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ON THE STATISTICAL THEORY OF HADRON MULTIPLE PRODUCTION

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К статистической теории множественного рождения адронов

Обсуждаются основные работы по статистической теории множественного рождения частиц. Ставится вопрос об однозначности решения проблемы на основе принципов статистической физики. Доказывается необходимость учета одномерного и теплового расширения системы. Получено соотношение для множественности процесса.

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On the Statistical Theory of Hadron Multiple Production

The basic investigations on the statistical theory of multiple particle production are surveyed. The problem is raised on the unambiguity of the general solution of the problem by basing on the principles of statistical physics. The necessity to take into account the one-dimensional and thermal expansion of a hadron cluster is proved. As a result, a general relation for multiplicity has been obtained.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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I. STATE OF THE PROBLEM

The idea of thermodynamic equilibrium of hadron radiation underlines the statistical theory of multiple particle production. There are deep physical reasons allowing one to expect a state close to statistical equilibrium in the system of produced hadrons.

The advantage of this idea is that it permits one to obtain the basic characteristics of the process without overcoming the difficulties of the quantum field theory of detailed description of multiple hadron production. The problem of statistical description of this process might seem to mean the simple application of the quantum theory of equilibrium radiation for the case of known hadron production. However, due to the microscopicity of the system under consideration unexpected difficulties appeared when fulfilling this programme which prevented obtaining the final and unambiguous solution of this problem. To be true, despite the unacceptability on the whole of earlier obtained solutions even the first studies [1,2] on the statistical theory of multiple production have provided the following important statements of the approach under development.

1. The initial state of the system of particles produced in a collision has dimensions (in the C-system) about those of the Lorentz-cotracted nucleon V_F (Fermi, 1950).

2. Only statistical equilibrium in the system expanded up to the critical volume V_c (at which new hadron formation may be neglected) refers directly to the experimentally observed characteristics of freely dissipating particles (Pomeranchuk, 1953).

The above statements are firmly established conditions of solving the problem of statistical equilibrium radiation of hadrons. The second condition determines unambiguously the mass composition of hadron radiation corresponding to the constant critical temperature $T_c \sim m$. As for multiplicity and also energy and angular distributions of particles, these characteristics of the process turn out to be also dependent on the manner of system expansion beginning from the initial volume V_F up to the critical one V_c. However, in numerous studies on the statistical theory of multiple production no unique solution of the mechanism of the Lorentz-contracted cluster of hadron matter has been found. The divergence of the results of the earlier proposed version of the theory is due to the choice of various but equally groundless schemes of the expansion of system primary volume.

Thus, Pomeranchuk /2/ has accepted the version of thermodynamic expansion of the system according to which this process occurs in all directions equally. The linear dependence of the mean multiplicity corresponding to this version upon energy in c.m.s. obviously contradicts experiment. But even without any experimental test one could see easily the groundlessness of the accepted assumption on the thermal system expansion. In the initial Lorentz-contracted volume of the nucleon it is impossible to reach the thermal equilibrium of hadrons whose dimensions exceed the longitudinal size of the initial system.

Another extreme assumption of the mechanism of hadron cluster expansion has been accepted in Landau's hydrodynamic approach /3/. The isentropic one-dimensional expansion along the axis of primary particle collision occurs until the system reaches the critical volume at which particle interaction stops. According to Landau the thermal three-dimensional expansion of the system

starts at the stage of free particle dissipation. Later this groundlessness of Landau's neglection of medium viscosity * was marked.

Another and the most important reason of the groundlessness of the results obtained according to this theory is that the equations of relativistic hydrodynamics are applied to the initially Lorentz-contracted microsystem ignoring the uncertainty principle. The groundlessness of such application of hydrodynamics was noted by D.I.Blokhintsev as far back as 1956 /4/. Nevertheless, this problem was further developed without taking into account the requirements of the fundamental principle of modern physics. This, e.g., concerns the versions of the hydrodynamic theory taking into account hadron medium viscosity. They eliminated the groundlessness of taking into consideration only the one-dimensional expansion of the system. Partial

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*Here we draw our attention to one more peculiarity of obtaining multiplicity according to Landau $N \sim (2E_c)^{1/2}$ (Fermi). This dependence has been obtained by means of the model taking into consideration system cooling down to Pomeranchuk's critical temperature $T_c \sim m$ not only due to erroneous neglection of medium viscosity but to indirect assumption in Landau's investigation to prove which one should go beyond proper thermodynamics or hydrodynamics. The reversible hydrodynamic expansion of the system starts according to Landau immediately after the collision of primary particles. Therefore, according to the conventional thermodynamic theory this process should retain entropy of the intial system of two colliding particles. Landau takes the entropy S $S_F \sim (2E_c)^{3/4} V_F^{1/4}$. This entropy was obtained by Fermi from the conditions that all collision energy $2E_c$ has been released in the volume of the Lorentz-contracted nucleon $V_F = V_0 \; M/E_c$ as statistically equilibrium hadron radiation. But this means that all the energy has been transferred to chaotic hadron motion and according to conventional thermodynamics after this no reversible one-dimensional expansion of the system can occur. Unfortunately, in numerous studies developing a hydrodynamic approach a necessity to search for the substantiation of the indirect assumption has not been discussed.

energy dissipation meant that both one-dimensional expansion and three-dimensional one of the system should take place. For the multiplicity these versions of the hydrodynamic approach gave results depending on the choice of the temperature dependence of the viscosity factor η. In particular, for $\eta \sim T^3$ the average multiplicity has been obtained to be proportional to $(2E_c)^{3/4}$ ref. /5/.

The same result for multiplicity being in good agreement with the latest experimental data has been recently obtained by the authors of another modification of the hydrodynamic theory of multiple production /6/. The power dependence of $N \sim (2E_c)^{3/4}$ has been found by these authors by varying the square of sound velocity in the equation of state.. On the whole, the sufficiently extensive and prolonged development of this branch of hydrodynamics performed against the requirements of the fundamental principle of uncertainty seems to be one of the greatest misunderstandings of modern theoretical physics. Sometimes such a retreat from the rigidity of the theoretical description is excused by the advocates of the hydrodynamic branch be referring to the good agreement of the obtained results with experimental data. Thus, e.g., in one of the latest surveys /7/ it has been emphasized that recent experimental data on multiple particle production confirm very well the prediction of hydrodynamic theory. However, the agreement of experiment with theoretical results is meant. But these results are not directly related with the accepted mechanism of hydrodynamic expansion of the hadron system. The important characteristic of the process, namely, the temperature of the system of dissipating hadrons, which determines the mass composition of produced particles depends only on the choice of the critical volume but not on the accepted expansion mechanism. Therefore, the agreement with the experimentally found composition of particles confirms only Pomeranchuk's idea on the decisive importance of the statistical equilibrium in the final state of the system of the correctness of the choice of the critical volume.

For the agreement of the theoretical dependence of multiplicity upon energy with experiment it is of crucial importance to take into consideration the very fact of certain energy transformation for the reversible process of one-dimensional expansion of the hadron system. This most important property of the process is not affected anyhow by the mechanism itself of one-dimensional expansion and by its relative role at certain stages of expansion. The origin of one-dimensional expansion is, undoubtedly, hidden in the most general elastic properties of compressed hadron matter. Their taking into account is not a prerogative of relativistic hydrodynamics only.

The choice of critical temperature essentially predetermines the basic characteristics of the angular and energy distribution of produced particles also. In particular, the constancy of the transverse momentum and the mean particle energy confirms the correctness of the choice of cticical temperature which is independent of the initial energy of colliding particles. To prove or reject the hydrodynamic effect of system expansion one needs to study rather subtle peculiarities of the energy spectrum and the angular distribution of particle produced. As far as the effects of properly hydrodynamic origin are concerned, there are no experimental data available confirming them. Experiment confirms directly only those theoretical predictions which are related to the initial statement of Pomeranchuk's statistical model on low critical temperature in the final equilibrium state of the hadron system.

Alongside, the linear increase of multiplicity with energy contradicts obviously experimental data. This unambiguously indicates the incompetence of Pomeranchuk's three-dimensional mechanism of expansion assuming the existence of statistical equilibrium in the entire system at each state of its expansion. The success of hydrodynamic theory is, undoubtedly, due to taking into consideration the energy expenditure for one-dimensional expansion of a hadron cluster. However, to correct Pomeranchuk's model in this way there is no need to apply to classical hydrodynamics which is in evident contradiction with the uncertaintly principle, (being applied to a hadron cluster contracted one-dimensionally).

II. NEW FORMULATION OF THE PROBLEM

The analysis of the previous versions of the statistical theory of multiple particle production results, first of all, in a necessity for searching for the universal formulation of the problem. The problem of equilibrium hadron radiation should have a definite solution as the quantum theory has a unique solution for "black body" radiation. Certainly, various models of primary particle interaction affect final results and one can speak about the uniqueness of formulating and settling only that part of the problem which concerns proper statistical physics. Therefore, the statistical problem should be formulated first of all for the simplest event of central collisions. As have been stated, discrepancies in the solutions of this problem are due to different assumptions on the statistical model of hadron cluster expansion. Consequently, it is a primary task to formulate a unique statistical approach for considering the expansion of a hadron cluster contracted one-dimensionally . When settling this problem no peculiarities of physical conditions arising from the microscopic sizes of the statistical system under consideration have been taken into account yet.

Fermi's initial assumption of the origin of equilibrium radiation of strongly interacting particles is the basis for the statistical approach to multiple particle production. However, this transfer of particle collision energy to statistical equilibrium radiation can occur only at a later stage, i.e., when as a result of one-dimensional expansion of the system, its volume acquires a spherically symmetrical shape. The one-dimensional expansion of the Lorentz-contracted volume to the spherically symmetric one V_0 in some coordinate system should have a necessary stage of development arising in the collision of non-equilibrium configuration of hadron matter. There is no need to connect this process to any certain mechanisms on the basis of the elasticity theory of the hydrodynamic version of the theory. We do not know any details of the one-dimensional process of reconstructing the spherical shape of the excited hadron cluster. Moreover, we have no theoretical basis for the detailed description of such a process in the microsystem obeying the laws of quantum field theory. However, to solve the problem raised in statistical theory general assumptions on the one-dimensional expansion of the hadron cluster are sufficient, without considering the detailed mechanism of this process. The application of relativistic hydrodynamics would be not only groundless but needless for the statistical solution of this problem.

Thus, all collision energy is transformed to the potential one of the non-equilibrium system of the compressed matter of stopped particles. A part of this energy should be necessarily contained in the elastic properties of compressed matter. The complete reversible transformation of energy to kinetic one of two particles corresponds to the elastic reaction channel when the "direct" collision of particles results in elastic backward scattering. However, in inelastic interactions a certain part of energy of compressed matter should be transformed to an elastic process of reconstructing equilibrium dimensions of produced hadrons. This condition can be considered a general requirement of the relativistic theory for existing equilibrium elementary objects whose dimensions can undergo one-dimensional compression according to Lorentz contraction due to their motion in the given inertial system of coordinates.

Pomeranchuk's statistical approach needs only one essential correction: thermodynamic expansion starts not from the initial state of the Lorentz-contracted volume but only when the cluster subsystem has reached the spherically symmetric volume about V_0 in the proper accompanying inertial system of coordinates as a result of one-dimensional expansion. Prior to this moment the cluster subsystem cannot be represented as statistical equilibrium radiation.

The refinement of Pomeranchuk's approach at this point considerably changes the final results. At the final state of expansion one must consider the system of statistically equilibrium hadron radiation with the same critical volume V_c, similar to Pomeranchuk's conception. However, the energy of chaotic hadron motion turns out considerably smaller since a certain part of collision energy is transformed in the reversible one-dimensional expansion of cluster systems. This correction to the value of thermodynamic energy results in the reduction of the total entropy of the system S, and respectively, average multiplicity N as $N \sim S$, while $S \sim W^{3/4} V^{1/4}$, where W is energy of thermal hadron motion.

The statistical approach outlined here must take into consideration the stage of one-dimensional expansion of the primary volume of the hadron system. Thus, it includes the most important part of Landau's statistical approach. Besides, along with excluding the hydrodynamic mechanism of one-dimensional expansion, our approach consists in the limitation of the one-dimensional stage of expansion to the volume V_0. Owing to this peculiarity the present statistical approach, by combining certain aspects of Pomeranchuk's and Landau's models, in the region of medium energies distinguishes, in a certain manner, the role of the first model. Thus, the proposed more rigid solution of the statistical problem of multiple hadron production agrees with the earlier choice of Pomeranchuk 's statistical model* made under the

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*Excluding completely from consideration the Fermi stage of the Lorentz-contracted state of hadron matter, E.L.Feinberg has used, in fact, a modified version of Pomeranchuk's model as he applied it directly to the isotropic state of the excited cluster, considering it to rest in the C-system only for the initial energy region of multiple particle production. To achieve the agreement of average multiplicity with experiment at high energies, the excited system was divided into two clusters moving in opposite directions along the axis of particle collision. For the Lorentz-factor of the cluster in the C-system the dependence $\gamma_{cl} \sim E^{1/2}$ was taken empirically. This made it possible to obtain Fermi's dependence for the average multiplicity upon collision energy /5c/.

effect of experimental results obtained for this energy region. Without considering in detail the one-dimensional expansion of the volume V, one must take into consideration only the limitation imposed by the law of impulse and energy conservation. A part of potential energy of the non-equilibrium system of compressed hadron cluster in one-dimensional expansion should be transformed to kinetic one of the directed motion of the hadron cluster. However, according to the law of impulse conservation the one-dimensional expansion of hadron matter should occur symmetrically in opposite directions. Consequently, such an expanding cluster cannot turn out to be a separate symmetric state satisfying cluster determination. Thus, not only at high but at an intermediate energies of colliding particles in strict theoretical consideration satisfying conservation laws one must take into account, at least, two clusters moving in opposite directions along the collision axis. The fact that in experiment two cluster production has been discovered at comparatively high energies, could be, apparently explained by the experimental difficulties of detecting cluster motion in the C-system at lower energies.

III. BASIC RELATIONSHIPS FOR THE MODIFIED VERSION OF THE THEORY

The statistical description of cluster transformation to dissipating hadrons becomes certain only if the value of the proper cluster energy in its rest system W has been found. We consider the simplest example for the determination of this quantity when the system of two similar clusters is produced in central nucleon collisions. This problem can be solved only by using an additional assumption on one-dimensional expansion of Lorentz-contracted cluster, e.g., the assumption which had been indirectly accepted in Landau's hydrodynamical model also. The entropy $S_F \sim (2E_c)^{3/4} V^{1/4}$ calculated for the intial state of the system compressed in collision can be taken only as a potential entropy since the thermodynamic explosion proposed by Fermi corresponding to the value of entropy is still impossible in the initial state of the system.

An additional physical assumption necessary to determine the proper cluster energy W, is to equate the initial entropy of two clusters to potential entropy calculated by Fermi

$$2S^0_{cl} = S_F \text{ or } 2W^{3/4} V_0^{1/4} = (2E_c)^{3/4} \left(\frac{V_0 M}{E_c}\right)^{1/4} . \qquad (1)$$

This equation gives for the central collisions the following relation

$$W = E_c^{2/3} \left(\frac{M}{2}\right)^{1/3} \qquad (2)$$

or

$$\gamma_{cl} = \left(\frac{2E_c}{M}\right)^{1/3} . \qquad (2a)$$

In a general case, taking into account the elasticity factor K, one obtains

$$W = K E_c^{2/3} \left(\frac{M}{2}\right)^{1/3} \qquad (3)$$

or

$$\gamma_{cl} = \left(\frac{2E_c}{M}\right)^{1/3} . \qquad (3a)$$

Further, the isotropic expansion of each equilibrium subsystem occurs. This gives rise to the increase of subsystem entropy and the appropriate increase of the number of hadrons in clusters.

Low critical temperature in a statistical sub-system allows one to only consider the pion component of radiation without complicating the conditions of statistical equilibrium in the final state of the system by taking into account a small admixture of heavier hadrons. However, with $T_c \sim m$ some complication arises which is due to the fact that particles forming equilibrium radiation cannot be considered ultra-relativistic. Therefore, in the statistical calculations of pion radiation one cannot proceed directly from Planck's formula.

Pion momentum distribution for the statistically equilibrium system having the full volume V_c at the temperature T_c is described by the formula, ref. /3b, 5c/

$$dN(\vec{p}) = \frac{3}{(2\pi)^3} V_c \frac{d\vec{p}}{e^{Z(1+X^2)^{1/2}} - 1} , \qquad (4)$$

where

$$Z = \frac{m}{T_c}, \quad X = \frac{|\vec{p}|}{m} .$$

The statistical weight was taken to be 3 in this relation. This corresponds to the assumption on the equal number of π^+, π^- and π^0 mesons produced.

On integrating over momenta distributions one finds that the total number of pions in one cluster is

$$\frac{N}{2} = \frac{3}{2\pi^3} V_c T_c^3 F_T , \qquad (5)$$

where the integral

$$F_T = Z^3 \int_0^\infty \frac{x^2 dx}{\exp Z(X^2+1)^{1/2} - 1}$$

is a slowly varying function of T_c. For $T_c = m$ this value is $F_m = 1.78$. Taking that the critical volume of the sub-system $V_c = V_0 N/2$ one obtains the following equation

$$T_c = \left(\frac{2\pi^2}{3F_m} \frac{1}{V_0}\right)^{1/3} . \qquad (6)$$

It follows from this equation that critical temperature completely depends upon the choice of the elementary volume $V_0 = 3/4\pi R^3$.

To make further some general changes of the final results of statistical calculations, introduce the only parameter $a \simeq 1$ which determines insufficient deviations of the hadron interaction radius $R = a/m$ from Fermi's formula $R_F = 1/m$. Relation (6) is as follows

$$T_c = \left(\frac{\pi}{2}\,\frac{1}{F_m}\right)^{1/3}\frac{m}{\alpha} \simeq \left(\frac{\pi}{2\cdot 1.78}\right)^{1/3}\frac{m}{\alpha} = 0.96\frac{m}{\alpha} \;*. \qquad (6a)$$

Initial momentum distribution (4) in the proper system of a separate cluster after transformation over the pion energy is as follows:

$$dN(E_\pi) = \frac{3}{2\pi^2} V_c T^2 Z^2 \frac{(y^2-1)^{1/2}\, y\, dE_\pi}{\exp Zy - 1}\,, \qquad (7)$$

where $y = E_\pi/m$.

Equating the total pion energy of the whole cluster to (3) and taking into account that $V_c = V^{\circ}\ N/2$ one obtains

$$K E_c^{2/3} \left(\frac{M}{2}\right)^{1/3} = \frac{3}{2\pi^2} V_0 \frac{N}{2} T_c^4\, \phi\,, \qquad (8)$$

where the integral

$$\phi_T = Z^4 \int_1^{\infty} \frac{(y^2-1)^{1/2}\, y^2\, dy}{\exp Zy - 1}$$

is a slowly varying function of T_c. For $T_c = m$ this value is $\phi_m = 5.78$.

Substituting (6) to obtained relation (8) it is possible to determine the average number of pions in the common

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*This result confirms the approximate equality $T_c = m$, which we have used when determining the integral $F = 1.78$. The low temperature of the system in the final state predetermines a small admixture of kaons and antinucleons in equilibrium radiation. The results of these predictions which are rather sensitive to the choice of critical temperature were of decisive importance in the experimental solution of the problem in favour of statistical theory versions taking into account the system expansion up to the critical volume /5c, p.563/.

system of two clusters corresponding to collisions with the given inelasticity factor K

$$N_k = \alpha(2/\pi)^{1/3}\, M/m\, \frac{F_m^{4/3}}{\phi_m} K(2E_c/M)^{2/3} \simeq$$

$$\simeq 2.16\,\alpha K(2E_c/M)^{2/3}\,. \qquad (9)$$

By averaging the associative multiplicity N_c over all the collisions one can obtain the average multiplicity

$$N \simeq 2.16\,\alpha \bar{K}(2E_c/M)^{2/3}\,. \qquad (10)$$

Assuming $\alpha = 1$, $K = 0.4$ and taking into account the factor 2/3 it is possible to obtain the average number of charged particles produced in clusters

$$N_\pm \simeq 0.58(2E_c/M)^{2/3}\,. \qquad (11)$$

Table

Comparison of Experimentally Obtained Average Multiplicity of Charged Particles in pp-Collisions with Calculation Results

$S\ (GeV)^2$	26.0	47.0	96.6	190	452	910	1990	2800
$N_\pm$ exp.	3.65 ± 0.05	4.30 ±0.04	5.32 ±0.13	6.34 ±0.14	7.0 ±1.1	9.3 ±1.6	10.9 ± 1.6	12.2 ± 1.8
$N_\pm$ + 2 pres. paper	3.8	4.2	4.8	5.5	5.6	7.9	9.6	10.5
Pomeranchuk's $N_\pm$ model	3.1	4.2	6.0	8.4	13.0	18.4	27.2	32.3

When comparing relation (11) with experimental data it should be borne in mind that they give the general number of detected charged particles including the primary protons and the particles of their fragmentation. As is seen from the Table, after increasing the obtained multiplicity (11) by two charged particles, corresponding to primary protons, $N_{\pm} + 2$ remains somewhat smaller than the number of charged particles observed in experiments performed with hydrogen bubble chambers and colliding beams /8/ (p.195). The Table presents for comparison the predictions of charged particle multiplicity, ref. /5c/ , according to the statistical Pomeranchuk's model without taking into account the cluster motion in the C-system, $N = 0.61 S^{1/2}$, where S is taken in $(GeV)^2$.

REFERENCES

1. *E.Fetmi. Prog. Theor. Phys., 5, 570 /1950/.*
2. *I.Ya.Pomeranchuk. DAN SSSR, 78, 889 /1951/.*
3. *L.D.Landau: a) Izv. AN SSSR, ser. fiz., 17, 51 /1953/; b) Proc. "Nauka", v. I, II, p. 259, Moscow, 1960.*
4. *D.I.Blokhintsev. Proc. CERN Symp., 1956, v. 2, p.155; JETP, 32, 350 /1957/.*
5. *E.L.Feinberg: a) "Quantum Field Theory and Hydrodynamics" (in Russian), Proc. Inst. of Phys. USSR Academy of Sciences, v. 29, 155 /1965/; b) UFN, 70, 333 /1960/; c) UFN, 104, 539 /1971/.*
6. *O.V.Zhirov, E.V.Shchuryak. Yad. Fiz., 21, 861 /1975/.*
7. *I.L.Rozental. UFN, 116, 271 /1975/.*
8. *V.S.Murzin, L.I.Sarycheva. "Mulptiple Processes at High Energies" (in Russian), Moscow, Atomizdat, 1974.*

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