_\mu - x^2 g_{\mu\nu})\frac{\partial}{\partial x^\nu} - \theta^+\gamma^\circ\gamma_\mu\gamma^\circ\frac{\partial}{\partial\xi^+} + 2x_\mu\left(d + \frac{1}{2}\theta^+\frac{\partial}{\partial\theta^+} - \frac{1}{2}\xi^-\frac{\partial}{\partial\xi^-}\right) - 2x^\nu\left[\theta^+\gamma^\circ\sigma_{\mu\nu}\gamma^\circ\frac{\partial}{\partial\theta^+} + \xi^-\gamma^\circ\sigma_{\mu\nu}\gamma^\circ\frac{\partial}{\partial\xi^-}\right]\right\},$$

$$\hat{S}^-_a = -8i\Sigma_{\mu\nu}(\sigma^{\mu\nu}\xi^-)_a - 8(\gamma^\nu\theta^+)_a\frac{\partial}{\partial x^\nu} - (8d - 12z)\xi^-_a - 8\xi^-\gamma^\circ\xi^-\left(\gamma^\circ\frac{\partial}{\partial\xi^-}\right)_a,$$

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*In what follows we use the notations introduced in paper /1/.

$$\hat{T}^{+}_{\alpha} = x_{\mu}(\gamma^{\mu}\hat{S}^{-})_{\alpha} - 8i\Sigma_{\mu\nu}(\sigma^{\mu\nu}\theta^{+})_{\alpha} + (8d + 12z)\theta^{+}_{\alpha} -$$

$$- 8\theta^{+}\gamma^{\circ}\theta^{+}(\gamma^{\circ}\frac{\partial}{\partial\theta^{+}})_{\alpha} - 16\theta^{+}_{\alpha}\xi^{-}\frac{\partial}{\partial\xi^{-}} .$$

Here $\Sigma_{\mu\nu}$ are the matrices of the generators of the finite-dimensional irreducible representation of the Lorentz-group, d and z - complex numbers.

In papers /1,2/ it was shown that in the cases of scalar and spinor representations provided certain relations between d and z hold the representation space contains invariant subspaces. These are the following:

a) the subspace of functions independent of ξ^{-}_{α} (these representations are often called chiral).

b) The subspace of functions linear in ξ^{-}_{α}.

In Section 2 we derive the conditions under which the results obtained in these papers are generalized to the case of similar conformal superalgebra representations but with arbitrary Lorentz structure.

In Section 3 we discuss the problem about the global transformation generated by the considered superalgebra. We also introduce a generalization of the well-known from the conformal symmetry K-inversion to the case of conformal superalgebra.

2. Consider a superfield, characterized by a set of Lorentz indices which transform according to a finite dimensional irreducible representation of the Lorentz group τ_{mn} (see for instance /3/). It is convenient to realize the finite dimensional representation of the Lorentz group in the space of spinors $\mathfrak{U}_{\{\alpha_1 \ldots \alpha_p\}\{\beta_1 \ldots \beta_q\}}$ symmetric under all permutations within the sets of indices $\{\alpha_1 \ldots \alpha_p\}\{\beta_1 \ldots \beta_q\}$. As is well known this representation is equivalent to the irreducible representation of the Lorentz group $\tau_{\frac{p}{2},\frac{q}{2}}$. So we consider a superfield

$$\mathfrak{U}_{\{\alpha_1 \ldots \alpha_p\};\{\beta_1 \ldots \beta_q\}} \equiv \mathfrak{U}_{\{\alpha_p\};\{\beta_q\}}(x_{\mu}, \theta^{+}_{\alpha}, \xi^{-}_{\alpha}), \qquad (2.1)$$

where $\{\alpha_p\} \equiv \{\alpha_1 \ldots \alpha_p\}$ and $\{\beta_q\} \equiv \{\beta_1 \ldots \beta_q\}$ are completely symmetrized sets of indices and

$$(1 - i\gamma_5)_{\alpha_i\delta}\, \mathfrak{U}_{\{\alpha_p(-i)|\delta\}\{\beta_q\}} = 0 \qquad i = 1, \ldots, p$$

$$(1 + i\gamma_5)_{\beta_i\delta}\, \mathfrak{U}_{\{\alpha_p\}\{\beta_q(-i)|\delta\}} = 0 \qquad i = 1, \ldots, q$$

Hereafter we use the following notations:

$$\{\alpha_p(-i)\} \equiv \{\alpha_1 \ldots \alpha_{i-1}, \alpha_{i+1}, \ldots, \alpha_p\}$$

$$\{\alpha_p(-i)|\gamma\} \equiv \{\alpha_1 \ldots \alpha_{i-1}, \gamma, \alpha_{i+1}, \ldots, \alpha_p\}$$

$$\{\beta_q(-j)\} \equiv \{\beta_1 \ldots \beta_{j-1}, \beta_{j+1}, \ldots, \beta_q\}$$

$$\{\beta_q(-j)|\delta\} \equiv \{\beta_1 \ldots \beta_{j-1}, \delta, \beta_{j+1}, \ldots, \beta_q\} .$$

For the given superfield the matrix of the finite dimensional representation of the Lorentz group $\Sigma_{\mu\nu}$ is the direct sum of the matrices $i\sigma_{\mu\nu}$, i.e.,

$$(\Sigma_{\mu\nu})_{\{\alpha_p,\alpha'_p\}\{\beta_q,\beta'_q\}} = i\sum_{k=1}^{p}(\sigma_{\mu\nu})_{\alpha_k\alpha'_k}\prod_{j\neq k}^{p}\delta_{\alpha_j\alpha'_j} + i\sum_{k=1}^{q}(\sigma_{\mu\nu})_{\beta_k\beta'_k}\prod_{j\neq k}\delta_{\alpha_j\alpha'_j} \qquad (2.2)$$

The power decomposition with respect to ξ^{-}_{α} has the following form (see Appendix A):

$$\mathfrak{U}_{\{\alpha_p\}\{\beta_q\}}(x, \theta^{+}, \xi^{-}) \equiv \Psi_{\{\alpha_p\}\{\beta_q\}}(x, \theta^{+}) + \frac{1}{q}\sum_{k=1}^{q}\chi_{\{\alpha_p\}\{\beta_q(-k)\}}\xi^{-}_{\beta_k} +$$

$$+ \frac{1}{q}\sum_{k=1}^{q} \phi^{\mu\nu}_{\{\alpha_p\}\{\beta_q(-k)\}}(\sigma_{\mu\nu}\xi^-)_{\beta_k} + \Phi_{\{\alpha_p\}\{\beta_q\}}(x,\theta^+)\xi^-\gamma^\circ\xi^- =$$

$$= \Psi_{\{\alpha_p\}\{\beta_q\}}(x,\theta^+) + \frac{1}{q}\sum_{k=1}^{q} \chi_{\{\alpha_p\}\{\beta_q(-k)\}}\xi^-_{\beta_k} + \qquad (2.3)$$

$$+ 2R_{\{\alpha_p\}\{\beta_q,\delta\}}(\gamma^\circ\xi^-)_\delta + \Phi_{\{\alpha_p\}\{\beta_q\}}(x,\theta^+)\xi^-\gamma^\circ\xi^- .$$

We use here the following identities:

$$\phi^{\lambda\rho}_{\{\alpha_p\}\{\beta_q(-k)\}} = R_{\{\alpha_p\}\{\beta_q(-k)|\gamma\delta\}}(\gamma^\circ\sigma^{\lambda\rho})_{\gamma\delta} \qquad (2.4$$

$$R_{\{\alpha_p\}\{\beta_q(-k)|\delta\gamma\}} = \frac{1}{2q}\sum_{k=1}^{q} \phi^{\mu\nu}_{\{\alpha_p\}\{\beta_q(-k)\}}\sigma_{\mu\nu}\gamma^\circ(1+i\gamma_5)_{\gamma\delta} .$$

Let us introduce the notions of diagonal, raizing and reducing operators with respect to ξ^-_α, as in paper /2/:

a) We call an operator $\hat{A}$ diagonal, if in the decomposition

$$\hat{A}\,\mathfrak{U}_{\{\alpha_p\}\{\beta_q\}} = \Psi'_{\{\alpha_p\}\{\beta_q\}} + \frac{1}{q}\sum_{k=1}^{p} \chi'_{\{\alpha_p\}\{\beta_q(-k)\}}\xi^-_{\beta_k} +$$

$$+ \frac{1}{q}\sum_{k=1}^{q} \phi'^{\mu\nu}_{\{\alpha_p\}\{\beta_q(-k)\}}(\sigma_{\mu\nu}\xi^-)_{\beta_k} + \Phi'_{\{\alpha_p\}\{\beta_q\}}\xi^-\gamma^\circ\xi^- , \qquad (2.5)$$

Ψ' is expressed by Ψ only; χ' and ϕ' by χ and ϕ respectively, and Φ' by Φ only.

b) The operator A is a reducing one if in the decomposition (2.5) Ψ' is expressed by χ and ϕ only; χ' and ϕ' by Φ only and $\Phi' = 0$.

c) We call the operator raizing if in the decomposition (2.5) $\Psi' = 0$; ; χ' and ϕ' are expressed by Ψ, and Φ', by χ and ϕ. It is evident that both subspaces of functions independent of ξ^-_α and of those which are only

linear in ξ^-_α should be invariant under the action of all generators, which do not contain raising operators. Having in mind the explicit for of the generators we see that for $\hat{M}_{\mu\nu}$, $\hat{\Pi}$, $\hat{P}_\mu$, $\hat{D}$, $\hat{K}_\mu$, $\hat{S}^+_\alpha$, $\hat{T}^-_\alpha$ these subspaces are invariant. It is just the generators $\hat{S}^-_\alpha$ and $\hat{T}^+_\alpha$ that spoil this invariance since they contain raising terms, too. It is easy to see that in fact it is enough to examine the generator $\hat{S}^-_\alpha$ only.

In accordance with the decomposition (2.3) it is convenient to consider the action of the generator $\hat{S}^-_A$ on the various terms in this decomposition separately. We have therefore:

$$[\hat{S}^-_A]_{\{\alpha_p,\alpha'_p\}\{\beta_q,\beta'_q\}}\Psi_{\{\alpha'_p\}\{\beta'_q\}} = 8(\gamma^\nu\theta^+)_A\frac{\partial}{\partial x^\nu}\Psi_{\{\alpha_p\}\{\beta_q\}} +$$

$$+ (-1)^{p+q+1}\frac{4}{q}(3q+d-\frac{3}{2}z)\sum_{k=1}^{q}\Psi_{\{\alpha_p\}\{\beta_q(-k)|A\}}\xi^-_{\beta_k} + \qquad (2.6)$$

$$+ (-1)^{p+q+1}\frac{4}{q}(q-d+\frac{3}{2}z)\sum_{k=1}^{q}(\sigma^{\mu\nu})^\rho_A\Psi_{\{\alpha_p\}\{\beta_q(-k)|\rho\}}(\sigma_{\mu\nu}\xi^-)_{\beta_k} .$$

$$[\hat{S}^-_A]_{\{\alpha_p,\alpha'_p\}\{\beta_q,\beta'_q\}}\frac{1}{q}\Big\{\sum_{k=1}^{q}\chi_{\{\alpha'_p\}\{\beta'_q(-k)\}}\xi^-_{\beta'_k} +$$

$$+ \sum_{k=1}^{q}\phi^{\mu\nu}_{\{\alpha'_p\}\{\beta'_q(-k)\}}(\sigma_{\mu\nu}\xi^-)_{\beta'_k}\Big\} =$$

$$= \frac{2}{q}(-1)^{p+q}(q-d+\frac{3}{2}z)\sum_{k=1}^{q}\chi_{\{\alpha_p\}\{\beta_q(-k)\}}\gamma^\circ(1+i\gamma_5)_{\beta_k A}\xi^-\gamma^\circ\xi^- + \qquad (2.7)$$

$$+ \frac{2}{q}(-1)^{p+q+1}(q+d+2-\frac{3}{2}z)\sum_{k=1}^{q}\phi^{\mu\nu}_{\{\alpha_p\}\{\beta_q(-k)\}}\sigma_{\mu\nu}\gamma^\circ(1+i\gamma_5)_{\beta_k A}\xi^-\gamma^\circ\xi^- +$$

$$+ 8(\gamma^\nu\theta^+)_\alpha\frac{\partial}{\partial x^\nu}\frac{1}{q}\Big\{\sum_{k=1}^{q}\chi_{\{\alpha_p\}\{\beta_q(-k)\}}\xi^-_{\beta_k} +$$

$$+ \sum_{k=1}^{q}\phi^{\mu\nu}_{\{\alpha_p\}\{\beta_q(-k)\}}(\sigma_{\mu\nu}\xi^-)_{\beta_k}\Big\} .$$

It is not necessary to consider the action of $\hat{S}_A^-$ on the last term of the decomposition (2.3).

These equalities show that if we set equal to zero certain structures in the decomposition (2.3) and coefficients in the right-hand sides of (2.6) and (2.7) we can provide the appearance of invariant subspaces. Thus, for instance we see that if the following equalities:

$$\phi^{\mu\nu}_{\{\alpha_p\}\{\beta_q(-k)\}} \equiv 0,$$
$$\Phi_{\{\alpha_p\}\{\beta_q\}} \equiv 0, \qquad (2.8)$$
$$d - q - \frac{3}{2}z = 0,$$

take place the action of the generator $\hat{S}_A^-$ on the superfield $\mathfrak{U}_{\{\alpha_p\}\{\beta_q\}}$ (which according to (2.8) is a linear function of ξ_α^-) gives no rise to terms quadratic in ξ^-, i.e., $\xi^-\gamma^\circ\xi^-$. At the same time it is easy to see that the second term in the r.h.s. of (2.6) does not appear. We stress that such a term does not exist in $\mathfrak{U}_{\{\alpha_p\}\{\beta_q\}}$ provided equalities (2.8) hold. That is why we say that the set of superfield with decomposition (2.3), for which the conditions (2.8) take place, form an invariant subspace, in which a certain representation of our superalgebra acts.

We can consider also other possibilities in a similar way. All the results which we can obtain by this procedure can be summarized in the following theorem:

Given representation of the conformal superalgebra characterized by the numbers p,, q,, d and z contains invariant subspaces in the following cases:

a) $q=0$, $d=\frac{3}{2}z$ and p arbitrary* (see Remark a)

$$\mathfrak{U}_{\{\alpha_p\}}(x,\theta^+,\xi^-) = \Psi_{\{\alpha_p\}}(x,\theta^+)$$

b) $q=0$, $d+2=\frac{3}{2}z$, p-arbitrary* (see Remark a)

$$\mathfrak{U}_{\{\alpha_p\}}(x,\theta^+,\xi^-) = \Psi_{\{\alpha_p\}}(x,\theta^+) + G_{\{\alpha_p\};\delta}(\gamma^\circ\xi^-)$$

c) $d-q=\frac{3}{2}z$, p- arbitrary

$$\mathfrak{U}_{\{\alpha_p\}\{\beta_q\}}(x,\theta^+,\xi^-) = \Psi_{\{\alpha_p\}\{\beta_q\}}(x,\theta^+) + \frac{1}{q}\sum_{k=1}\chi_{\{\alpha_p\}\{\beta_q(-k)\}}\,\xi^-_{\beta_k}$$

d) $q=1$, $d+3=\frac{3}{2}z$, p- arbitrary

$$\mathfrak{U}_{\{\alpha_p\};\beta}(x,\theta^+,\xi^-) = \Psi_{\{\alpha_p\};\beta}(x,\theta^+) + \phi^{\mu\nu}_{\{\alpha_p\}}(x,\theta^+)(\sigma_{\mu\nu}\,\xi^-)_\beta\,.$$

<u>Remarks</u>: a) The above considerations concern the case $q\neq 0$. We insist on the fact, that the results are generalized to the case $q=0$ without any modifications. This is readily proved by the explicit calculations.

b) It seems that under the condition $d+3q=\frac{3}{2}z$

$$\mathfrak{U}_{\{\alpha_p\}\{\beta_q\}}(x,\theta^+,\xi^-) = \Psi_{\{\alpha_p\}\{\beta_q\}}(x,\theta^+) +$$
$$+\frac{1}{q}\sum^{q}\phi^{\mu\nu}_{\{\alpha_p\}\{\beta_q(-k)\}}(x,\theta^+)(\sigma_{\mu\nu}\,\xi^-)_{\beta_k} + \Phi_{\{\alpha_p\}\{\beta_q\}}(x,\theta^+)\xi^-\gamma^\circ\xi^-$$

forms an invariant subspace, too. But $\hat{T}_\alpha^-$ contains the reducing operator $(\gamma^\circ\frac{\partial}{\partial\xi^-})_\alpha$ and therefore gives rise to the structure $\tau_{\frac{p}{2},\frac{q-1}{2}}$ via $\Phi_{\{\alpha_p\}\{\beta_q\}}\,\xi^-\gamma^\circ\xi^-$.

It seems somewhat astonishing that the conditions under which the representation space contains invariant subspaces do not depend on p. The latter is a corollary of the character of the considered representations. If in addition to the given representations we consider the "conjugate" ones (see paper /2/) then the symmetry with respect to p and q is restored. In paper /2/ examples of the above representation were considered. Here we give one more example, namely the vector superfield V^μ. In the latter case $p=q=1$ and

$$\mathfrak{U}_{\alpha;\beta}(x,\theta^+,\xi^-) = V^\mu(x,\theta^+,\xi^-)\gamma_\mu\gamma^\circ(1+i\gamma\ )_{\alpha\beta}$$

$$V^\mu(x,\theta^+,\xi^-) = -\frac{1}{8}\mathfrak{U}_{\alpha;\beta}(x,\theta^+,\xi^-)\gamma^\circ\gamma^\mu(1-i\gamma_5)^{\alpha\beta}$$

with the following invariant subspaces:

$$V^\mu(x,\theta^+,\xi^-) = A^\mu(x,\theta^+) + \Psi_\delta(x,\theta^+)(\gamma^\mu\xi^-)_\delta \text{ with } d-1=\frac{3}{2}z$$

$$W^\mu(x,\theta^+,\xi^-) = B^\mu(x,\theta^+) + G_{k,\delta}(x,\theta^+)(\gamma^\mu\gamma^k\gamma^\circ\xi^-)_\delta \text{ with } d+3=\frac{3}{2}z.$$

Thus we see that the vector superfield cannot contain invariant subspaces of the chiral type. The latter shows that the chiral superfields only are not sufficient to describe all representations with simplest Lorentz structures.

3. In this section the problem on the global transformations generated by our superalgebra is discussed. As is well-known, if V_g is a representation of a given group acting in the space of the field operators $\mathfrak{U}$, then

$$V_{g^{-1}}\mathfrak{U}(x,\dots)V = T_g\mathfrak{U}(x,\dots), \qquad (3.1)$$

where T_g is a representation of the same group in the space of functions of just the same variables that the field $\mathfrak{U}$ depends on. To each generator $\hat{O}$ of the superalgebra (1.1) a one-parameter finite transformation $e^{i\alpha\hat{O}}$

corresponds which is an element of the representation T_g. We find here explicitly these elements.

But instead of eq. (1.1) we use here new expressions for the superalgebra generators which are obtained from the old ones by the substutution

$$\eta^+_\alpha = \theta^+_\alpha + (\hat{x}\xi^-)_\alpha, \qquad\qquad (3.2)$$
$$\xi'^-_\alpha = \xi^-_\alpha .$$

In this way we find

$$\hat{S}^+_\alpha = i(\gamma^\circ\frac{\partial}{\partial\eta^+})_\alpha, \qquad \hat{T}^-_\alpha = i(\gamma^\circ\frac{\partial}{\partial\xi^-})_\alpha,$$

$$\hat{D}_\mu = -i\frac{\partial}{\partial x^\mu} - i\xi^-\gamma^\circ\gamma_\mu\gamma^\circ\frac{\partial}{\partial\eta^+},$$

$$\hat{M}_{\mu\nu} = \Sigma_{\mu\nu} + i\{x_\mu\frac{\partial}{\partial x^\nu} - x_\nu\frac{\partial}{\partial x^\mu} - \eta^+\gamma^\circ\sigma_{\mu\nu}\gamma^\circ\frac{\partial}{\partial\eta^+} - \xi^-\gamma^\circ\sigma_{\mu\nu}\gamma^\circ\frac{\partial}{\partial\xi^-}\},$$

$$\hat{D} = -i\{\frac{\partial}{\partial k} + x^\mu\frac{\partial}{\partial x^\mu} + \frac{1}{2}[\eta^+\frac{\partial}{\partial\eta^+} - \xi^-\frac{\partial}{\partial\xi^-}]\},$$

$$\hat{K}_\mu = 2x^\nu\Sigma_{\mu\nu} + i\{(2x_\mu x_\nu - x^2 g_{\mu\nu})\frac{\partial}{\partial x^\nu} + 2x_\mu\frac{\partial}{\partial k}\} - i\eta^+\gamma^\circ\gamma_\mu\gamma^\circ\frac{\partial}{\partial\xi^-},$$

$$\hat{\Pi} = -i\frac{\partial}{\partial\lambda} + \eta^+\frac{\partial}{\partial\eta^+} + \xi^-\frac{\partial}{\partial\xi^-},$$

$$\hat{S}^-_\alpha = -8i\Sigma_{\mu\nu}(\sigma^{\mu\nu}\xi^-)_\alpha + 8(\gamma^\nu\eta^+)_\alpha\frac{\partial}{\partial x^\nu} - 8(\gamma^\nu\hat{x}\xi^-)_\alpha\frac{\partial}{\partial x^\nu} -$$
$$-(8\frac{\partial}{\partial k} - 12i\frac{\partial}{\partial\lambda})\xi^-_\alpha - 8\xi^-\gamma^\circ\xi^-(\gamma^\circ\frac{\partial}{\partial\xi^-})_\alpha - 16\xi^-_\alpha\eta^+\frac{\partial}{\partial\eta^+},$$

$$\hat{T}^+_\alpha = -8i\Sigma_{\mu\nu}(\sigma^{\mu\nu}\eta^+)_\alpha - 8i\Sigma_{\mu\nu}[(x^\mu\gamma^\nu - x^\nu\gamma^\mu)\xi^-]_\alpha +$$

$$+\,8(\hat{x}\gamma^{\nu}\eta^{+})_{a}\frac{\partial}{\partial x^{\nu}}+\left(8\frac{\partial}{\partial k}+12i\frac{\partial}{\partial\lambda}\right)\eta_{a}^{+}-8\eta^{+}\gamma^{\circ}\eta^{+}\left(\gamma^{\circ}\frac{\partial}{\partial\eta^{+}}\right)_{a}- \tag{3.3}$$

$$-16\eta_{a}^{+}\xi^{-}\frac{\partial}{\partial\xi^{-}}-8(\gamma^{\mu}\xi^{-})_{a}\left[(2x_{\mu}x_{\nu}-x^{2}g_{\mu\nu})\frac{\partial}{\partial x^{\nu}}+2x_{\mu}\frac{\partial}{\partial k}\right].$$

We note here that the operator $\hat{P}_{\mu}$ has obtained an additional term acting on the Grassman variables. Therefore this representation is evidently not the physical one. But on the other hand most of the generators are essentially simplified which justifies the use of such a representation. We exploit it in what follows since all the results can be reexpressed in terms of the physical representation due to the nondegeneracy of the substitution (3.2). Besides, in the expressions (3.3) we turn back to the variables k and λ in accordance with paper [1], substituting d and z by $\frac{\partial}{\partial k}$ and $i\frac{\partial}{\partial\lambda}$, respectively. Therefore the superfield $\mathfrak{U}$ becomes a function of two more variables k and λ so that the representations with given d and z have to be separated at the end according to the equation:

$$\mathfrak{U}(x,\eta^{+},\xi^{-},k,\lambda)=e^{dk-i\lambda z}\,\mathfrak{U}_{(d,z)}(x,\eta^{+},\xi^{-}). \tag{3.4}$$

We first of all find the finite transformations of the arguments $(x_{\mu},\eta_{a}^{+},\xi_{a}^{-},k,\lambda)$ but we write out only the ones corresponding to the generators $\hat{S}_{a}^{-}$ and $\hat{T}_{a}^{+}$. The rest of the transformations are either well-known (see for instance [4, 5]) or they can be obtained trivially from the corresponding generators. (For example $\hat{P}_{\mu}$, $\hat{S}_{a}^{+}$, $\hat{T}_{a}^{-}$ evidently generate translations and so on). Using the

explicit form of the operators $\hat{S}_{a}^{-}$ and $\hat{T}_{a}^{+}$ it is not difficult to obtain the following relations:

a) for $\hat{S}_{a}^{-}$

$$\begin{aligned}
x_{\mu} &\to (1+16i\beta^{-}\gamma^{\circ}\xi^{-})^{-1/2}\,x_{\nu}\Lambda^{\nu}_{\mu}(\beta^{-},\xi^{-})+8i\beta^{-}\gamma^{\circ}\gamma_{\mu}\eta^{+}\\
\eta_{a}^{+} &\to \frac{\eta_{a}^{+}}{(1+16i\beta^{-}\gamma^{\circ}\xi^{-})},\\
\xi_{a}^{-} &\to \xi_{a}^{-}+8i\xi^{-}\gamma^{\circ}\xi^{-}\beta_{a}^{-},\\
k &\to k-8i\beta^{-}\gamma^{\circ}\xi^{-}+32\beta^{-}\gamma^{\circ}\beta^{-}\xi^{-}\gamma^{\circ}\xi^{-},\\
\lambda &\to \lambda-12\beta^{-}\gamma^{\circ}\xi^{-}-48i\beta^{-}\gamma^{\circ}\beta^{-}\xi^{-}\gamma^{\circ}\xi^{-},
\end{aligned} \tag{3.5}$$

where the matrix

$$\Lambda^{\nu}_{\mu}(\beta^{-},\xi^{-})=\delta^{\nu}_{\mu}+16i\beta^{-}\gamma^{\circ}\sigma^{\nu}_{\mu}\xi^{-}-48\beta^{-}\gamma^{\circ}\beta^{-}\xi^{-}\gamma^{\circ}\xi^{-}\delta^{\nu}_{\mu}$$

(see Appendix B) belongs to a finite dimensional representation of the Lorentz group.

b) For $\hat{T}_{a}^{+}$ we have

$$\begin{aligned}
x_{\mu} &\to \frac{y_{\mu}+c_{\mu}y^{2}}{\rho(y,c)},\\
\eta_{a}^{+} &\to \eta_{a}^{+}+8i\eta^{+}\gamma^{\circ}\eta^{+}\beta_{a}^{+},\\
\xi_{a}^{-} &\to (1+16i\beta^{+}\gamma^{\circ}\eta^{+})^{-1}\,\xi_{a}^{-},\\
k &\to k+8i\beta^{+}\gamma^{\circ}\eta^{+}-16i\beta^{+}\gamma^{\circ}\hat{x}\xi^{-}-32\beta^{+}\gamma^{\circ}\beta^{+}\eta^{+}\gamma^{\circ}\eta^{+}-\\
&\quad -128\beta^{+}\gamma^{\circ}\beta^{+}\eta^{+}\gamma^{\circ}\hat{x}\xi^{-}+64x^{2}\beta^{+}\gamma^{\circ}\beta^{+}\xi^{-}\gamma^{\circ}\xi^{-},\\
\lambda &\to \lambda-12\beta^{+}\gamma^{\circ}\eta^{+}-48i\beta^{+}\gamma^{\circ}\beta^{+}\eta^{+}\gamma^{\circ}\eta^{+},
\end{aligned} \tag{3.7}$$

where c_{μ}, y_{μ} and $\rho(y,c)$ are determined in the following way

$$y_\mu = (1 + 16i\beta^+\gamma^\circ\eta^+)^{1/2}\, x_\nu \Lambda^\nu{}_\mu(\beta^+,\eta^+),$$

$$c_\mu = 8i\beta^+\gamma^\circ\gamma_\mu\xi^- , \qquad (3.8)$$

$$\rho(y,c) = 1 + 2y\cdot c + y^2c^2.$$

Let us consider now functions of these variables. Substituting the argument transformations from formulae (3.5) and (3.7) into the function $\mathfrak{U}(x,\eta^+,\xi^-,k,\lambda)$ and separating the factor $e^{dk-i\lambda z}$ we find for the scalar superfield:

$$e^{i\beta^-\gamma^\circ\hat{S}^-}\mathfrak{U}_{(d,z)}(x,\eta^+,\xi^-) =$$

$$=(1+16i\beta^-\gamma^\circ\xi^-)^{-\frac{1}{2}(d-\frac{3}{2}z)}\,\mathfrak{U}_{(d,z)}\{(1+16i\beta^-\gamma^\circ\xi^-)^{-1/2}x_\nu\Lambda^\nu{}_\mu(\beta^-,\xi^-)+$$

$$+8i\beta^-\gamma^\circ\gamma_\mu\eta^+,\ (1+16i\beta^-\gamma^\circ\xi^-)^{-1}\eta^+_a,\ \xi^-_a+8i\xi^-\gamma^\circ\xi^-\beta^-_a\},$$

$$e^{i\beta^+\gamma^\circ\hat{T}^+}\mathfrak{U}_{(d,z)}(x,\eta^+,\xi^-) =$$

$$= (1+16i\beta^+\gamma^\circ\eta^+)^{\frac{1}{2}(d+\frac{3}{2}z)}\,\rho^{-d}(y,c)\,\mathfrak{U}_{(d,z)}\{\frac{y_\mu+c_\mu y^2}{\rho(y,c)},\ \eta^+_a+$$

$$+8i\eta^+\gamma^\circ\eta^+\beta^+_a,\ (1+16i\beta^+\gamma^\circ\eta^+)^{-1}\xi^-_a\}. \qquad (3.10)$$

In the case of a superfield of arbitrary Lorentz structure it is necessary to separate from the exponents $e^{i\beta^-\gamma^\circ\hat{S}^-}$ and $e^{i\beta^+\gamma^\circ\hat{T}^+}$ all multiplicative terms which do not contain derivatives. Such terms are evidently finite-dimensional matrices. So we find:

$$(e^{i\beta^-\gamma^\circ\hat{S}^-})_{AB} = W_{AB}(\beta^-,\xi^-)\,e^{i\beta^-\gamma^\circ\hat{S}'^-}, \qquad (3.11)$$

$$(e^{i\beta^+\gamma^\circ\hat{T}^+})_{AB} = W_{AC}(\beta^+,\eta^+)K_{CB}(y,c)\,e^{i\beta^+\gamma^\circ\hat{T}'^+}, \qquad (3.12)$$

where $\hat{S}'^-_a$ and $\hat{T}'^+_a$ denote these terms in the generators which are homogeneous functions of the derivatives with respect to all the arguments. $(\Sigma_{\mu\nu})_{AB}$ are matrices of the generators of the finite dimensional irreducible representation of the Lorentz group, y_μ and c_μ are given by eq. (3.8) and the matrices W_{AB} and K_{AB} have the following form:

$$W_{AB}(\beta^-,\xi^-) = \delta_{AB} + 8(\Sigma_{\mu\nu})_{AB}\beta^-\gamma^\circ\sigma^{\mu\nu}\xi^- +$$

$$+ 32(\Sigma_{\mu\nu}\Sigma_{\lambda\rho})_{AB}\beta^-\gamma^\circ\sigma^{\mu\nu}\xi^-\beta^-\gamma^\circ\sigma^{\lambda\rho}\xi^-, \qquad (3.13)$$

$$K_{AB}(y,c) = \delta_{AB} + 16(\Sigma_{\mu\nu})_{AB}\,y^\mu c^\nu +$$

$$+ 128(\Sigma_{\mu\nu}\Sigma_{\lambda\rho})_{AB}\,y^\mu y^\lambda c^\nu c^\rho.$$

In order to write the final result for the matrices W_{AB} and K_{AB} it is necessary to calculate the action of the matrices $(\Sigma_{\mu\nu}\Sigma_{\lambda\rho})_{AB}(\beta^-\gamma^\circ\sigma^{\mu\nu}\xi^-)(\beta^-\gamma^\circ\sigma^{\lambda\rho}\xi^-)$ and $(\Sigma_{\mu\nu}\Sigma_{\lambda\rho})_{AB}\,y^\mu y^\lambda c^\nu c^\rho$ on the superfield. For this purpose it is convenient to use the realization of the finite-dimensional irreducible Lorentz group representation in the spinor space $\mathfrak{U}_{\{\alpha_p\}\{\beta_q\}}$.

We recall that in this case the matrices $\Sigma_{\mu\nu}$ act on the field $\mathfrak{U}_{\{\alpha_p\}\{\beta_q\}}$ in the following way:

$$(\Sigma_{\mu\nu})_{\{\alpha_p \alpha'_p\}\{\beta_q, \beta'_q\}} \; \mathfrak{U}_{\{\alpha'_p\}\{\beta'_q\}} = \tag{3.14}$$
$$= i \sum_{k=1}^{p} (\sigma_{\mu\nu})_{\alpha_k \delta} \mathfrak{U}_{\{\alpha_p(-k)\delta\}\{\beta_q\}} + i \sum_{k=1}^{q} (\sigma_{\mu\nu})_{\beta_k \delta} \mathfrak{U}_{\{\alpha_p\}\{\beta_q(-k)\delta\}},$$

After some calculation it is not difficult to verify the below written final relations;

$$W_{\{\alpha_p,\alpha'_p\}\{\beta_q,\beta'_q\}}(\bar\beta^-, \bar\xi^-) = \delta_{\{\alpha_p,\alpha'_p\}\{\beta_q,\beta'_q\}} +$$
$$+ 8(\Sigma_{\mu\nu})_{\{\alpha_p,\alpha'_p\}\{\beta_q,\beta'_q\}} \beta^- \gamma^\circ \sigma^{\mu\nu} \xi^- - \tag{3.15}$$
$$- 8q(q+2)\delta_{\{\alpha_p,\alpha'_p\}} \sum_{k=1}^{q} \delta_{\{\beta_q(-k), \beta'_q(-k)\}} (1-i\gamma_5)_{\beta_k \beta'_k} \beta^- \gamma^\circ \beta^- \bar\xi^- \gamma^\circ \bar\xi^- =$$
$$= \delta_{\{\alpha_p,\alpha'_p\}} \prod_{k=1}^{q} \exp\{8(\sigma_{\mu\nu})_{\beta_k \beta'_k} \beta^- \gamma^\circ \sigma^{\mu\nu} \xi^-\},$$

$$W_{\{\alpha_p,\alpha'_p\}\{\beta_q \beta'_q\}}(\beta^+, \eta^+) = \delta_{\{\alpha_p,\alpha'_p\}\{\beta_q,\beta'_q\}} +$$
$$+ 8(\Sigma_{\mu\nu})_{\{\alpha_p,\alpha'_p\}\{\beta_q,\beta'_q\}} \beta^+ \gamma^\circ \sigma^{\mu\nu} \eta^+ - \tag{3.16}$$
$$- 8p(p+2)\delta_{\{\beta_q,\beta'_q\}} \sum_{k=1}^{p} \delta_{\{\alpha_p(-k), \alpha'_p(-k)\}} (1+i\gamma_5)_{\alpha_k \alpha'_k} \beta^+ \gamma^\circ \beta^+ \eta^+ \gamma^\circ \eta^+ =$$
$$= \delta_{\{\beta_q,\beta'_q\}} \prod_{k=1}^{p} \exp\{8(\sigma_{\mu\nu})_{\alpha_k \alpha'_k} \beta^+ \gamma^\circ \sigma^{\mu\nu} \eta^+\},$$

$$K_{\{\alpha_p,\alpha'_p\}\{\beta_q,\beta'_q\}}(y,c) = \delta_{\{\alpha_p,\alpha'_p\}\{\beta_q,\beta'_q\}} + \tag{3.17}$$
$$+ y^\mu c^\nu [\delta_{\{\beta_q,\beta'_q\}} \sum_{k=1}^{p} \delta_{\{\alpha_p(-k), \alpha'_p(-k)\}} (\gamma_\mu \gamma_\nu)_{\alpha_k \alpha'_k} +$$
$$+ \delta_{\{\alpha_p,\alpha'_p\}} \sum^{q} \delta_{\{\beta_q(-k), \beta'_q(-k)\}} (\gamma_\mu \gamma_\nu)_{\beta_k \beta'_k}] + 32[(\Sigma_{\mu\lambda} \Sigma^{\lambda}_{\nu})_{\{\alpha_p,\alpha'_p\}\{\beta_q,\beta'_q\}} -$$
$$- \frac{p+q}{2} \; \frac{p+q+2}{2} \delta_{\{\alpha_p,\alpha'_p\}\{\beta_q,\beta'_q\}} g_{\mu\nu}] y^\mu y^\nu c^2 =$$
$$= \rho^{-\frac{p+q}{2}}(y,c) \prod_{k=1}^{p} (\delta_{\alpha_k \alpha'_k} + y^\mu c^\nu (\gamma_\mu \gamma_\nu)_{\alpha_k \alpha'_k}) \prod_{k=1}^{q} (\delta_{\beta_k \beta'_k} + y^\nu c^\nu (\gamma_\mu \gamma_\nu)_{\beta_k \beta'_k}).$$

Let us consider some examples.

a) For the case of spinor superfield (spin 1/2) eqs. (3.15)-(3.17) give:

$$W_{\alpha\beta}(\beta^-, \xi^-) = \delta_{\alpha\beta} + 8i(\sigma_{\mu\nu})_{\alpha\beta} \beta^- \gamma^\circ \sigma^{\mu\nu} \xi^- - 24\beta^- \gamma^\circ \beta^- \xi^- \gamma^\circ \xi^- (1-i\gamma_5)_{\alpha\beta},$$
$$W_{\alpha\beta}(\beta^+, \eta^+) = \delta_{\alpha\beta} + 8i(\sigma_{\mu\nu})_{\alpha\beta} \beta^+ \gamma^\circ \sigma^{\mu\nu} \eta^+ - 24\beta^+ \gamma^\circ \beta^+ \eta^+ \gamma^\circ \eta^+ (1+i\gamma_5)_{\alpha\beta}, \tag{3.18}$$
$$K_{\alpha\beta}(y,c) = \rho^{-1/2}(y,c)[\delta_{\alpha\beta} + y^\mu c^\nu (\gamma_\mu \gamma_\nu)_{\alpha\beta}]. \tag{3.19}$$

b) For the vector superfield we have analogously

$$\Lambda^\nu_\mu(\beta^-, \xi^-) \equiv W^\nu_\mu(\beta^-, \xi^-) = \delta^\nu_\mu + 16i\beta^- \gamma^\circ \sigma^\nu_\mu \xi^- - \tag{3.20}$$
$$- 48\beta^- \gamma^\circ \beta^- \xi^- \gamma^\circ \xi^- \delta^\nu_\mu,$$
$$\Lambda^\nu_\mu(\beta^+, \eta^+) \equiv W^\nu_\mu(\beta^+, \eta^+) = \delta^\nu_\mu + 16i\beta^+ \gamma^\circ \sigma^\nu_\mu \eta^+ -$$
$$- 48\beta^+ \gamma^\circ \beta^+ \eta^+ \gamma^\circ \eta^+ \delta^\nu_\mu, \tag{3.21}$$
$$K^\rho_\lambda(y, c) = \rho^{-1}(y,c)\{\delta^\rho_\lambda + 2(y_\lambda c^\rho - y^\rho c_\lambda + y.c\delta^\rho_\lambda) +$$
$$- \frac{1}{2}(2y_\lambda y^\rho - y^2 \delta^\rho_\lambda) c^2\}.$$

We note that it is possible to generalize the well-known from conformal symmetry transformation R-inversion to the case of conformal supersymmetry. For this purpose we should consider simultaneously the algebra (1.1) and its "conjugated". Let us make a substitution, which we denote (for convenience only) as H-operation

$$x_\mu \xrightarrow{H} y_\mu = \frac{x_\mu}{x^2},$$
$$\theta^+_\alpha \xrightarrow{H} \theta'^+_\alpha = -\frac{1}{x^2}(\hat{x}\theta^-)_\alpha, \quad \theta^-_\alpha \xrightarrow{H} \theta'^-_\alpha = -\frac{1}{x^2}(\hat{x}\theta^+)_\alpha, \tag{3.22}$$
$$\xi^-_\alpha \rightarrow \xi'^-_\alpha = (\hat{x}\xi^+)_\alpha + \theta^-_\alpha, \qquad \xi^+_\alpha \rightarrow \xi'^+_\alpha = (\hat{x}\xi^-)_\alpha + \theta^+_\alpha.$$

The inverse transformation has the form:

$$y_\mu \xrightarrow{H} x_\mu = \frac{y_\mu}{y^2},$$
$$\theta'^-_\alpha \xrightarrow{H} \theta^-_\alpha = -\frac{1}{y^2}(\hat{y}\theta'^+)_\alpha, \quad \theta'^+_\alpha \xrightarrow{H} \theta^+_\alpha = -\frac{1}{y^2}(\hat{y}\theta'^-)_\alpha, \tag{3.23}$$
$$\xi'^+_\alpha \xrightarrow{H} \xi^+_\alpha = (\hat{y}\xi'^-)_\alpha + \theta'^+_\alpha, \quad \xi'^-_\alpha \xrightarrow{H} \xi^-_\alpha = (\hat{y}\xi'^+)_\alpha + \theta^-_\alpha.$$

Now let J^R be the algebra generators after R-inversion. The automorphism

$$J^R \overset{R}{\leftrightarrow} J$$

in our case has the form:

$$\begin{aligned} &\hat{P}^R_\mu = \hat{K}_\mu, && \hat{M}^R_{\mu\nu} = \hat{M}_{\mu\nu}, \\ &\hat{D}^R = -\hat{D}, && \hat{K}^R_\mu = \hat{P}_\mu, \\ &\hat{S}^{\pm R}_\alpha = \hat{T}^\pm_\alpha, && \hat{T}^{\pm R}_\alpha = \hat{S}^\pm_\alpha, \\ &\hat{\Pi}^R = -\hat{\Pi}. \end{aligned} \tag{3.24}$$

Using the above defined H-operation it is possible to combine eqs. (3.24) and to obtain the final formula for the R-inversion in the case of conformal superalgebra

$$J \xrightarrow{R} J^R = H\tilde{J}H, \tag{3.25}$$

where $\tilde{J}$ are generators of the representation "conjugate" to (1.1).

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APPENDIX A

The most general expansion of the superfield $\mathcal{U}_{\{\alpha_p\}\{\beta_q\}}(x,\theta^+,\xi^-)$ (see formula 2.3) in powers of ξ^-_α has the form:

$$\mathcal{U}_{\{\alpha_p\}\{\beta_q\}}(x,\theta^+,\xi^-) = \Psi_{\{\alpha_p\}\{\beta_q\}}(x,\theta^+) + \\ + R_{\{\alpha_p\}\{\beta_q\}\delta}(x,\theta^+)(\gamma^\circ\xi^-)_\delta + \Phi_{\{\alpha_p\}\{\beta_q\}}(x,\theta^+)\,\xi^-\gamma^\circ\xi^-, \tag{A.1}$$

where $\{\ldots\}$ denotes, as in the text above total symmetrization over the spinor indices. The index δ in $R_{\{\alpha_p\}\{\beta_q\}\delta}$ is of the same type as the indices $\{\beta_q\}$, but is not symmetrized with the latter ones. Therefore the structure R belongs to a reducible representation of the Lorentz group. We decompose the structure R into irreducible Lorentz structures.

$$R_{\{\alpha_p\}\{\beta_q\}\delta}(x,\theta^+) = \frac{1}{q}\sum_{k=1}^{q}\chi_{\{\alpha_p\}\{\beta_q(-k)\}}\,\xi^-_{\beta_k} + \\ + \frac{1}{q}\sum^{q}\phi^{\mu\nu}_{\{\alpha_p\}\{\beta_q(-k)\}}(\sigma_{\mu\nu}\xi^-)_{\beta_k}, \tag{A.2}$$

where

$$\chi_{\{\alpha_p\}\{\beta_q(-k)\}} = R_{\{\alpha_p\}\{\beta_q(-k)[\gamma\}\delta]}(\gamma^\circ)^{\gamma\delta}, \tag{A.3}$$

$$\phi^{\mu\nu}_{\{\alpha_p\}\{\beta_q(-k)\}} = R_{\{\alpha_p\}\{\beta_q(-k)|\gamma\delta\}}(\gamma^\circ\sigma^{\mu\nu})^{\gamma\delta} \tag{A.4}$$

and they belong to the irreducible representations $\tau_{\frac{p}{2},\frac{q-1}{2}}$ and $\tau_{\frac{p}{2},\frac{q+1}{2}}$, respectively. It is important for us to have an inverse formula to formula (A.4) as it turns out to be convenient sometimes to deal with the totally symmetrized quantity $R_{\{\alpha_p\}\{\beta_q\delta\}}$.

For this purpose it is enough to multiply both sides of eq. (A.4) by $\sigma_{\mu\nu}\gamma^\circ(1+i\gamma_5)_{\gamma\delta}$, then symmetrize with respect to k and make use of the following identity:

$$\frac{1}{2}[\gamma^\circ\sigma_{\mu\nu}(1-i\gamma_5)]_{\beta_k\delta}[\sigma^{\mu\nu}\gamma^\circ(1+i\gamma_5)]_{\gamma\delta} = (1+i\gamma_5)_{\beta_k\gamma}(1+i\gamma_5)_{\delta\rho} - \frac{1}{2}\gamma^\circ(1-i\gamma_5)_{\beta_k\delta}\gamma^\circ(1+i\gamma)_{\gamma\delta}. \qquad (A.5)$$

As a result we have

$$R_{\{\alpha_p\}\{\beta_q(-k)\gamma\rho\}} = \frac{1}{2q}\sum_{k=1}\phi^{\mu\nu}_{\{\alpha_p\}\{\beta_q(-k)\}}\sigma_{\mu\nu}\gamma^\circ(1+i\gamma_5)_{\gamma\rho}. \qquad (A.6)$$

In calculating the action of the generator $\hat{S}^-_A$ on the superfield it is convenient to use the following identities which can be found by the general method of expanding an arbitrary spinor in covariant Lorentz structures

$$(\sigma_{\mu\nu})_{\beta_k}^{\ \ \rho}\Psi_{\{\alpha_p\}\{\beta_q(-k)\rho\}}(\sigma^{\mu\nu}\xi^-)_A = \qquad (A.7)$$
$$= \frac{3}{4}\Psi_{\{\alpha_p\}\{\beta_q(-k)\rho\}}(\gamma^\circ\xi^-)^\rho\gamma^\circ(1+i\gamma_5)_{\beta_k A} -$$
$$-\frac{1}{2}\Psi_{\{\alpha_p\}\{\beta_q(-k)\rho\}}(\gamma^\circ\sigma^{\mu\nu}\xi^-)^\rho(\sigma_{\mu\nu}\gamma^\circ)_{\beta_k A},$$

$$\Psi_{\{\alpha_p\}\{\beta_q\}}\xi^-_A = \frac{1}{q}\sum_{k=1}^{q}\Big\{\frac{1}{4}\Psi_{\{\alpha_p\}\{\beta_q(-k)\rho\}}(\gamma^\circ\xi^-)^\rho\gamma^\circ(1+i\gamma_5)_{\beta_k A} +$$
$$+\frac{1}{2}\Psi_{\{\alpha_p\}\{\beta_q(-k)\rho\}}(\gamma^\circ\sigma^{\mu\nu}\xi^-)^\rho(\sigma_{\mu\nu}\gamma^\circ)_{\beta_k A}\Big\}, \qquad (A.8)$$

$$(\sigma^{\mu\nu}\xi^-)_\alpha(\sigma_{\mu\nu})_{\beta\gamma} = [\gamma^\circ(1+i\gamma_5)]_{\beta\alpha}(\gamma^\circ\xi^-)_\gamma - \frac{1}{2}(1-i\gamma_5)_{\beta\gamma}\xi^-_\alpha. \qquad (A.9)$$

The same procedure gives the possibility to obtain two more identities:

$$(\sigma_{\mu\nu})_A^{\ \ \rho}\Psi_{\{\alpha_p\}\{\beta_q(-k)\rho\}}(\sigma^{\mu\nu}\xi^-)_{\beta_k} =$$
$$= -\frac{3}{4}\Psi_{\{\alpha_p\}\{\beta_q(-k)\rho\}}(\gamma^\circ\xi^-)^\rho\gamma^\circ(1+i\gamma_5)_{\beta_k A} - \qquad (A.10)$$
$$-\frac{1}{2}\Psi_{\{\alpha_p\}\{\beta_q(-k)\rho\}}(\gamma^\circ\sigma^{\mu\nu}\xi^-)^\rho(\sigma_{\mu\nu}\gamma^\circ)_{\beta_k A},$$

$$\Psi_{\{\alpha_p\}\{\beta_q(-k)A\}}\xi^-_{\beta_k} = -\frac{1}{4}\Psi_{\{\alpha_p\}\{\beta_q(-k)\rho\}}(\gamma^\circ\xi^-)^\rho\gamma^\circ(1+i\gamma_5)_{\beta_k A} + \qquad (A.11)$$
$$+\frac{1}{2}\Psi_{\{\alpha_p\}\{\beta_q(-k)\rho\}}(\gamma^\circ\sigma^{\mu\nu}\xi^-)^\rho(\sigma_{\mu\nu}\gamma^\circ)_{\beta_k A}.$$

It is easily seen that eqs. (A.10) and (A.11) can be solved with respect to the structure in their right-hand sides. Then we have

$$\Psi_{\{\alpha_p\}\{\beta_q(-k)\rho\}}(\gamma^\circ\xi^-)^\rho\gamma^\circ(1+i\gamma)_{\beta_k A} = -\Psi_{\{\alpha_p\}\{\beta_q(-k)A\}}\xi^-_{\beta_k} -$$
$$-(\sigma_{\mu\nu})_A^{\ \ \rho}\Psi_{\{\alpha_p\}\{\beta_q(-k)\rho\}}(\sigma^{\mu\nu}\xi^-)_{\beta_k}, \qquad (A.12)$$

$$\Psi_{\{\alpha_p\}\{\beta_q(-k)\rho\}}(\gamma^\circ\sigma^{\mu\nu}\xi^-)^\rho(\sigma_{\mu\nu}\gamma^\circ)_{\beta_k A} = \frac{3}{2}\Psi_{\{\alpha_p\}\{\beta_q(-k)A\}}\xi^-_{\beta_k} -$$
$$-\frac{1}{2}(\sigma_{\mu\nu})_A^{\ \ \rho}\Psi_{\{\alpha_p\}\{\beta_q(-k)\rho\}}(\sigma^{\mu\nu}\xi^-)_{\beta_k}. \qquad (A.13)$$

Substituting (A.12) and (A.13) in (A.7) and (A.8) and taking their sum with the proper coefficients we find formula (2.6). To obtain the expression (2.7) one has to use eq. (A.9).

APPENDIX B

Let σ_i $(i=1,2,3)$ be the Pauli matrices. As is well known, they satisfy the following commutation and anticommutation relations, respectively:

$$[\sigma_i, \sigma_k] = 2\epsilon_{ik\ell}\sigma_\ell, \qquad \{\sigma_i, \sigma_k\} = 2\delta_{ik}. \tag{B.1}$$

An arbitrary 2x2-matrix and in particular a unitary one can be decomposed in terms of σ_i and the unit matrix:

$$\mathfrak{U}(a) = \frac{1}{\sqrt{1+\frac{a^2}{4}}}\left(1+\frac{i}{2}a_j\sigma_j\right). \tag{B.2}$$

As is known, the matrices $\mathfrak{U}(a)$ belong to SU(2) and using eqs. (B.2) and (B.1) it is easy to find the multiplication law of the parameters a_j. Taking the product

$$\mathfrak{U}(a_j)\mathfrak{U}(\beta_j) = \mathfrak{U}((a\beta)_j)$$

we obtain:

$$(a\beta)_j = \frac{a_j + \beta_j - \frac{1}{2}\epsilon_{jk\ell}\, a_k \beta_\ell}{1-\frac{1}{4}(a_j\beta_j)}. \tag{B.3}$$

Let us now consider the matrices $\sigma^{\mu\nu} = \frac{1}{4}[\gamma^\mu, \gamma^\nu]$, where the γ^μ are the Dirac matrices in Majorana representation. In particular we are interested in $\sigma^{i0}(1-i\gamma_5) = S^i$. It can be shown that these matrices satisfy commutation and anticommutation relations analogous to (B.1):

$$[S^i, S^k] = 2i\epsilon_{ik\ell}S^\ell, \qquad \{S^i, S^k\} = (1-i\gamma_5)\delta^{ik}. \tag{B.4}$$

The relations (B.4) and (B.1) are practically the same since in the subspace where S^i act the matrix $\frac{1}{2}(1-i\gamma_5)$ plays the role of the unit matrix. Therefore

$$V(a) = \frac{1}{\sqrt{1+\frac{a^2}{4}}}\left(1+\frac{i}{2}a_i S^i\right) \tag{B.5}$$

have just the properties of the matrices $\mathfrak{U}$ in (B.2). In particular they also form a representation of the group SU(2) and what is most important the multiplication law of their parameters is the same as (B.3). Let us now consider the case when the parameters of $V(a)$ have for example, the following form:

$$a_i = N(\beta^-\gamma^\circ\sigma^{i\circ}\xi^-), \tag{B.6}$$

where β^- is the spinor parameter of the global $\hat{S}^-$-transformation, N - an arbitrary number and ξ^--grassmann variable which transforms according to :

$$\xi^- \to \xi^- + 8i\beta^-\bar{\xi}^-\gamma^\circ\xi^-$$

(see (3.5)).

Under the action of the $\hat{S}^-$-transformation the parameter a_j in the form (B.6) change, too. If we perform a global S^--transformation with a parameter β_1^-, we obtain

$$a_i' = N(\beta^-\gamma^\circ\sigma^{i\circ}\xi^-) + 8iN\beta^-\gamma^\circ\sigma^{i\circ}\beta_1^-\xi^-\bar{\gamma}^\circ\xi^-. \tag{B.7}$$

Using now eq. (B.3) it is easy to show the following relation:

$$(a_1 a')_i = N(\beta^- + \beta_1^-)\gamma^\circ\sigma^{i\circ}\xi^-, \tag{B.8}$$

where $a_{1i} = N\beta_1^-\gamma^\circ\sigma^{i\circ}\xi^-$.

Let $T(a)$ be an arbitrary finite representation of the group SU(2) We substitute in $T(a)$ the parameters a_i from (B.6). After that $T(a)$ become functions of the grassmann variables β^- and ξ^-:

$$T(a) \equiv \tau(\beta^-, \xi^-).$$

Let e^T be an arbitrary element in the space of the representation $T(a)$:

$$e^{T\prime} = T(a)e^T \equiv \tau(\beta^-, \xi^-)e^T :$$

Performing another transformation with a parameter β_1^- on the element $e^{T'}$, we have

$$e^{T''} = T(\alpha_{1i})T(\alpha'_i)e^T = T((\alpha_1\alpha')_i)e^T,$$

where α' and α_1 are determined from eqs. (B.7) and (B.9), respectively. Due to eq. (B.8) we obtain the following relation:

$$\tau(\beta_1^-, \xi^-)\tau(\beta^-, \xi^- + 8i\beta^-\xi^-\gamma^\circ\xi^-) = \tau(\beta_1^- + \beta^-, \xi^-). \qquad (B.10)$$

The matrices (3.16) for different irreducible $\Sigma_{\mu\nu}$ coincide with the discussed here matrices τ. In particular, the matrix $\Lambda^\nu{}_\mu$ which enters into the transformation law of x_μ is a realization of the vector representation of the group SU(2), and the matrix W_{AB} which appears in the transformation law of spinor superfields is identical with the considered here matrix $V(\alpha)$.

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