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GAUGE THEORIES AS THEORIES OF SPONTANEOUS BREAKDOWN

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1. There exists a close analogy between the Goldstone fields and the gauge fields [1]. Goldstonions arise inevitably if a symmetry is spontaneously broken (the Goldstone theorem [2,3]), so do gauge fields when a symmetry is localized (the Yang-Mills-Utiyama theorem [4,5]). They both transform by relevant groups inhomogeneously.

In the present note we demonstrate that this analogy is not accidental. Any gauge theory is proved to arise from spontaneous breakdown of symmetry subject to certain infinite parameter group, the corresponding gauge field being the Goldstonion by which this breakdown is accompanied. Thus the Yang-Mills-Utiyama theorem turns out a particular case of the Goldstone theorem.

Our paper develops and generalizes results of Borisov and one of the authors (V.I.O.) [6] who realized that the gravitation theory is the theory of spontaneously broken affine and conformal symmetries, the gravitation field $h_{\mu\nu}$ being the Goldstonion connected with proper affine transformations. To ours there is close in the spirit also the approach of Ferrari, Picasso [7] and Brandt, Ng [8]. They treat an Abelian gauge field (photon) as the Goldstonion corresponding to the gauge transformation with linear phase.

Another approach was suggested recently by Cho and Freund [9]. They deduce non-Abelian Yang-Mills fields on the N-dimensional compact group from the spontaneous breakdown of affine symmetry in $4+N$ dimensions. Their approach is nonminimal for it requires extra dimensions, is restricted to the non-Abelian case only, and leads to a lot of superfluous (scalar) Goldstonions along with the Yang-Mills fields. Our scheme is free of such troubles.

2. The first statement is that any gauge transformation can be considered as a constant parameter transformation of some group K having the infinite number of generators. The commutation relations between these generators are defined uniquely by the algebra of the initial finite parameter group. We confine ourselves to gauge functions which are decomposable in the Taylor series near $x_\mu = 0$ and consider first internal symmetries.

The proof is as follows. We represent an element of some local symmetry group $G^\ell(x)$ in the form

$$g^\ell(x) = \exp\{ i a^a(x) Q^a \} = \exp\{ i a^a(0)\, Q^a + i \sum_{n \geq 1} \frac{1}{n!} a^a_{\mu_1 \dots \mu_n} Q^a_{\mu_1 \dots \mu_n} \}, \quad (1)$$

where Q^a are the generators of the corresponding finite parameter group G^0 and

$$a^a_{\mu_1 \dots \mu_n} = \partial^x_{\mu_1} \dots \partial^x_{\mu_n} a^a(x)|_{x=0}$$

$$Q^a_{\mu_1 \dots \mu_n} = x_{\mu_1} \dots x_{\mu_n} Q^a \qquad (2)$$

Thus the gauge group $G^\ell(x)$ (in some vicinity of $x_\mu = 0$) is a particular realization of infinite constant parameter group K which is generated by operators $Q^a_\mu, \dots, Q^a_{\mu_1 \dots \mu_n}, \dots$ together with Q^a. The commutation relations between themselves and with the 4-momentum operator $P_\mu = -i\partial^x_\mu$ can be written using their representation (2)

$$[Q^a_{\mu_1 \dots \mu_k}, Q^\beta_{\mu_{k+1} \dots \mu_n}] = i C^\gamma_{a\beta} Q^\gamma_{\mu_1 \dots \mu_n} \qquad (3)$$

$$[P_\rho, Q^a_\mu] = -i\delta_{\rho\mu} Q^a \qquad (4)$$

$$[P_\rho , Q^\alpha_{\mu_1 \dots \mu_n}] = -i(\delta_{\rho\mu_1} Q^\alpha_{\mu_2 \dots \mu_n} + \dots + \delta_{\rho\mu_n} Q^\alpha_{\mu_1 \dots \mu_{n-1}}), (n \geq 2) \quad (5)$$

where $C^\gamma_{\alpha\beta}$ are the structure constants of the subgroup G^0. The transformation properties of $Q^\alpha_{\mu_1 \dots \mu_n}$ under G^0 and under the homogeneous Lorentz group L are obvious. Together with the Poincare group $\mathcal{P}$ the group K forms the semi-direct product $\mathcal{K} = K \subset\!\!\!\times \mathcal{P}$. Once the Lie algebra of $\mathcal{K}$ is found one can forget about particular representation (2) and regard this group as given by its Lie algebra solely.

3. Our main result is the following Theorem.

The gauge theory associated with the gauge local group $G^\rho(x)$ can be obtained by the Nambu-Goldstone realization [2,10] *of the symmetry under the group $\mathcal{K}$, the subgroup $G^0 \times \mathcal{P}$ being the vacuum stability subgroup. By this procedure the gauge field turns out to be Goldstonion corresponding to the generator Q^α_μ.*

The most natural and direct approach to the Nambu-Goldstone realization of a symmetry is the nonlinear realization method thoroughly worked out in [11,12]. According to [11,12] we have to parametrize the quotient space $\mathcal{K} / G^0 \times L$ by the fields $b^\alpha_\mu(x), \dots, b^\alpha_{\mu_1 \dots \mu_n}(x)$.. quantum numbers of which coincide with those of the generators $Q^\alpha_\mu, \dots Q^\alpha_{\mu_1 \dots \mu_n}$, and then we have to consider the left action of the group $\mathcal{K}$ in the quotient space:

$$G(x,b) = e^{i x_\mu P_\mu} e^{i \sum_{n \geq 1} \frac{1}{n!} b^\alpha_{\mu_1 \dots \mu_n} Q^\alpha_{\mu_1 \dots \mu_n}}, \quad k\, G(x,b) = G(x,b') e^{i U^\alpha(x,k) Q^\alpha} \quad (6)$$

We are interested only in the action of the group K because the Poincare group acts on fields and on x_μ in the standard way.

It follows from (6) that the fields $b^{\alpha}_{\mu}(x),\dots b^{\alpha}_{\mu_1\dots\mu_n}(x)$.. transform inhomogeneously:

$$Q^{\alpha}_{\mu}:\ \delta b^{\alpha}_{\mu}=a^{\alpha}_{\mu}+O^{\alpha}_{1}(x,b);\quad (O^{\alpha}_{1}(x,b)=-C^{\alpha}_{\beta\gamma}b^{\beta}_{\mu}\,x_{\rho}a^{\gamma}_{\rho})$$

$$Q^{\alpha}_{\mu_1\dots\mu_n}:\ \delta b^{\alpha}_{\mu_1\dots\mu_n}=a^{\alpha}_{\mu_1\dots\mu_n}+O^{\alpha}_{n}(x,b). \tag{7}$$

Thus, these fields are Goldstonions . Other fields $\Psi(x)$ transform by the representations of stability subgroup G^{0} but with parameters-functions $U^{\alpha}(x,k)$:

$$\Psi'(x)=e^{iU^{\alpha}(x,k)Q^{\alpha}_{\Psi}}\,\Psi(x). \tag{8}$$

Under the stability group all the fields

$\Psi(x), b^{\alpha}_{\mu}(x),\dots b^{\alpha}_{\mu_1\dots\mu_n}(x)\dots$

transform according to their representations in this group linearly and homogeneously.

The invariant Lagrangians are written down in a standard way and they involve fields $\Psi(x)$, covariant derivatives $D_{\rho}\Psi(x)$ and the Goldstonion covariant derivatives $\nabla_{\rho}b^{\alpha}_{\mu_1\dots\mu_n}$ /11,12/. The covariant derivatives are determined following general prescriptions of /11,12/:

$$G^{-1}(x,b)\partial_{\rho}G(x,b)=iP_{\rho}+i\sum_{n\geq 1}\frac{1}{n!}\nabla_{\rho}b^{\alpha}_{\mu_1\dots\mu_n}Q^{\alpha}_{\mu_1\dots\mu_n}+i\,\mho^{\alpha}_{\rho}Q^{\alpha}, \tag{9}$$

$$D_{\rho}\Psi(x)=(\partial_{\rho}+i\,\mho^{\alpha}_{\rho}Q^{\alpha}_{\Psi})\,\Psi(x). \tag{10}$$

Due to a specific structure of the group K they involve Goldstonions polynomially and one can easily find their explicit form:

$$\nabla_{\rho}b^{\alpha}_{\mu}(x)=\partial_{\rho}b^{\alpha}_{\mu}(x)+b^{\alpha}_{\rho\mu}(x)-\frac{1}{2}C^{\alpha}_{\beta\gamma}b^{\beta}_{\rho}(x)b^{\gamma}_{\mu}(x), \tag{11}$$

$$D_\rho \Psi(x) = (\partial_\rho + i b^a_\rho (x) Q^a_\Psi) \Psi(x). \tag{12}$$

For brevity we do not write covariant derivatives of $b^a_{\mu_1 \dots \mu_n}$ $(n \geq 2)$.

It is important that the symmetric and antisymmetric parts of $\nabla_\rho b^a_\mu$:

$$\nabla_{\{\rho} b^a_{\mu\}} = \partial_\rho b^a_\mu + \partial_\mu b^a_\rho + 2 b^a_{\rho\mu} , \tag{13}$$

$$\nabla_{[\rho} b^a_{\mu]} = \partial_\rho b^a_\mu - \partial_\mu b^a_\rho - C^a_{\beta\gamma} b^\beta_\rho b^\gamma_\mu \tag{14}$$

transform independently of each other. Covariant quantities $\nabla_{[\rho} b^a_{\mu]}$ and $D_\mu \Psi$ do not contain Goldstonions with more than one vector index. We see, therefore, that the invariant Lagrangians can be constructed out of $\nabla_{[\rho} b^a_{\mu]}$, $D_\rho \Psi$ and Ψ only:

$$\mathcal{L}^{inv}_{(K)} = \mathcal{L}^{inv}_{(L \times G^0)} \left(\nabla_{[\rho} b^a_{\mu]}, D_\rho \Psi, \Psi \right).$$

Further, $\nabla_{[\rho} b^a_{\mu]}$ coincides exactly with the covariant Yang-Mills field strength and the couplings of the Goldstonion $b^a_\mu(x)$ to other fields $\Psi(x)$ are identical with the couplings of the Yang-Mills gauge field . Thus, we can identify $b^a_\mu(x)$ with this gauge field.

Finally, the tensor Goldstonions $b^a_{\mu_1 \dots \mu_n}$ $(n \geq 2)$ turn out to be unessential. One can eliminate them in a covariant manner by means of the inverse Higgs phenomenon /13/. Namely, they all can be expressed in terms of

$b^{\alpha}_{\mu}(x)$ and its derivatives by equating to zero the symmetric parts of covariant derivatives $\nabla_{\{\rho} b^{\alpha}_{\mu_1 \dots \mu_n\}}$ $(n \geq 1)$*.

For instance:

$$\nabla_{\{\rho} b^{\alpha}_{\mu\}} = 0 \to b^{\alpha}_{\rho\mu} = -\frac{1}{2}(\partial_{\rho} b^{\alpha}_{\mu} + \partial_{\mu} b^{\alpha}_{\rho})$$

Thus, there is one essential, "true" Goldstonion b^{α}_{μ} which coincides with the Yang-Mills canonical field. All other Goldstonions are merely certain functions of this field and its derivatives. Note that the inhomogeneous term a^{α}_{μ} in Eq. (7) can result only from the transversal part of field b^{α}_{μ} and not from the longitudinal part. Therefore in fact just the transversal, physical component of b^{α}_{μ} is the Goldstonion ** connected with generator Q^{α}_{μ}. Correspondingly, just the invariance under the transformation with this generator plays the crucial role (for the Abelian case it has been pointed out in [7,8]). The longitudinal component is also of the Goldstonion type but it is expressed in terms of Goldstonions $\partial_{\mu} b^{\alpha}_{\mu}$, $\partial_{\rho}\partial_{\mu} b^{\alpha}_{\mu}$, ... associated with generators $Q^{\alpha}_{\mu_1\mu_2} \dots Q^{\alpha}_{\mu_1 \dots \mu_n}$ $(n \geq 2)$.

Above we take the subgroup G^0 be unbroken. If the global symmetry under G^0 itself is spontaneously broken, say, down to some subgroup H with generators V^{α}, we have to consider realizations in the quotient space

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* Of course, it is not necessary to do so. One may keep these fields but then they spontaneously acquire mass through absorbing symmetrical combinations of vector Goldstonion derivatives (i.e , the direct Higgs phenomenon occurs). The resulting massive tensor fields are not of the Goldstonion type (they transform homogeneously, like other fields $\Psi(x)$), and, besides, their masses are not related. Therefore they are extra fields and are not of need for constructing invariant Lagrangians.

** The statement of paper [11] that Goldstonions of spin >1/2 do not exist is based on two demands: 1) explicit relativistic invariance and 2) the positive definiteness of the metric in the space of states. This statement does not refer to any gauge theory because it is impossible to fulfill there these both demands simultaneously.

$\mathcal{K}/L \times H$ and, respectively, to use, instead of $G(x,b)$ (6), the quantity

$$G(x,b,\xi) = G(x,b)\exp\{i\xi^i Z^i\}$$

where Z^i are those generators from G^0 which do not enter into H, ξ^i are relevant Goldstonions. In this case our method reproduces automatically the covariant derivatives $\nabla_\mu \xi^i(x)$ of fields $\xi^i(x)$ and the covariant curls of fields $b_\mu^\alpha(x)$ (in contrast to the standard covariant strengths (14) they transform nonlinearly, like $\nabla_\mu \xi^i(x)$). Goldstonions connected with generators Z_μ^i are no longer true ones since they can acquire mass by the Higgs mechanism, i.e., by absorbing the Goldstonions $\xi^i(x)$. The only true Goldstonions in this case are those associated with generators V_μ^α . These coincide with the Yang-Mills massless fields corresponding to the subgroup H.

Up to now we considered internal symmetries. However a similar analysis can be performed also in the case when the initial subgroup G^0 determines the space-time symmetry. The difference is that in such a case group $\mathcal{K}$ has no longer simple form of the semidirect product $K \subset\!\!\times \mathcal{P}$.

4. Thus, any gauge theory is a theory of some spontaneously broken symmetry. Therefore the spontaneously broken symmetry is more profound and general concept than the gauge symmetry.

It should be emphasized once more that, from the formal point of view, the only difference between the Yang-Mills theories and, say, nonlinear chiral dynamics lies in the fact that the former is nonlinear realizations of infinite parameter groups ($\mathcal{K}$) whereas the latter is those of finite parameter group ($SU_2 \times SU_2$). In this connection it is natural to ask whether the linear σ -model of the Yang-Mills theory exists, by analogy with the linear SU_2- σ-model. As the group $\mathcal{K}$ has the infinite number of generators its linear representations (if exist) will be infinite-dimensional. Therefore the relevant σ -model will contain, parallel with the Yang-Mills fields, the infinite set of resonances, in particular, the tensor Goldstonions $b^{a}_{\mu_1 \dots \mu_n}$ $(n \geq 2)$ which are no longer to be eliminated. We believe that such a σ-model could be closely related to dual models of strong interactions. Further, there arises the problem of restoration of the symmetry under the group K at high temperature [15]. It is interesting also to realize the connection between the present approach and gauge theories on a lattice [16]. All these questions will be examined elsewhere.

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