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THE ANALYSIS OF THE TOTAL CROSS SECTIONS OF THE $\pi N \to \pi\pi N$ REACTIONS NEAR THRESHOLD

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THE ANALYSIS OF THE TOTAL CROSS SECTIONS OF THE $\pi N \to \pi\pi N$ REACTIONS NEAR THRESHOLD

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1. *Isotopic Analysis of the* $\pi N \to \pi\pi N$ *Reactions at 270 MeV*

On measuring the cross sections of the $\pi^- p \to \pi^\circ \pi^\circ n$ reaction /1/ tha total cross sections of all five channels of the $\pi N \to \pi\pi N$ reaction have become known. These cross sections are expressed by four isotopic amplitudes and two phases. At energies close to the threshold one may use T-invariance and S-matrix unitarity /2/ to estimate phases. Thus, it follows that to an accuracy of "π" these phases are equal to the proper phases in the elastic channel /3/. The uncertainty "π" is easily eliminated by comparing theoretical expressions with experimental cross section values.

It worth noting that when comparing the cross sections at energies lower than 300 *MeV* the difference of the reaction threshold for various channels is of inportance. Therefore, the cross section must be compared at energies corresponding to similar kinetic energies in c.m.s.

The total cross sections of the $\pi N \to \pi\pi N$ reactions are expressed through the modules of isospin amplitudes and the relative phases as follows:

$$\sigma(\pi^- p \to \pi^+\pi^- n) = \frac{2}{9}[\,|F_{10}|^2 - \sqrt{\frac{2}{5}}\,|F_{10}|\,|F_{32}|\cos\phi_{32,10} + \qquad (1)$$

$$+ \frac{1}{10}|F_{32}|^2 + \frac{1}{2}(|F_{11}|^2 + |F_{31}|^2 - 2\,|F_{11}|\,|F_{31}|\cos\phi_{31,11})],$$

$$\sigma(\pi^- p \to \pi^\circ\pi^\circ n) = \frac{2}{9}[\frac{1}{2}|F_{10}|^2 + \sqrt{\frac{2}{5}}\,|F_{10}|\cdot|F_{38}|\cos\phi_{32,10} +$$

$$+\frac{1}{5}|F_{32}|^2], \qquad (2)$$

$$\sigma(\pi^- p \to \pi^- \pi^\circ p) = \frac{1}{10}|F_{32}|^2 + \frac{1}{9}[\frac{1}{2}|F_{31}|^2 + 2|F_{11}|^2 + \tag{3}$$

$$+ 2|F_{31}||F_{11}|\cos\phi_{31,11}],$$

$$\sigma(\pi^+ p \to \pi^+ \pi^\circ p) = \frac{1}{10}|F_{32}|^2 + \frac{1}{2}|F_{31}|^2, \tag{4}$$

$$\sigma(\pi^+ p \to \pi^+ \pi^+ n) = \frac{2}{5}|F_{32}|^2. \tag{5}$$

The isotopic amplitudes have been written in the $(\pi\pi)N$ -presentation, the first index is a double value of the isotopic spin of the whole system, while the second one is the isotopic spin of the $(\pi\pi)$ -subsystem. As a result of fitting the following values of isotopic amplitudes in $(mb)^{1/2}$ units have been obtained: $F_{10} = 1.47 \pm 0.09$, $F_{32} = 0.31 \pm 0.12$, $F_{11} = 0.29 \pm 0.11$, $F_{31} = 0.40 \pm 0.33$. The *Table* presents the experimental values of the total cross sections of the $\pi N \to \pi\pi N$ reactions and the results of fitting, the isotopic relations between channels (1)-(5) being used.

Table

No.	Reaction channel	Energy T_π(MeV)	Experiment σ mb	Fitting results σmb
1.	$\pi^- p \to \pi^+ \pi^- n$	283	0.38±0.09 [17]	0.43±0.07
2.	$\pi^- p \to \pi^\circ \pi^\circ n$	270	0.32±0.04 [1]	0.31±0.03
3.	$\pi^- p \to \pi^- \pi^\circ p$	276	0.08±0.08 [27]	0.07±0.07
4.	$\pi^+ p \to \pi^+ \pi^\circ p$	275	0.05±0.03 [22]	0.05±0.03
5.	$\pi^+ p \to \pi^+ \pi^+ n$	275	0.026±0.055 [22] 0.020	0.04±0.03

As is seen from the *Table*, the experimental data on the cross sections are in agreement with isotopic invariance.

2. *Comparison with the Theory Based on the Broken Chiral* SU2 × SU2 *Symmetry and the Hypothesis of the Partial Conserved Axial-Vector Current*

An attempt aimed at describing the cross sections of the $\pi N \to \pi\pi N$ reaction in the framework of the field theory [4] based on the statical Chew and Low model [5] has turned useless. New possibilities appeared in connection with the theory based on the chiral symmetry of strong interactions [6]. The first calculations of the cross sections of $\pi N \to \pi\pi N$ reactions in the framework of chiral theory have been performed by Chang [7] in 1967. The comparison of the results of these calculations with the experimental data on the $\pi^- p \to \pi^+ \pi^- n$ reaction obtained in Dubna have shown that calculations performed according to Weinberg"s model are in good agreement with the experiment. Weinberg has stressed it in his review talk at the Vienna conference in 1968 [6].

Chang"s calculations have been performed by the current commutator method. Nearly simultaneously Olssen and Turner [8,9] have made calculations by the effective Lagrangian method. Diagrams contributing near threshold have been taken into consideration (they are given in ref. [10]). The cross sections were defined according to the formula

$$\sigma = |a|^2 \times K^2 \times \text{ phase volume}$$

valid near threshold. Here a is the amplitude of the proper $\pi N \to \pi\pi N$ reaction channel, K is the momentum of the primary pion (c.m. system). The authors introduced the parameter ξ * determining the chiral-

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* The ξ parameter is included in one of the Lagrangian terms corresponding to the diagram of one-pion exchange in the following way:

$$\mathcal{L}_{\pi\pi N} = (g/2M)^2 (g_V/g_A)^2 [-\phi^2(\partial^\mu\phi)^2 + \frac{1}{2}(1-\frac{1}{2}\xi)m_\pi^2(\phi^2)^2] + \dots$$

symmetry-breaking term and stated that the calculations of the cross sections agree with the experiment with $\xi=0$, in accordance with the Weinberg version of chiral symmetry violation (the Σ-model)[9].

In his further investigation[9] he has given the values of K^2 phase volume with respect to energy for the various channels of the $\pi N \to \pi\pi N$ reaction. The comparison has shown that the cross section values calculated by formula (1) do not correspond to the curve presented by Olsson and Turner[8]. To put in agreement the values of the cross sections[8] and the phase volumes[9] in refs.[11,12,22] the values of phase volumes given in ref.[9] have been multiplied by 2. It turned out that the cross sections of two channels of the $\pi^- p \to \pi^+\pi^- n$ and $\pi^+ p \to \pi^+\pi^+ n$ reactions near threshold van be described by the values of the parameter $\xi = -0.18 \pm 0.20$[12] close to Weinberg"s version. However, in fact, it is not phase volumes that turned out to be erroneous[9] but the theoretical values of cross sections[8], as has been noted by Rockmore[13]. Therefore, the theoretical curves of the cross sections were recalculated by ourselves. They are shown in *Figs. 1* and *2* together with the calculations of Kim ze Pkhen and Zollner[4] and Chang[7].

As is seen from *Fig. 1*, experimental data on the $\pi^- p \to \pi^+\pi^- n$ reaction agree best of all with the calculations of Chang[7]. As for the calculations by the effective Lagrangian method, the near threshold values of the cross sections (up to 230 *MeV*) are best of all described with $\xi = -1.4 \pm 0.2$ but not with $\xi=0$. This agrees qualitatively with the results of refs.[14] and [10], where -1 and -2, respectively, have been obtained for ξ.

However, as is seen from *Fig. 2*, the cross section of another channel of the $\pi^+ p \to \pi^+\pi^+ n$ reaction cannot be described with $\xi = -1.4$.

The cross sections of the channel $\pi^- p \to \pi^0\pi^0 n$[1] are presented in *Fig. 3*. It is seen that the experimental values of the cross section at 270 *MeV* do not coincide with one of the quoted theoretical models. (Chang[7] has calculated only the cross section of the $\pi^- p \to \pi^+\pi^- n$ and $\pi^- p \to \pi^- \pi^0 p$ channels). The experimental values turned

out to be too large comparing with the theory with $\xi = 0$ or -1.4.

Thus, the effective Lagrangian method, by taking into account only the simplest diagrams in the "soft" pion model does not allow the description of the total cross sections of the $\pi N \to \pi\pi N$ reaction near threshold. The $\pi^- p \to \pi^0\pi^0 n$ channel turned out too sensitive to the selection of the theoretical version. It is of interest to compare the cross sections of this channel with theory at lower energies.

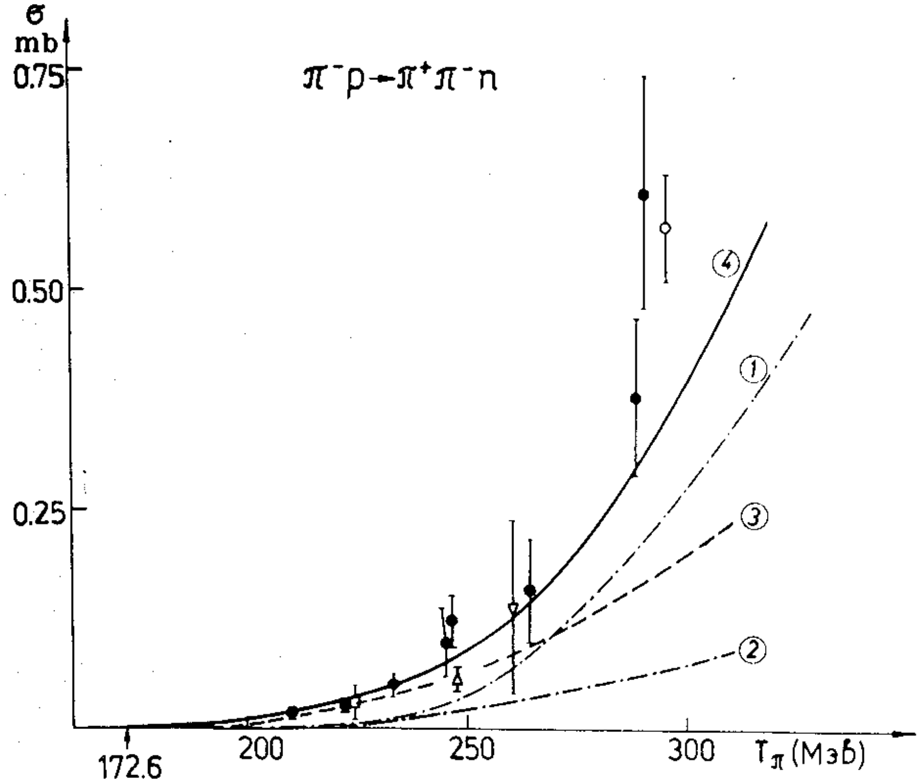

Fig. 1. Total cross sections of the $\pi^- p \to \pi^+\pi^- n$ *reaction, the points* ● [15,16,17], ○ [18], ∇ [19], Δ [20], □ [21]. *The curves: 1 - calculations according to the statical model*[4]; *2* - SU2×SU2, *the effective Lagrangian method for ref.*[9] $\xi = 0$, *3* - SU2×SU2, *the effective Lagrangian method*[9] $\xi = -1.4$, *4* - SU2×SU2, *the current commutator method*[7].

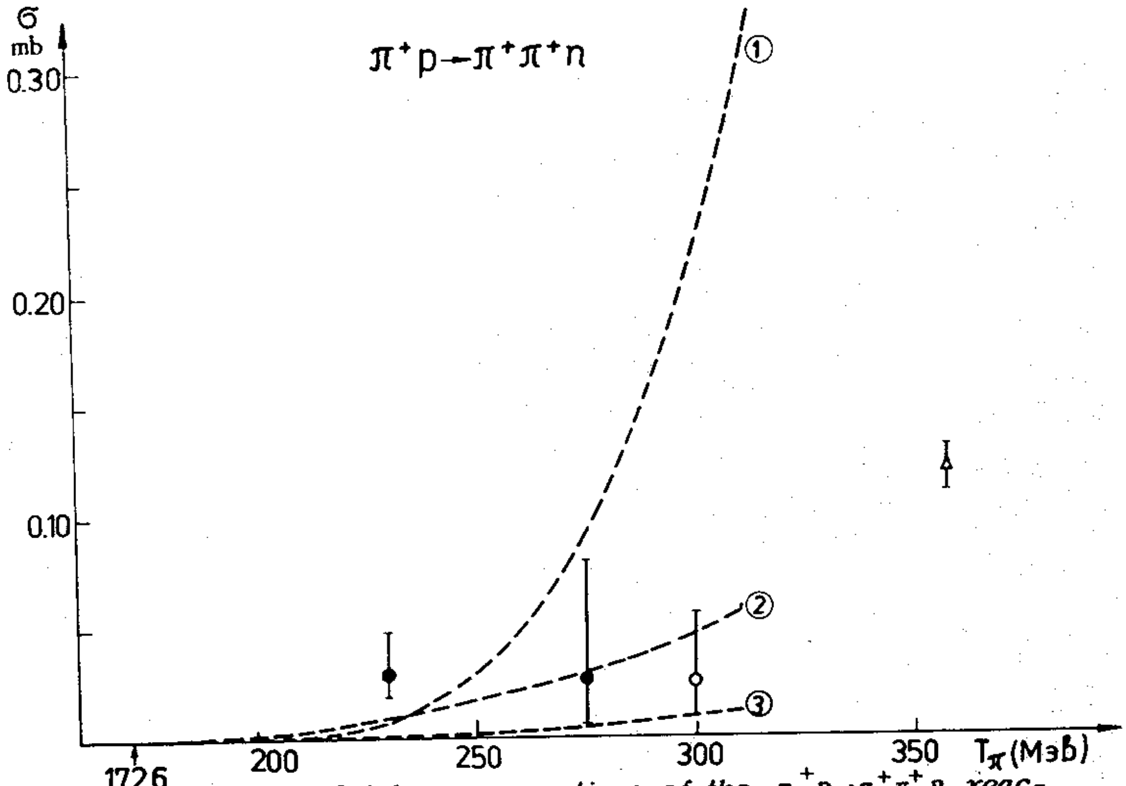

Fig. 2. Total cross sections of the $\pi^+ p \to \pi^+ \pi^+ n$ *reaction,* ● /22/ , o /23/, Δ /24/. *Notations 1,2,3, are the same as in Fig. 1.*

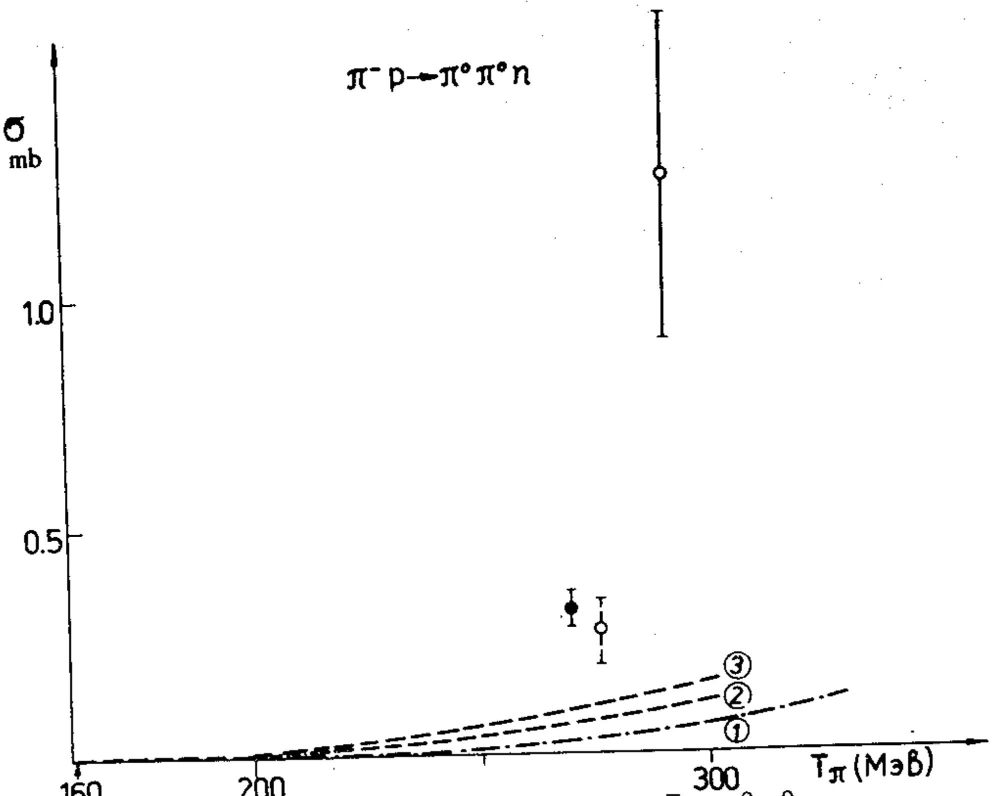

Fig. 3. Total cross section of the $\pi^- p \to \pi^0 \pi^0 n$ *reaction,* ● /1/, ǫ /25/, ǫ /26/. *The notations 1,2,3 are the same as in Fig. 1.*

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