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Summary

On the basis of the simplified version of the relativistic three-dimensional two-body equation (the quasi-potential equation in terms of rapidities) the composite model of mesons is built. The considerations are made in terms of the relativistic relative distance r , which is canonically conjugated to rapidities in the sense of the Fourier analysis on the Lorentz group. The interaction between quarks is described by the linear potential. The calculations give the satisfactory description of masses and leptonic widths of vector mesons. The energies of radial and orbital excitations in the systems $p\bar{p}$, $n\bar{n}$, $\lambda\bar{\lambda}$, $c\bar{c}$ are found. Between the radial excitations of S-states of the $c\bar{c}$ -system (the ground state being identified with Ψ (3.095)) there are states with masses 3.68 GeV (Ψ'), 4.16 GeV (Ψ''), 4.57 GeV (Ψ''') and 5.91 GeV (Upsilon Υ).

The model is also considered, which is based on the quantum field theory with the fundamental length l_0 . The geometrical nature of the variable, which stands here for the rapidities, reflects the new properties of interactions on a super small distances. The hypothesis, that the quark mass coincides with "maximon" mass $M = \hbar/l_0 c = \sqrt{\hbar^3/cG_F}$ =300 GeV (G_F is Fermi constant), leads to the correct mesonic mass spectrum.

1. INTRODUCTION

The aim of this paper is to investigate the composite model of the mesons with spin ≥ 1 based on the relativistic quasipotential type equation, which has been proposed by the authors[1]. Recently interest in the dynamical equations, describing the composite quark models[2-4], has increased due to the discovery of a number of new resonances.

After the suggestion had been made that Ψ (3095) and Ψ'(3684) are the bound states of charmed quark c and its antiquark $\bar{c}$ [5], a number of models was considered[5]-[10] with linear potential of the form:

$$V(r) = gr - V_0 . \tag{1.1}$$

We take particular interest in paper[10] which deals with the quasipotential equation[11]-[12] in relativistic configurational representation[13],[14]. The finite-difference Schroedinger equation with the step equal to the Compton wave length $\hbar/mc$ of quark was solved there using the Laplace transformation. The lowest radial excitations correspond to Ψ, Ψ' and ρ, ρ' mesons.

However, in analising the finite-difference equation one runs into some difficulties, the main of which is connected with the complexity of boundary conditions. In detailed calculations both of radial and orbital excitations in the system of quark and antiquark and of the decay widths, the relativistic equation proposed in[1] proved to be more suitable. Consider it in more detail.

For the wave function $\Psi_q(\vec{p})$ of a two-particle system with equal masses m this equation has the form:

$$(\chi_q^2-\chi_p^2)\Psi_q(\vec{p})=\frac{\chi_q}{sh\chi_q}\left(\int V(\vec{p},\vec{k};E_q)\Psi_q(k)\,d\Omega_k\right)\frac{1}{(2\pi)^3}, \quad (1.2)$$

where

$$d\Omega_k=d^3k\Big/\sqrt{1+\frac{k^2}{m^2}}\ ,$$

$$2E_q=\sqrt{s}=2m\,ch\,\chi_q=\mu \quad \text{(the bound state mass)}, \quad (1.3)$$

$$\hbar=c=1.$$

For the state with fixed orbital momentum ℓ the equation (1.2) is reduced in the relativistic configurational r -representation[14],[15] to the differential one:

$$\left[\frac{1}{m}\frac{d^2}{dr^2}+m\chi_q^2-m R_\ell(rm,\chi_q)\right]\varphi_{q\ell}(r)=\gamma_\ell(rm,\chi_q)V(r)\varphi_{q\ell}(r) \quad (1.4)$$

with the boundary conditions

$$\varphi_{q\ell}(0)=\varphi_{q\ell}(\infty)=0. \quad (1.5)$$

The explicit expressions for centrifugal potential $R_\ell(r,\chi_q)$ and the coefficient $\gamma_\ell(r,\chi_q)$ can be found in[1].

In principle, the quasipotential $V(r)$ is determined by the field-theoretical diagrams. However, the arguments, favouring the potental (1.1), are essentially beyond the framework of perturbation theory[16]. Besides the relativistic configurational r -space strongly differs in its nature from the usual one. Its connection with the momentum representation is realized by the "Shapiro

transformation"[14],[15] with the kernels

$$\langle\vec{p}\,|\,\vec{r}\rangle=\left(\frac{p_0-\vec{p}\vec{n}}{m}\right)^{-1-imr}, \qquad \vec{r}=r\vec{n} \quad (1.6)$$

turning to the usual plane waves only in the nonrelativistic limit $m\to\infty$:

$$\langle\vec{p}\,|\,\vec{r}\rangle\xrightarrow[m\to\infty]{} e^{i\vec{p}\vec{r}}. \quad (1.7)$$

Taking into account these arguments, we do not derive the potential (1.1) from the field theory but simply postulate it. In maintaining such an insertion of interaction directly in the relativistic r-space equation (1.4), the analogy with quantum mechanics, where the potentials are also postulated, is essential.

Note that the relativistic configurational r -representation was also employed in[17] to study the properties of the hadron wave function in the parton model.

2. The Case of Light Quarks

Consider first the state with zeroth orbital momentum. This is the case when

$$R_0(r,\chi_q)=0, \quad \gamma_0(r,\chi_q)=\chi_q/sh\chi_q \quad (2.1)$$

and the equation (1.4) with the potential (1.1) is reduced to the Airy equation, the wave function $\varphi_n(r)$ corresponding to the bound states having the form

$$\varphi_n(r)=\left(\frac{gm\chi_n}{sh\chi_n}\right)^{1/3}Ai\left[\left(\frac{gm\chi_n}{sh\chi_n}\right)^{1/3}\left(r-\frac{V_0}{g}-\frac{m}{g}\chi_n\,sh\chi_n\right)\right], \quad (2.2)$$

where n is the principal quantum number, and $Ai(z)$ is the Airy function. For (2.2) the normalization condition:

$$\int|\varphi_n(r)|^2dr=1 \quad (2.3)$$

is fulfilled. Keeping (1.5) we have the equation for the spectrum of rapidities:

$$Z_n = -\left(\frac{m\, \chi_n}{g^2\, sh\, \chi_n}\right)^{1/3} \left(V_0 + m\, \chi_n\, sh\, \chi_n \right), \tag{2.4}$$

where Z_n are the Airy function zeros.

Comparing the ground 1^3S state and the first radial excitation 2^3S in $p\bar{p}, n\bar{n}, \lambda\bar{\lambda}$ and $c\bar{c}$ systems to vector mesons (see tables II-7) we obtain the quark masses and parameters of the potential g and V_0 (compare[6]-[8]). In the case of $p\bar{p}$-system two versions of 2^3S-state are considered, viz, ρ'(1.6) and ρ' (1.25). The experimental data on the masses of resonances are taken from papers[18].

If one assumes the slope g of the potential to be universal for all quarks and puts $V_0 = 0$ in $c\bar{c}$-system, this is sufficient to determine unambiguously the quark masses and the depths V_0 (the latter are not universal).

Our calculations indicate , however, that the lowest states of the S -wave spectrum ($n < 5$) vary slowly with the slope.

We adopted the dependence of the slope on the quark system and changed it in the limits: 0.064 GeV$^2 \leq g_p^{II} \leq$ 0.094 GeV^2, 0.064 GeV$^2 \leq g_{p,n}^{III} \leq$ 0.22 GeV^2, 0.014 GeV$^2 \leq g_\lambda \leq$ 0.094 GeV^2, 0.15 GeV$^2 \leq g_c \leq$ 0.25 GeV^2. This corresponds to the following regions in which quark masses change: 0.17 GeV $\leq m_p^{II} \leq$ 0.63 GeV, 0.036 GeV $\leq m_p^{III} \leq$ 0.8 GeV, 0.03 GeV $\leq m_n^{III} \leq$ 0.59 GeV, 0.012 GeV $\leq m_\lambda \leq$ 0.73 GeV and 0.34 GeV $\leq m_c \leq$ 1.85 GeV.

The maximum difference in the masses of resonances equals now 0.01 GeV in the ground state and 0.17 GeV in the state with n =4. To choose the parameters we used an additional criterion, namely, the requirement of a good description of the leptonic

widths of vector mesons which are highly sensitive to the slope variations.

Let us calculate the leptonic width Γ_L of a meson in the approximation of annihilation at rest:

$$\Gamma_L = \lim_{v \to 0} \left[\sigma(s,v) j \right], \tag{2.5}$$

where $\sigma(s, v)$ is the total cross section of the annihilation in a given channel, v is the velocity, j is the relative flux equal to

$$j = \chi_n \, |\Psi_n(0)|^2 ; \tag{2.6}$$

$$\Psi_n(z) = \frac{1}{\sqrt{4\pi}} \; \frac{\varphi_n(z)}{z} . \tag{2.7}$$

In the decay of orthostates into leptons the one-photon annihilation is known to be the lowest allowed electromagnetic process. Calculating its cross section in the framework of the model with twelve coloured quarks, (see, e.g.[19]) one obtains finally:

$$\Gamma_L = \frac{\chi_n}{sh\,\chi_n} \; \frac{4\alpha^2 g m}{\mu_n{}^2} e_Q^2 , \tag{2.8}$$

where e_Q^2 is an effective charge, defined by the "quark content of meson"; $e_Q^2 = 1/2 , 1/18 , 1/9 , 4/9$ for ρ^0, ω^0, ϕ^0 and ψ mesons, resp.

The results of our computations together with experimental data for the widths[18] are presented in table I.

In the case of $\ell \neq 0$ the equation (1.4) has no analytic solution. The energy levels were evaluated at computer.

For numerical calculations of the discrete spectrum of the equation (1.4) together with the boundary conditions (1.5) there are applicable all methods developed for the Schroedinger equation. The calculation process is slightly complicated in our case due to dependence of the coefficients $R_\ell(z,\chi)$ and $\gamma_\ell(z,\chi)$ on energy.

In accordance with[1] these functions are defined at arbitrary values of ℓ via the free solutions $s_\ell(z,\chi)$ and $c_\ell(z,\chi)$. The latter were computed using the reccurence relations:

$$(iz+\ell)y_{\ell-1} + (iz-\ell-1)y_{\ell+1} = i\,cth\chi\,(2\ell+1)\,y_\ell,$$

where

$$y_\ell = s_\ell(z,\chi) \quad \text{or} \quad c_\ell(z,\chi),$$

$$s_0(z,\chi) = \sin z\chi, \qquad c_0(z,\chi) = \cos z\chi,$$

$$s_{-\ell-1}(z,\chi) = (-1)^{\ell+1}c_\ell(z,\chi).$$

To obtain the numerical solution it is necessary to take into account the asymptotic behavior of the solution at the origin and at infinity.

Because of the finiteness of $R_\ell(z,\chi)$ and $\gamma_\ell(z,\chi)$ at $z=0$ the asymptotic form

$$\varphi_\ell(z) \sim const \times z$$

of the wave function in this region is essentially different from the nonrelativistic one $z^{\ell+1}$.

The behavior at $z \gg 1$:

$$\gamma_\ell(z,\chi) \sim \chi/sh\chi, \qquad R_\ell(z,\chi) \to 0$$

yields the following estimate for the wave function for the potential (1.1):

$$\varphi_n(z) \sim \left(\frac{g m \chi_n}{sh\chi_n}\right)^{1/3} \frac{t_n^{-1/4}}{2\sqrt{\pi}}\, e^{-\zeta_n} \sum_{n=0}^{\infty} (-1)^K C_K \zeta_n^{-K}, \tag{2.9}$$

where

$$t_n = \left[\left(\frac{g m \chi_n}{sh\chi_n}\right)\left(z - \frac{V_0}{g} - \frac{m\chi_n\, sh\chi_n}{g}\right)\right],$$

$$\zeta_n = \frac{2}{3}t_n^{3/2}, \quad C_K = \Gamma(3K+1/2)/54^K K!\,\Gamma(K+1/2).$$

For numerical integration of the equation (1.4) at the interval $0 \le z \le R$ we used the standard JINR program INTSTP. The obtained solution was sewed at $z=R$ with the asymptotical one (2.9).

For $\ell=1$ simplified expressions for $R_1(z,\chi)$ and $\gamma_1(z,\chi)$[1] were chosen.

The energy levels are computed with accuracy 0.01 GeV for $\ell = 0,1$ and 0.05 GeV for $\ell = 2,3$.

Our results are presented in tables II-V. The masses of the levels, marked by stars, are there the input parameters. The experimentally observed resonances with appropriate masses, spins and parities are given in parenthesis.

In figs.(1)-(4) there are plotted curves for the dependence of $\gamma_\ell(zm,\chi)$, $m R_\ell(zm,\chi)$ and of the effective quasipotential

$$U_\ell^{eff}(zm,\chi) = m R_\ell(zm,\chi) + (gz - V_0)\gamma_\ell(zm,\chi)$$

on variable z for the $c\bar{c}$ pair at fixed rapidities and $\ell = 1,2,3$.

It is evident from these pictures that the curves $m R_\ell(zm,\chi)$ have the narrow minima for $z \lesssim 1/m$. Such a form of centrifugal barrier yields the peculiar behaviour of the effective quasipotential $U_\ell^{eff}(zm,\chi)$ which has no nonrelativistic analog*).

*) In quantum mechanics the energy levels are splitted in the potential consisting of two wells separated by a barrier. The value of the splitting is defined by the barrier height and width. It may happen that in our case with a suitable combination of parameters the fine structure of levels can be obtained corresponding to the resonances with the double peak(K-mesons, $A_2(1300)$).

3. The Case of Superheavy Quarks

In this section the quark-antiquark system is studied within the field-theoretical scheme in which the momenta of quanta off the mass shell belong to the de-Sitter space /20/,/21/.

$$\left(\frac{\hbar}{\ell_0}\right)^2 p_4^2 + p_0^2 - \vec{p}^{\,2} = \left(\frac{\hbar}{\ell_0}\right)^2, \qquad (3.1)$$

where ℓ_0 is the fundamental length.

Following to ref./22/, let us identify ℓ_0 with the length scale arising in the weak interaction theory:

$$\ell_0 = \sqrt{\frac{G_F}{\hbar c}} \simeq 6\times 10^{-4}\, fm, \qquad (3.2)$$

$$M = \frac{\hbar}{\ell_0 c} \simeq 300\ GeV.$$

The quanta possessing the mass M (maximons), play in the theory with the de-Sitter space the basic role*).

That is an attractive idea to identify quark with maximon. In such an approach these particles originating due to the properties of the geometry of the momentum space, are at the same time the fundamental constituents of hadrons.

Consider the model of ψ -mesons represented as the bound states of the quarks c and $\bar{c}$ with mass M. With slight changing of the arguments of ref./21/ we obtain the quasipotential equation which does not differ by form from (1.4), but includes the rapidity χ_q of another geometrical nature:

$$q_4 = Mc\, ch\chi_q. \qquad (3.3)$$

The masses of all known resonances are much smaller than M, that is, χ_q is of order $i\pi/2$ for lowlying excitations.

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*) The mass M is limiting for virtual quanta/21/. The term "maximon" is taken from paper /23/.

Let the interaction between quarks be described by the potential (1.1). From the equality (2.4) for spectrum of the states with $\ell = 0$ we obtain immediately $V_0 \approx \frac{\pi}{2} M$. It is easy to see that the absolute value $2|\vec{p}|$ of the relative momentum equals to

$$2|\vec{p}| \simeq 2Mc, \qquad (3.4)$$

within accuracy of $(\mu/M)^2$. Thus the quarks motion is of a relativistic character.

The results of numerical computation of radial and orbital energy levels in the system $c\bar{c}$ are given in table VI, the masses of ψ (3.095) and ψ'(3.684) being the input parameters. The parameters g and V_0 are determined in this case unambiguously. In table VII analogous results are presented for the $p\bar{p}$ -system.

The knowledge of the relativistic bound state wave function permits one to obtain further information about its structure. For example, the computation of the values of mean-square radius $\langle r^2\rangle^{1/2}$ of the composite system $\psi(c\bar{c})$ (see table X) yields

$$\langle r^2\rangle^{1/2} \simeq 0.05\, fm. \qquad (3.5)$$

It is remarkable that the uncertainty principle in the relativistic r -space*)

$$\langle r^2\rangle^{1/2}\langle\Delta\chi_q\rangle \simeq \ell_0 \qquad (3.6)$$

results in the correct estimate for $\langle r^2\rangle^{1/2}$.

It follows from (3.4) that the mean value of rapidity equals $i\pi/2$ within accuracy of $(\mu/M)^2$. Hence

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*) The rapidity χ_q and the relativistic relative distance r are canonically conjugated in the sense of the Shapiro transformation. The relation (3.6) was first applied to estimate the mean value of quark momentum in the potential well in ref./13/. This relation is essentially used in the relativistic scheme describing the data on high energy hadron-hadron scattering /24/.

$$\frac{\mu}{M} = ch\, i\left(\frac{\pi}{2} - \Delta\chi\right) = \sin\Delta\chi \simeq \Delta\chi . \qquad (3.7)$$

Substituting (3.7) into (3.6) with account of (3.2) we obtain the estimate:

$$\langle z^2 \rangle^{1/2} \approx l \frac{M}{\mu} \simeq 0.06\, fm \qquad (3.8)$$

(cf. with (3.5)).

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Table I

The values of leptonic widths of some vector resonances (see (2.8)). In the last column the numbers of the tables for which the corresponding widths are input parameters are given*)

n	Resonance	Γ_L (KeV)	Γ_L^{exp} (keV)	$\chi_n/sh\chi_n$	
I	$\rho(0.77)$	6.3I	6.5 ± 0.5	I.02	II
2	$\rho'(I.25)$	2.0I		0.83	II
3	$\rho'(I.6)$	I.07	I.0 ÷ 2.65	0.76	II
I	$\rho(0.77)$	5.4	6.5 ± 0.5	0.83	III
2	$\rho'(I.6)$	0.88	I.0 ÷ 2.65	0.58	III
I	$\omega(0.78)$	0.42	0.76 ± 0.I7	0.74	
I	$\phi(I.02)$	I.5I	I.34 ± 0.08	I.I	
I	$\psi(3.095)$	4.6	4.8 ± 0.6	I.05	
2	$\psi'(3.684)$	3.07	2.2 ± 0.6	0.99	
3	$\psi''(4.I5)$	2.3I	4.0 ± I.2	0.95	

*) Note, that the factor $\chi_n/sh\chi_n$, accounting in the present scheme for the relativistic character of motion essentially differs from 1 for many states.

Table II

Radial and orbital excitations in the $\rho\bar{\rho}$ -system.
The values of the parameters are: m_ρ = 0.41 GeV, g_ρ = 0.084 GeV^2, V_0 = 0.65 GeV

n	$\ell = 0$	$\ell = 1$	$\ell = 2$	$\ell = 3$
I	0.77$^{x)}$ $\rho(0.77)$	I.03 $\delta(0.97)$ $A_1(I.I)$ $A_2(I.3I)$	I.25 $\rho'(I.25)$?	I.43
2	I.25$^{x)}$ $\rho'(I.25)$	I.44 $X(I.44)$ $F_1(I.54)$	I.6I $g(I.68)$	I.78
3	I.62 $\rho'(I.6)$	I.78 $X(I.795)$	I.96 $S(I.93)$?	2.08 $S(I.93)$?
4	I.94 $\rho(2.I)$	2.08	2.28 $U(2.36)$?	2.36
5	2.23 $\rho(2.275)$	2.36 $U(2.36)$?	2.5I	2.62
6	2.49	2.6I	2.8I	2.86
7	2.74	2.85	2.98	3.09
8	2.97	3.08	3.20	3.30
9	3.I9	3.30	3.40	
I0	3.4I	3.5I	3.6I	

<u>Table III</u>

Radial and orbital excitations in the $p\bar{p}$- and $n\bar{n}$ -systems (another input parameters).
The values of the parameters are: $m_p = 0.23$ GeV, $m_n = 0.19$ GeV, $g_{p,n} = 0.15$ GeV^2, $V_0 = 0.83$ GeV.

n	$\ell = 0$		$\ell = $ I	$\ell = 2$	$\ell = 3$
I	0.77$^{x)}$ ρ(0.77)	0.784$^{x)}$ ω(0.78)	I.I8 δ(0.97) A_1(I.I) A_2(I.3I)	I.52 g(I.68)	I.78 S(I.93)?
2	I.60$^{x)}$ ρ'(I.6)	I.66$^{x)}$ ω(I.66)	I.9I	2.20 U(2.36)?	2.48
3	ρ(2.275) 2.23	2.34	U(2.36)? 2.50	2.75	3.00
4	2.78	2.93	3.03	3.28	3.49
5	3.28	3.47	3.5I	3.75	3.88
6	3.74	3.97	3.96	4.I8	4.I0
7	4.I8	4.44	4.39	4.58	4.65
8	4.59	4.89	4.79	4.98	5.I0
9	4.99	5.32	5.I8	5.39	5.54
I0	5.37	5.73	5.55	5.62	

<u>Table IV</u>

Radial and orbital excitations in the $\lambda\bar{\lambda}$ -system.
The values of the parameters are: m_λ =0.72 GeV, g_λ =0.084 GeV^2, V_0 =0.87 GeV.

n	$\ell = 0$	$\ell = $ I	$\ell = 2$	$\ell = 3$
I	I.02$^{x)}$ ϕ(I.0I9)	I.26 S^*(0.99) D(I.28) f'(I.5I)	I.34	I.55
2	I.43$^{x)}$ χ(I.43)	I.6I	I.79	I.9I S(I.93)?
3	I.76	I.9I S(I.93)?	2.05	2.I7
4	2.04	2.I7	2.33	2.45
5	2.29	2.4I	2.54	2.65
6	2.52	2.63	2.76	2.86
7	2.74	2.84	2.96	3.04
8	2.94	3.04	3.I4	3.23
9	3.I3	3.22	3.32	3.4I
I0	3.3I	3.4I	3.54	

Table V

Radial and orbital excitations in the $c\bar{c}$-system. The values of the parameters are: m_c =1.81 GeV, g_c =0.24 GeV^2, V_0 =1.25 GeV.

n	ℓ = 0	ℓ = I	ℓ = 2	ℓ = 3
I	3.09 x) ψ(3.095)	3.43	3.74	3.97
2	3.68 x) ψ'(3.68)	3.94	4.I9	4.39
3	4.I6 ψ''(4.I5)	4.36	4.58	4.77
4	4.57 ψ'''(4.4I)	4.74	4.98	5.I3
5	4.94	5.I2	5.29	5.46
6	5.28	5.45	5.64	5.82
7	5.60	5.76	5.92	
8	5.9I Upsilon?	6.06		
9	6.I9	6.34		
IO	6.47	6.6I		

Table VI

Radial and orbital excitations in the system of superheavy $c\bar{c}$-quarks.
(g_c =1.52 GeV^2, V_0 =470.3 GeV)

n	ℓ = 0	ℓ = I
I	3.I0 x) ψ(3.I)	3.49
2	3.70 x) ψ'(3.7)	4.00
3	4.I7 ψ''(4.I5)	4.44
4	4.60 ψ'''(4.4I)	4.84
5	4.99	5.2I
6	5.35	5.56
7	5.69	5.89
8	6.02 Upsilon?	6.2I
9	6.33	6.5I
IO	6.63	6.80

Table VII

Two variants of the input parameters in the system of superheavy $\rho\bar{\rho}$-quarks.

g_ρ =1.1 GeV^2, V_0 = 471.2 GeV g_ρ =2.51 GeV^2, V_0 =471.6 GeV

n	ℓ = 0	ℓ = 0
I	0.77 x) ρ(0.77)	0.77 x) ρ(0.77)
2	I.25 x) ρ'(I.25)	I.60 x) ρ'(I.6)
3	I.64 ρ'(I.6)	2.28 ρ(2.275)
4	I.99 ρ(2.I)	2.88
5	2.30 ρ(2.275)	3.42
6	2.60	3.93
7	2.88	4.4I
8	3.I4	4.87
9	3.39	5.3I
IO	3.64	5.73

Table VIII

The values of the mean radius $\bar{r}$ and the mean-square radius $\langle r^2\rangle^{1/2}$ for the lowest mesonic states in the model with light quarks*).

μ (GeV)	χ	$\bar{r}_{fm}$	$\langle r^2\rangle^{1/2}$	μ (GeV)	χ	$\bar{r}_{fm}$	$\langle r^2\rangle^{1/2}_{fm}$
0.77 (II)	0.35i	0.94	I.02	0.77 (III)	I.08	0.99	I.09
I.25 (II)	0.98	I.72	I.90	I.60 (III)	I.89	I.96	2.65
I.62	I.3	2.47	2.7I	2.23	2.24	2.9I	3.20
I.94	I.5I	3.I0	3.4I	2.78	2.46	3.74	4.II
I.02 (IY)	0.78i	0.78	0.86				
I.43 (IY)	0.059i	I.48	I.63				
I.76	0.66	I.93	2.06				
2.04	0.89	2.4	2.62				

*) The numbers in brackets indicate the tables with the corresponding input parameters.

Table IX

The values of the mean radius $\bar{r}$ and the mean-square radius $\langle r^2\rangle^{1/2}$ for the ψ -mesons family in the model with light quarks.

μ (GeV)	χ	$\bar{r}_{fm}$	$\langle r^2\rangle^{1/2}_{fm}$
3.095	0.55i	0.4I	0.46
3.68	0.I8	0.72	0.8I
4.I6	0.54	0.95	I.05
4.57	0.7I	I.20	I.32
4.94	0.83	I.43	I.57
5.28	0.92	I.67	I.85
5.60	I.00	I.90	2.I0

Table X

The values of the mean radius $\bar{r}$ and the mean-square radius $\langle r^2\rangle^{1/2}$ for the ψ -mesons family in the model with superheavy quarks.

μ (GeV)	χ	$\bar{r}_{fm}$	$\langle r^2\rangle^{1/2}_{fm}$
3.09	I.5656i	0.039	0.048
3.68	I.5647i	0.069	0.077
4.I6	I.5639i	0.096	0.I05
4.57	I.5632i	0.I32	0.I42

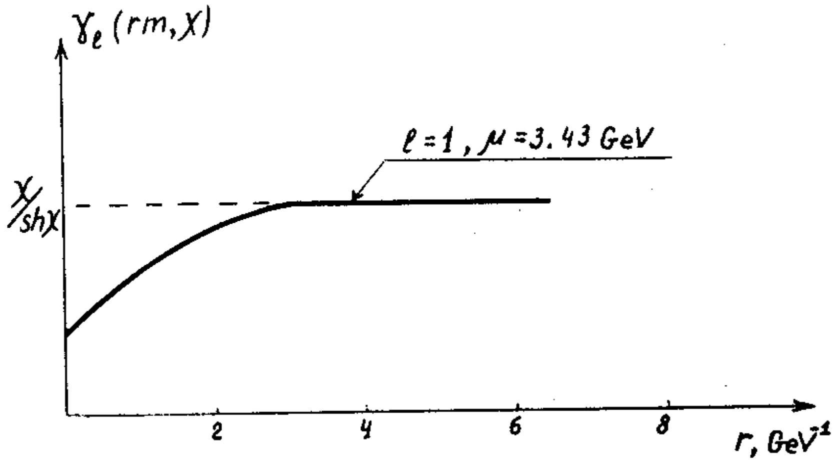

Fig.1. The typical behaviour of $\gamma_\ell(rm,\chi)$. This curve is slightly sensitive to variations of ℓ and χ.

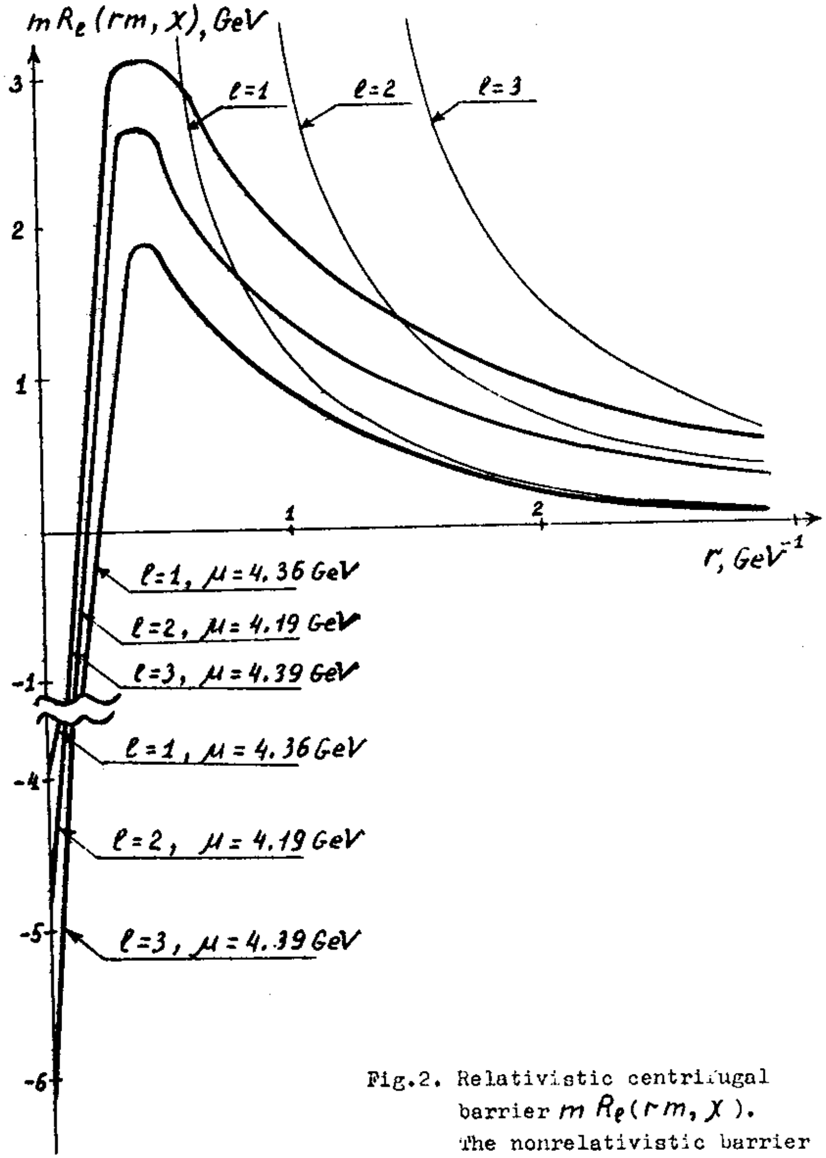

Fig.2. Relativistic centrifugal barrier $mR_\ell(rm,\chi)$. The nonrelativistic barrier $\ell(\ell+1)/mr^2$ at the same ℓ is also plotted.

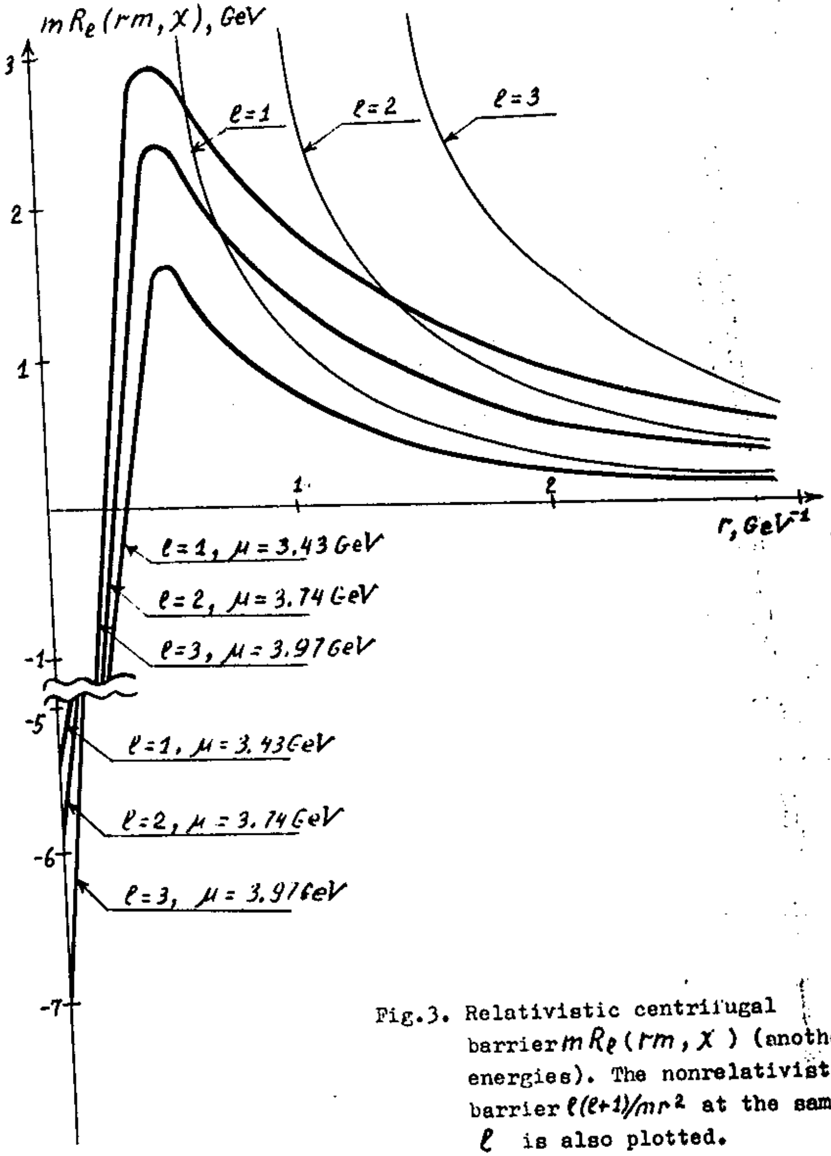

Fig.3. Relativistic centrifugal barrier $mR_\ell(rm,\chi)$ (another energies). The nonrelativistic barrier $\ell(\ell+1)/mr^2$ at the same ℓ is also plotted.

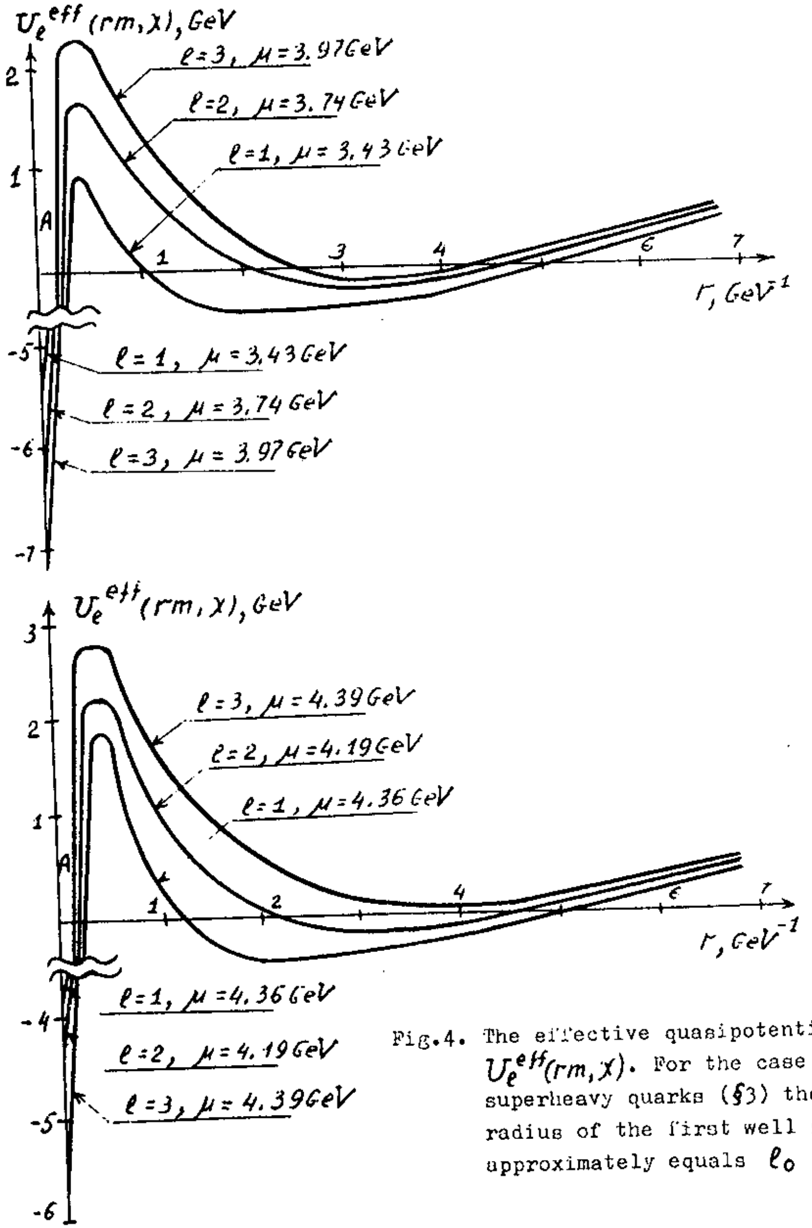

Fig.4. The effective quasipotential $U_\ell^{eff}(rm,\chi)$. For the case of superheavy quarks (§3) the radius of the first well (A) approximately equals ℓ_0 .

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