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# MASSIVE LEPTON PAIR PRODUCTION IN MASSIVE QUANTUM ELECTRODYNAMICS

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*INTRODUCTION*

The study of lepton pair production in hadron-hadron interactions may provide important indication about the structure of hadrons and about the range of applicability of several popular theoretical ideas, e.g., partons, "multiperipheralism", current algebra and light-cone dominance. If we consider the production of lepton pair by virtual photon then the lowest order in the electromagnetic coupling constant, the process

$$p + p \to (\text{Virtual } \gamma) + \text{anything}$$

$$\ell^{+} + \ell^{-} + \text{anything}$$

represents the inclusive production of a virtual photon and it has the same kinematical structure as one particle inclusive reaction. It will be interesting to see whether the cross section shows the same features that is observed in purely hadronic inclusive cross section both from the theoretical point of view and experimental point of view.

From the point of view of the parton model Drell and Yan /1/ predict the differential cross section in a simple scaling form for the $pp \to \mu^{+} + \mu^{-}$ + anything as

$$\frac{d\sigma}{dQ^2} = \left(\frac{4\pi\alpha^2}{3Q^2}\right)\frac{1}{Q^2}\tau W(\tau),$$

where $\tau = \frac{Q^2}{s} < 1$.

In the parton model the scaling is obtained by impulsive approximation, i.e., by assuming that the annihilating partons are moving as free particles.

The general idea of the Drell-Yan model is that the production of a massive photon in an inclusive cross section can be visualized as in *fig. 1*.

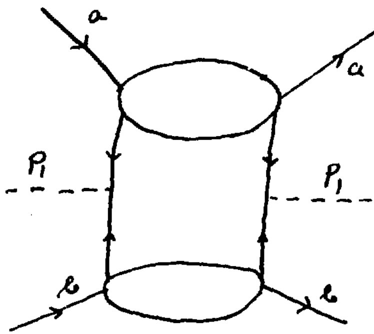



*Fig. 1. The Drell-Yan structure.*

The photon is produced in these types of diagrams only when a quark and antiquark annihilates. Drell and Yan conjectured that the process like (1) can be viewed in an approximate infinite momentum frame as the annihilation of point like constituents into $\mu^+ + \mu^-$ pairs.

Recently, Halliday /2/ considered massive $\mu$ pair production in massive electrodynamics like the diagrams shown in *fig.1* and has found

$$\frac{d\sigma}{dQ^2} = \left[\frac{d\sigma}{dQ^2}\right]_{D.Y.} f(r, \xi),$$

where

$$f(r,\xi) \equiv \int d\alpha\, d\beta\, \delta(\alpha\beta - r)\, \exp\left[\left(\frac{81\xi}{2\ln\xi}\ln^2\alpha\right)^{1/3} + \left(\frac{81\xi}{2\ln\xi}\ln^2\beta\right)^{1/3}\right]$$

and as $Q^2 \to \infty$, $\xi \to -\infty$, he has obtained an asymptotic answer marginally greater than the Drell-Yan formula.

Sanda and Suzuki /3/ applied the current algebra technique to study the hadronic inclusive reactions and massive lepton pairs from hadron-hadron collisions. They predicted a linear growth of the cross section in s for massive lepton pair production. They have calculated cross section for massive lepton pair by ignoring the current conservation. Brandt and Preparata and their collaborators /4/ used light cone expansion technique to calculate the massive lepton pair from proton-proton collisions . They also predicted the same linear growth of the cross section in s.

The massive lepton pair production in high energy hadron collisions was studied also by Subbarao /5/ in ABFST multiperipheral model. The cross section for point electromagnetic coupling is given by

$$\frac{d\sigma}{dQ^2} \simeq \frac{1}{Q^4} f\left(\frac{s}{Q^2}\right),$$

where $Q^2 \gg M^2, \sqrt{s}$ is the centre of mass energy of the colliding protons, $Q$ is the mass of the lepton pair, and $M$ is the nucleon mass.

Thacker /6/ studied also $pp \to \mu^+ + \mu^- +$ anything in massive electrodynamics by using general method for treating the typical phase space integration with arises in a subsequent model calculation and showed cancellation of a linear growth of the cross section in s among several graphs. The nature of cancellation emphasizes the importance of taking proper account of gauge invariance. In the previous paper /7/ when we have studied $pp \to \left(\frac{p}{\pi}\right) + x$, we have shown the cancellation occurs for a linear growth of cross section among other graphs in massive electrodynamics. For the inclusive process $pp \to \left(\frac{p}{\pi}\right) + x$ in a low order Feynman diagram we have found for large $p_T$ and $\sqrt{s}$

$$E\frac{d^3\sigma}{d^3p_1} \propto \frac{1}{p_T^4} \ln^2 s\, f(x_1, y_1),$$

where $f$ is a function of $x_1$, $y_1$. In view of this calculation we would like to study the production of massive $\mu$ pairs in high energy proton-proton scattering. We will show that for the graph shown below the differential

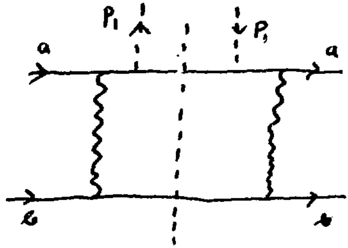



*Fig. 2. Low order graphs for* $p + p \to \ell^+ + \ell^- + X$.

cross section for $p + p \to \mu^+ + \mu^- +$ anything can be written as

$$\frac{d\sigma}{dQ^2} = \left(\frac{4\pi\alpha^2}{3}\right) \cdot \frac{g^4 \tau^2}{(Q^2)^2} f(x_1, y_1, \tau).$$

This behaviour is similar to the behaviour of Drell-Yan mechanism.

## II. KINEMATICS

The process of interest $p + p \to \gamma$ (virtual) + anything
$\to \ell^+ + \ell^- +$ anything

is shown below

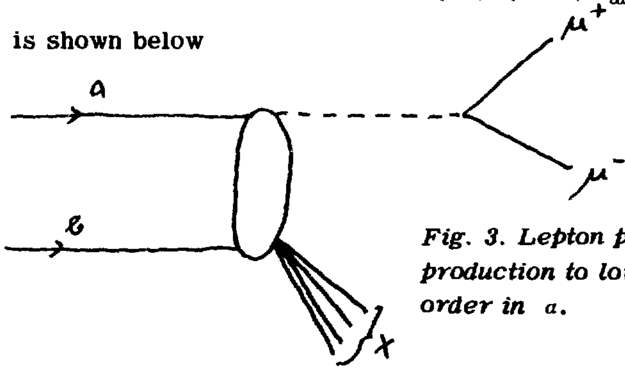



*Fig. 3. Lepton pair production to lowest order in* $\alpha$.

The initial hadrons (a,b) have momentum $a_\mu$ and $b_\mu$. We are interested in the region where $s \to \infty$ and $p_l^2 = \tau s = Q^2 \to \infty$

$$s = 2ab \quad , \quad sx_l = 2bp_l , \quad sy_l = 2ap_l .$$

The possibility of the mass and energy of the final hadronic state requires $x_l < 1$, $y_l < 1$, $1 - x_l - y_l + \tau > 0$ and

$$x_l y_l = \tau + \frac{p_T^2}{s} .$$

The differential cross section for $\mu^+ + \mu^-$ pair can be written as

$$\frac{d\sigma}{dp_l^2} = \frac{\alpha^2}{8\pi^4} \cdot \frac{1}{\{s(s-4M^2)\}^{1/2}} \cdot \frac{1}{p_l^2} W_{\mu\nu} L^{\mu\nu} , \tag{2.1}$$

where $L_{\mu\nu}$ is the leptonic tensor defined by

$$L_{\mu\nu} = \int \frac{d^3 q_l}{2q_{10}} \frac{d^3 q_2}{2q_{20}} \delta^4(q_1 + q_2 - p_l) T_\gamma [\hat{q}_1 \gamma_\mu \hat{q}_2 \gamma_\nu ] \tag{2.2}$$

and the hadronic tensor $W_{\mu\nu}$ defined by

$$W_{\mu\nu} = \Sigma (2\pi)^4 \delta^4 (a + b - p_l - p_n) \langle ab | J_\mu (0) | n \rangle \langle n | J_\nu (0) | ab \rangle , \tag{2.3}$$

where spin average of the proton is implied.

Since leptonic tensor is conserved we can write

$$L_{\mu\nu} = (p_{l\mu} p_{l\nu} - g_{\mu\nu} p_l^2) L \tag{2.4}$$

where $L = \dfrac{2\pi}{3}$.

Again since the hadronic tensor is also conserved only $g_{\mu\nu} p_l^2$ will contribute. Therefore the differential cross section for $\mu^+ + \mu^-$ pair can be written as

$$\frac{d\sigma}{dQ^2} = \left( \frac{1}{12\pi^3} \frac{\alpha^2}{[s(s-4M^2)]^{1/2}} (-W_\mu^\mu) \right) . \tag{2.5}$$

Now we shall calculate $W_\mu^\mu$ for the lowest order Feynman diagrams which are shown below

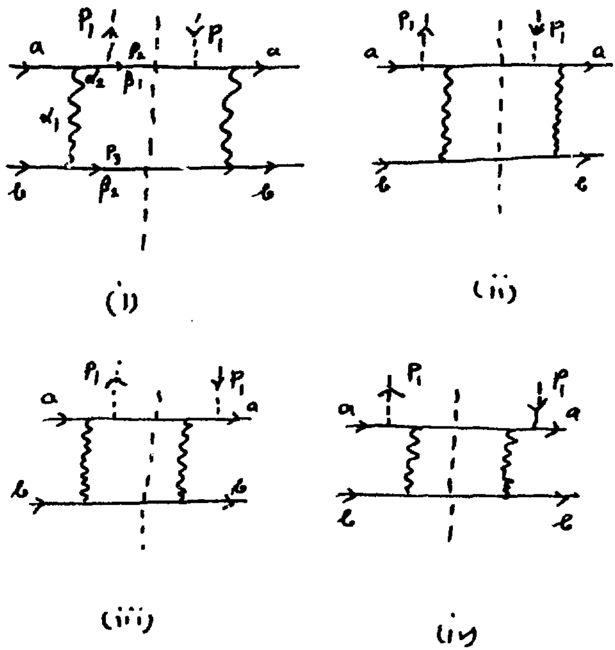



*Fig. 4. Lowest order graphs for* $p + p \to \ell^{+} + \ell^{-} + X$.

In our calculation we shall omit a common constant term which will appear for every diagram. We study the $W^{\mu}_{\mu}$ function as a function of $s, x, y,$ and $\tau$

$$W^{\mu}_{\mu} = \int |M_{fi}|^{2}\, \delta^{4}(a + b - p_{1} - p_{2} - p_{3})\, d^{4}p_{2}\, d^{4}p_{3}\, \delta(p_{2}^{2} - m^{2})\, \delta(p_{3}^{2} - m^{2}),$$

where

$$M_{fi} = [\bar{u}(a)\{\gamma_{\nu} \frac{\hat{p}_{1} + \hat{p}_{2} + m}{(p_{1} + p_{2})^{2} - m^{2} + i\epsilon} \gamma_{\mu} + \gamma_{\nu} \frac{(\hat{a} - \hat{p}_{1}) + m}{(a - p_{1})^{2} - m^{2} + i\epsilon} \gamma_{\mu}\} u(p_{2})] \frac{1}{(b - p_{3})^{2} - \mu^{2} + i\epsilon} \times [\bar{u}(b)\gamma_{\nu} u(p_{3})], \tag{2.7}$$

and

$$|M_{fi}|^2 = \frac{1}{\{(b-p_3)^2-\mu^2+i\epsilon\}^2} \mathrm{Sp}\{(\hat{a}+m)M(\hat{p}_2+m)\bar{M}\}\mathrm{Sp}[(\hat{b}+m)\gamma_\nu(\hat{p}_3+m)\gamma_{\nu'}], \tag{2.8}$$

where

$$M = \gamma_\nu \frac{(\hat{p}_1+\hat{p}_2)+m}{(p_1+p_2)^2-m^2+i\epsilon}\gamma_\mu + \gamma_\nu \frac{(\hat{a}-\hat{p}_1)+m}{(a-p_1)^2-m^2+i\epsilon}\gamma_\mu . \tag{2.9}$$

We shall neglect mass terms in the numerator because these do not contribute to the leading term. Equation (2.7) can be written as

$$|M_{fi}|^2 = \frac{1}{\{(b-p_3)^2-\mu^2+i\epsilon]^2} \mathrm{Sp}[\hat{b}\gamma_\nu\hat{p}_3\gamma_{\nu'}] \times \tag{2.10}$$

$$\times \mathrm{Sp}\Big[\frac{\hat{a}\gamma_\nu\hat{s}_1\gamma_\mu\hat{p}_2\gamma_{\mu'}\hat{s}_1\gamma_{\nu'}}{G_1^2} + \frac{\hat{a}\gamma_\nu\hat{s}_1\gamma_\mu\hat{p}_2\gamma_{\nu'}s_2\gamma_{\mu'}}{G_1 G_2} +$$

$$+ \frac{\hat{a}\gamma_\mu\hat{s}_2\gamma_\nu\hat{p}_2\gamma_\mu\hat{s}_1\gamma_{\nu'}}{G_1G_2} + \frac{\hat{a}\gamma_\mu\hat{s}_2\gamma_\nu\hat{p}_2\gamma_{\nu'}s_2\gamma_{\mu'}}{G_2^2}\Big] =$$

$$= A + B + C + D ,$$

where $s_1 = p_1 + p_2$, $s_2 = a - p_1$, $G_1 = (p_1+p_2)^2 - m^2 + i\epsilon$,

$G_2 = (a-p_1)^2 - m^2 + i\epsilon$.

where A, B, C and D represent *figures 4 (i), (ii), (iii), (iv)*, respectively.

For the delta function and propagator we shall use $\alpha$ representation, e.g.,

$$\frac{1}{\{(b-p_3)^2-\mu^2+i\epsilon\}} = \frac{1}{i}\int_0^\infty d\alpha_1 e^{i\alpha_1\{(b-p_3)^2-\mu^2+i\epsilon\}}, \tag{2.11}$$

$$\delta(p_2^2-m^2)=\frac{1}{2\pi}\Big[\int_0^\infty e^{i\beta_1(p_2^2-m^2+i\epsilon)}d\beta_1+$$

$$+\int_0^\infty e^{-i\beta_1(p_2^2-m^2-i\epsilon)}d\beta_1\Big].$$

**To make computations easier before calculating spur we integrate over $p_2$, then we will have for *figure 4 (i)***

$$\int A\,\delta(p_2^2-m^2)\,\delta(p_3^2-m^2)\,d^4p_3\Big|_{p_2=a+b-p_1-p_3}=$$

$$=\frac{1}{(2\pi)^2}\int\alpha_1\alpha_2\,d\beta_1 d\beta_2 d\alpha_1 d\alpha_2\,(A^0)\ \times$$

$$\times e^{i\{p_3^2(\alpha_1+\alpha_2+\beta_1+\beta_2)-2p_3\{(\alpha_1+\alpha_2)b+\alpha_2 a+\beta_1(a+b-p_1)\}}\,d^4p_3\ \times$$

$$\times e^{is\{\alpha_2+\beta_1(1-x_1-y_1+\tau)\}}\,e^{i\{(\beta_1+\beta_2+\alpha_2)(-m^2+i\epsilon)-\alpha\,(\mu^2-i\epsilon)\}}\ .\ .$$

**Similarly there will be three other terms interacting $\beta_1,\beta_2$ by $-\beta_1,-\beta_2$.**

**From (2.9) we have**

$$A=-64[2(ab)\{(ab)(2p_3a+p_3b+p_3p_1-p_3^2)-$$

$$-(ap_1)(2ap_3+bp_3)-(bp_1)(ap_3+bp_3)\}-\qquad(2.12)$$

$$-(ap_3)\{2(ab)(ap_3)-(ab)(p_1p_3)+4(bp_3)(ab)+(ap_3)(bp_1-bp_3)\}-$$

$$-2(bp_3)\{(ab)(bp_3)-(ap_3)(bp_3+ap_1-p_1p_3)\}+$$

$$+p_3^2\{(ab)(4ap_3+3p_3b+p_3p_1-p_3)+ap_3(bp_1-bp_3)\}.$$

**After integration over $p_3$, we can write (2.11) as,**

$$\frac{1}{(2\pi)^2}\int\frac{\alpha_1\alpha_2\,d\alpha_1\,d\alpha_2}{X^2}\,d\beta_1 d\beta_2\,\mathfrak{D}(\alpha_1,\alpha_2,\beta_1)\times$$

$$\times e^{is\{\beta(1-x_1-y_1+\tau)(1-\frac{\beta_1}{X})+\frac{\alpha_2(\beta_1+\beta_2)}{X}-\frac{(\alpha_1+\alpha_2)\beta_1}{X}(1-x_1)+\frac{\alpha_2\beta_1}{X}(1-y_1)\}}\times$$

$$\times e^{i\{(\beta_1+\beta_2+\alpha_2)(-m^2+i\epsilon)-\alpha_1(\mu^2-i\epsilon)\}}, \tag{2.13}$$

where $\mathfrak{D}$ is a function of $\alpha_1$, $\alpha_2$ and $\beta_1$ and $X = \alpha_1 + \alpha_2 + \beta_1 + \beta_2$.

Now applying the Mellin transform, we get

$$\frac{1}{(2\pi)^2}\int\frac{\alpha_1\alpha_2 d\alpha_1\, d\alpha_2\;\, d\beta_1\, d\beta_2}{X^2}\mathfrak{D}(\alpha_1,\alpha_2,\beta_1)\times \tag{2.14}$$

$$\times\frac{i}{2}\int_{\delta-i\infty}^{\delta+i\infty}\frac{dj(-isF)^j}{}\,e^{i\{(\beta_1+\beta_2+\alpha_2)(-m^2+i\epsilon)-\alpha_1(\mu^2-i\epsilon)\}},$$

where

$$F=\beta(1-x_1-y_1+\tau)(1-\frac{\beta_1}{X})+\frac{\alpha_2(\beta_1+\beta_2)}{X}-\frac{(\alpha_1+\alpha_2)}{X}\beta_1(1-x_1)+$$

$$+\frac{\alpha_2\beta_1}{X}(1-y_1).$$

After integration over $p_3$, the function $\mathfrak{D}(\alpha_1, \alpha_2, \beta_1)$ contains a term without $\beta_1$ and $\alpha_2$. In that case if we scale

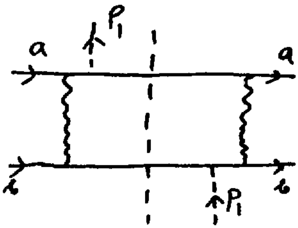
$(\alpha_1, \beta_2) = \lambda(\alpha_1\,\beta_2)$

and integrating over $\lambda$ and taking $\lambda \to 0$ we will get

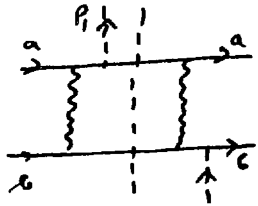
$W^\mu_\mu$ ~ constant.

We see tha+ when we add up three other diagrams the terms $W^\mu_\mu$ ~ constant are cancelled. This is due to gauge invariance.

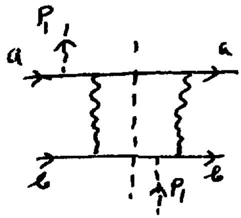
Similarly, we have the same result interchanging a by b. The diagrams which are shown below will also contribute to the differential cross section for $p + p \to \ell^+ + \ell^- +$ anything



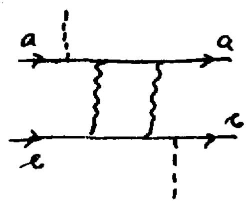

(v)




(vi)




(vii)




(viii)

But they will give us $W^{\mu}_{\mu} \sim \frac{1}{s}$.

So we see that in massive quantum electrodynamics $W^{\mu}_{\mu} \sim \frac{1}{s}$ and which gives the differential cross section for $p+p \to \ell^+ + \ell^- +$anything for the lowest order is of the form

$$\frac{d\sigma}{dQ^2} \approx \frac{a^2 g^4}{(Q^2)^2} \tau^2 f(x_1, y_1, \tau) \ln(\frac{s}{\mu^2}) .$$

This behaviour is the same as in scalar gluon theory, but the function $f(x_1, y, \tau)$ is different from the scalar gluon theory.

## *III. DISCUSSIONS*

In the previous calculation for $p + p \to \binom{p}{\pi} +$ anything we have seen that for $\sigma \sim s$ cancellation occurs among different graphs. This is due to gauge invariance and current conservation. Our calculation shows that to get the gauge invariant result, it is necessary to sum the whole set of diagrams granted in a well defined way.

In the quark parton model Bjorken scaling is obtained by applying impulse approximation, i.e., by assuming that the partons (quarks) are moving as free particles. From the field theoretical point of view one of the most interesting field theory to explain the Bjorken scaling in deep inelastic electron scattering is the asymptotically free theory /8/ of non-Abelian Yang-Mills field, although in that theory Bjorken scaling is violated logarothmically . Again if the strong interactions are described by a field theory of this kind, it may be possible to calculate some quantities which depend on the details of strong interaction dynamics using perturbation theory. Scaling is also obtained in massive neutral vector gluon theory in deep inelastic electron scattering when $g^2 \ln q^2 \ll 1$ /9/ ..

However we have demonstrated here that in massive neutral vector gluon theory the inclusive processes such as $p + p \to \binom{p}{\pi} + X$ and $p + p \to (\ell^+ + \ell^-) + X$ give

$$E \frac{d^3\sigma}{d^3p} \sim \frac{1}{p_T^4} \tag{3.1}$$

and

$$\frac{d\sigma}{dQ} \sim \frac{1}{Q^3}.$$

At present, experimental result [10] (see *fig.5*) for $p + p \to \mu^+ + \mu^- + X$ shows the behaviour

$$\frac{d\sigma}{dQ} \sim \frac{1}{Q^{5.4}}. \tag{3.2}$$

Simularly for the non-Abelian gauge theory we expect the same result like equation (3.1) but which differs only by constant that depends on the group structure.

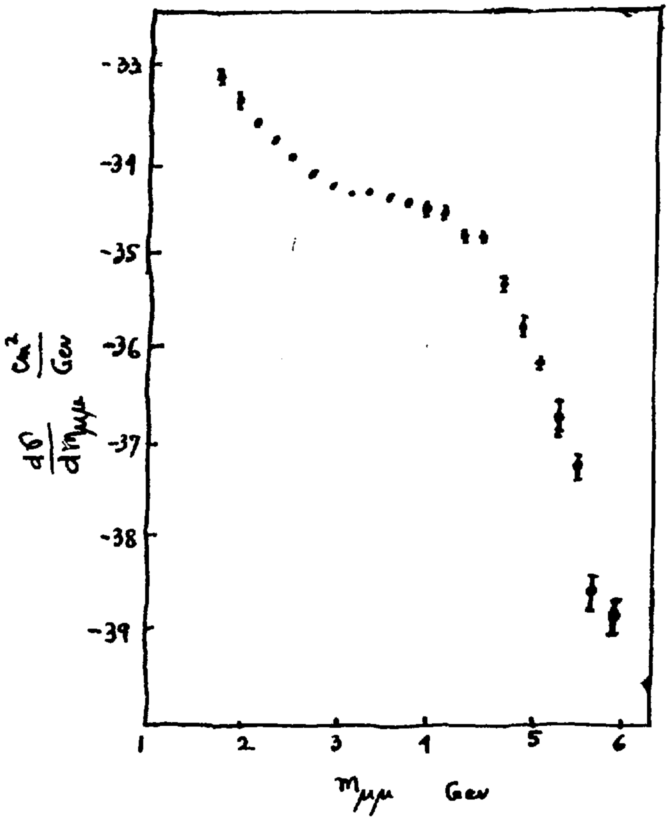


*Fig. 5. Experimental result for the process* $p + p \to \mu^+ + \mu^- +$ *anything.*

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