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1. The question of oscillations in neutrino beams has been discussed for some time [1-7]. To start with, we list below the various schemes which have been treated recently.

i) In reference [6] neutrino oscillations were considered on the basis of a theory [8,6] of the weak interaction of four leptons, which is fully analogous to the theory of the weak interaction of four quarks [9]. In this scheme the two neutrinos fields ν and ν', describing particles with definite masses m and m′ different from zero, enter the interaction through the two orthogonal combinations

$$\nu_e = \nu\cos\theta + \nu'\sin\theta,$$

$$\nu_\mu = -\nu\sin\theta + \nu'\cos\theta,$$

where θ is a mixing angle, which a priori has nothing to do with the hadron Cabibbo angle. In this scheme ν_e and ν_μ are not stationary states and the oscillations $\nu_e \leftrightarrows \nu_\mu$, $\bar\nu_e \leftrightarrows \bar\nu_\mu$ arise, the origin of which can be traced to the mass difference $|m - m'|$. Let us emphasize here that, first, in this scheme the overall number of neutral lepton states is 8 (2 four-component neutrinos) and that, second, in the case of maximum mixing ($\theta = \frac{\pi}{4}$), the properly averaged intensity of ν_e at large distances from a source of ν_e (let us say the Sun) is equal to 1/2 of the intensity expected when oscillations are absent.

ii) In references [5, 7] a scheme was considered which is a generalization of the proceeding case to that of N four-component neutrinos with masses different from zero ($4N$ states). Here too oscillations will take place, as the particles entering the weak interaction will not be described by stationary states and will be orthogonal mixtures of the mass eigenstates. As an example, if the mixing is maximum, the average intensity at the Earth of ν_e from the Sun is equal to $1/N$ of the intensity of solar neutrinos expected in the absence of oscillations. Let us note that in references [10, 11] arguments are given in favour of the hypothesis that in nature there are more than two neutral leptons.

iii) In the preceeding schemes the lepton charge is supposed to be strictly conserved. In reference [2] the case is considered of lepton charge violation. It is assumed that there exist two types of neutrinos with only four states. This means, in fact, that the bare particles have masses equal to zero; neutrino masses arise through the interaction which violates the lepton charge. In such a scheme the particles with definite masses are two Majorana neutrinos with different masses, while ν_e and ν_μ are no longer stationary states. The only oscillations which arise are $\nu_e \leftrightarrows \nu_\mu$ and $\bar{\nu}_e \leftrightarrows \bar{\nu}_\mu$.

The schemes i) and iii) are clearly different in their physical content, but as far as oscillations are concerned, they give identical results. For example they both lead to a decrease of the average intensity of ν_e at the Earth from the Sun by a factor of two at most with respect to the "expected" intensity.

2. In the present paper we discuss a scheme which is a generalization of all the preceeding schemes. Let us assume that the masses of ν_e and ν_μ are different from zero (8 states in all). The usual charged current has the form

$$j_\alpha = (\bar{\nu}_{eL}\gamma_\alpha e_L) + (\bar{\nu}_{\mu L}\gamma_\alpha \mu_L), \qquad (1)$$

where

$$\nu_{eL} = \frac{1+\gamma_5}{2}\nu_e$$

and so on. Obviously the current (1) conserves muon and lepton charges. Only the left components of the lepton fields participate in the above currents; however , the right components of neutrino fields could participate in the weak interaction together with new charged leptons, as it is required by vector-like theories [10, 11]. Let us now suppose that in the Hamiltonian an additional term is present which violates lepton and muon charges. As we shall see, this implies that the particles with definite masses are Majorana neutrinos (obviously four, as we started with two four-component spinors).

If CP invariance is assumed for simplicity, the Hamiltonian describing the above mentioned violations of leptons charges is

$$H_I = \bar{\nu}^c_R M_1 \nu_L + \bar{\nu}^c_L M_2 \nu_R + \bar{\nu}_L M_3 \nu_R + \text{h.c.}, \qquad (2)$$

where $\nu = \begin{pmatrix}\nu_e \\ \nu_\mu\end{pmatrix}$ (here ν_e and ν_μ are the neutrino fields, $\nu_{L,R} = \frac{1 \pm \gamma_5}{2}\nu$)

$$M_i = \begin{pmatrix} m^{(i)}_{ee} & m^{(i)}_{\mu e} \\ m^{(i)}_{\mu e} & m^{(i)}_{\mu\mu} \end{pmatrix} (i = 1, 2), \quad M_3 = \begin{pmatrix} 0 & m^{(3)}_{e\mu} \\ m^{(3)}_{\mu e} & 0 \end{pmatrix},$$

ν^c is the charge-conjugated spinor, and the parameters m_{ee} etc., have the dimensions of a mass and characterise the various lepton charge violations. Let us remark here that parity conservation of the interaction (2) would imply that $M_1 = M_2$ and $m^{(3)}_{\mu e} = m^{(3)}_{e\mu}$.

The first term of expression (2) is the Hamiltonian introduced in reference [2]:

$$\bar{\nu}^c_R M_1 \nu_L + \text{h.c.} = m^{(1)}_{ee}\bar{\nu}^c_{eR}\nu_{eL} + m^{(1)}_{\mu\mu}\bar{\nu}^c_{\mu R}\nu_{\mu L} +$$

$$+ m^{(1)}_{\mu e}(\bar{\nu}^c_{eR}\nu_{\mu L} + \bar{\nu}^c_{\mu R}\nu_{eL}) + h.c. \qquad (3)$$

The second and third terms in expression (2) are connected with the starting assumptions (two four component massive neutrinos). In addition to the Hamiltonian (2), the present scheme implies a lepton charge conserving mass term

$$H_0 = \bar{\nu}_L M_0 \nu_R + h.c., \qquad (4)$$

where

$$M = \begin{pmatrix} m_e & 0 \\ 0 & m_\mu \end{pmatrix} \qquad (5)$$

and m_e, m_μ are the bare masses of the four component neutrinos. It is easy to see that the Hamiltonian $H = H_0 + H_I$ can be written:

$$H = \bar{\chi}_1 M_1 \chi_1 + \bar{\chi}_2 M_2 \chi_2 + \bar{\chi}_1 M_{12} \chi_2 + \bar{\chi}_2 M^T_{12} \chi_1, \qquad (6)$$

where

$$M_{12} = \frac{1}{2}(M_0 + M_3) \qquad (7)$$

and

$$\chi_1 = \nu_L + \nu^c_R = \begin{pmatrix} \nu_{eL} + \nu^c_{eR} \\ \nu_{\mu L} + \nu^c_{\mu R} \end{pmatrix}, \qquad \chi_2 = \nu_R + \nu^c_L = \begin{pmatrix} \nu_{eR} + \nu^c_{eL} \\ \nu_{\mu R} + \nu^c_{\mu L} \end{pmatrix} \qquad (8)$$

are four Majorana fields. Compactly the expression (6) has the form

$$H = \bar{\chi} M \chi, \qquad (9)$$

where $\chi = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}$ and M is the 4x4 matrix

$$M = \begin{pmatrix} M_1 & M_{12} \\ M^T_{12} & M_2 \end{pmatrix}. \qquad (10)$$

The diagonalization of this matrix gives

$$H = \sum_{i=1}^{4} m_i \bar{\phi}_i \phi_i . \qquad (11)$$

Here m_i are eigenvalues of the matrix (10), and

$$\phi_i = \sum_{\sigma=1}^{4} U_{i\sigma} \chi_\sigma , \qquad (12)$$

where U is the orthogonal 4x4 matrix which diagonalizes the matrix (10). It is seen that particles with definite masses m_i are four Majorana neutrinos described by the fields ϕ_i. The neutrino fields which do participate in the usual weak interaction are a coherent superposition of ϕ_i and the corresponding particles ν_e, ν_μ, $\bar{\nu}_e$, $\bar{\nu}_\mu$ are no more described by stationary states. According to (12), the state vector at the time t of a beam, which initially $(t=0)$ consists of ν_e or ν_μ, is

$$\sum_{\sigma'=1,2} \mathfrak{A}_{\sigma\sigma'}(t)|\nu_{\sigma' L}> + \sum_{\sigma'=3,4} \mathfrak{A}_{\sigma\sigma'}(t)|\bar{\nu}_{\sigma' L}>, \quad (\sigma = 1,2), \qquad (13)$$

where

$$\mathfrak{A}_{\sigma\sigma'}(t) = \sum_{i=1}^{4} U_{i\sigma}\, e^{-iE_i t}\, U_{i\sigma'} \qquad (14)$$

and E_i is the energy in the lab. syst. The state vector of a beam which initially consists of $\bar{\nu}_e$ or $\bar{\nu}_\mu$ is clearly

$$\sum_{\sigma'=1,2} \mathfrak{A}_{\sigma\sigma'}(t)|\bar{\nu}_{\sigma' R}> + \sum_{\sigma'=3,4} \mathfrak{A}_{\sigma\sigma'}(t)|\nu_{\sigma' R}>. \quad (\sigma = 1,2). \qquad (15)$$

From expressions (13) and (15) one can easily obtain the probability of finding any given type of neutrinos, (interacting or not interacting through the usual weak interaction (1)), at the time t for any initial $(t=0)$ neutrino state. For example the intensity $I_{\nu_e}(R,p)$ of usual electron neutrinos with impulse p at a distance R from a source of ν_e is *:

$$I_{\nu_e}(R,p) = I^0_{\nu_e}(R,p)[\sum_i U^4_{i1} + \sum_{i \neq k} U^2_{i1} U^2_{k1} \cos 2\pi \frac{R}{L_{ik}}], \qquad (16)$$

where $I^0_{\nu_e}(R,p)$ is the intensity of ν_e which would be expected in the absence of oscillations,

$$L_{ik} = \frac{4\pi p}{|m_i^2 - m_k^2|} \qquad (17)$$

are oscillation lengths. The physical consequences of oscillation phenomena described by the expression (16) and by similar expressions will be observable only if the oscillation lengths are comparable with or smaller than the distance from the source to the detector.

3. Recent measurements /12/ of the β spectrum of ^3H yield a value $m_{\bar{\nu}_e} < 35$ *eV*; interpreted according to the point of view expressed in ref. /16/, (substantial mixing), they suggest that the masses of all the Majorana neutrinos are less than 35 *eV*. Reactor experiments /13/, in which the cross section for the process $\nu_e + n \to \bar{e} + p$ was measured, imply that $|m_i - m_k| \leq 1$ *eV*.

In order to observe some consequences of oscillation phenomena one should work with relatively low energy neutrinos at sufficiently large distances between the detector and the neutrino source, that is one should consider experiments at reactor, meson factories and especially solar neutrino experiments /14-16/*.

This last case can be treated in terms of expression (16). According to any realistic situation one must average the expression (16) over the neutrino spectrum, the dimensions of the active Sun region and so on. The relation between the average intensity $\overline{I_{\nu_e}}$ and the intensity $\overline{I^{\circ}_{\nu_e}}$ expected in absence of oscillations can be written as

$$\overline{I_{\nu_e}} = \delta \overline{I^{\circ}_{\nu_e}}. \qquad (18)$$

In our case

$$\delta = \sum_{i=1}^{4} U^4_{i1}. \qquad (19)$$

It is easy to see that $\delta_{min} = 1/4$, which is obtained when all the values U^2_{i1} are equal.

Such maximum decrease in the expected intensity of ν_e is connected with oscillations of the type $\nu_e \leftrightarrows \nu_\mu$, $\nu_e \leftrightarrows \bar{\nu}_{eL}$, $\nu_e \leftrightarrows \bar{\nu}_{\mu L}$. As far as the weak interaction involving charged currents is concerned, low energy particles ν_μ, $\bar{\nu}_{eL}$, $\bar{\nu}_{\mu L}$ are "sterile" **, so that the intensity of detectable neutrinos decreases at most by a factor four, as it was already stated above. Note that in the scheme ii) with four types of neutrinos a value $\delta_{min} = 1/4$ is obtained as well as in our scheme with only two types of neutrinos.

Thus the problem as to whether neutrinos taking place in the weak interaction are a coherent superposition

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*We denote here usual electron and muon neutrinos and antineutrinos ν_e, ν_μ, $\bar{\nu}_e$, $\bar{\nu}_\mu$. New particles will be denoted in a self-explanatory way, for example, ν_{eR}, etc.

*Some consequences of neutrino oscillations for the neutrino astronomy of the Sun have been previously discussed in ref. /1-7/.

** "Sterility", however, is a relative notion; low energy particles ν_μ, $\bar{\nu}_{eL}$, $\bar{\nu}_{\mu L}$ could be scattered due to possible neutral currents and in principle might be detectable in the future. Incidentally, in addition to ν_μ at high energy $\bar{\nu}_{eL}$, $\bar{\nu}_{\mu L}$ might be active because of possible right charged currents.

of massive particles can be solved, in principle, by measuring the intensity I_{ν_e} of solar neutrinos and comparing it with the intensity $\overline{I^{\circ}_{\nu_e}}$ which would be expected in absence of oscillations. Clearly the last intensity must be estimated in a reliable way. This requirement implies that one must detect neutrinos from the main thermonuclear reaction on the Sun $p + p \to d + e^{+} + \nu_e$ (and or from the reaction $e^{-} + p + p \to d + \nu_e$); fortunately the possibility of fullfilling such a program is becoming real with the recent development of the Ga-Ge radiochemical method of detecting low energy neutrinos /17, 18/.

4. So far we have been discussing the case of two four-component neutrinos under the assumption that in the Hamiltonian a term violating the lepton charges is present. This implies that the particles with definite masses are four Majorana neutrinos. If N four-component neutrinos participate in the weak interaction under the same assumptions, the field of every one of them will be a mixture of 2N Majorana fields *. The minimum mean intensity of non-sterile neutrinos, let us say solar ν_e, is 1/2N of the intensity expected in the absence of oscillations, that is $\delta_{min} = 1/2N$. Should oscillations really exist in nature and should N turn out to be uncomfortably large, the future of neutrino astronomy probably will have to rely on the detection of neutral current processes, a very difficult task indeed.

5. In conclusion let us stress that the main points related to oscillation phenomena are: finite neutrino masses, neutrino mixing, lepton charge violation, number of neutrino types. Thus the questions which might be answered in experiments based on the neutrino oscillation ideology directly concern the very nature of neutrinos.

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*In this scheme the analogy between leptons and quarks is not lost. The quarks and the charged leptons, which do have baryon and/or electrical charges, participate in the weak interaction as four-component spinors; neutral leptons, not having any charge, are (two component) Majorana fermions, whereby only mixtures of them do enter the weak interaction. It would be attractive from an easthetic point of view if in the lepton family all the neutral mass eigenstates were described by Majorana fields.

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