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RADIATIVE CORRECTIONS
TO THE PROCESS $\mu^+\mu^- \to H\gamma$

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1 Introduction

The search for Higgs particles is one of important goals of the future high–energy colliders [1]. Most interesting and crucial parameters are the values of the Higgs couplings to other fundamental particles. A measurement of the latter would allow us to make a choice between different Higgs schemes.

Presumably, Higgs boson may be revealed at the forthcoming LHC collider, but the precision will be insufficient for the aims outlined above. Future muon colliders can provide a possibility to study the Higgs properties in detail. The idea to collide $\mu\mu$-lepton beams was discussed long ago [2]. Its recent development and a discussion of physics goals are given, for example, in paper [3]. Nowadays there are two designs for $\mu\mu$ colliders:
1) $\sqrt{s} = 500$ GeV, $\qquad \mathcal{L} = 10^{33} \ cm^{-2}s^{-1}$, $\qquad \mathcal{L}_{tot} = 50$ fb^{-1}/year ;
2) $\sqrt{s} = 4$ TeV, $\qquad \mathcal{L} = 10^{35} \ cm^{-2}s^{-1}$, $\qquad \mathcal{L}_{tot} = 200 \div 1000$ fb^{-1}/year.

A $\mu\mu$ collider has definite advantages as compared to e^+e^- one:
1) beamstrahlung and bremsstrahlung essentially absent ;
2) $m_\mu \gg m_e$;
3) focus problems absent, energy resolution $\sim 0.1\%$, small diameter due to absence of synchrotron radiation (cost decreases).

At the same time there are also disadvantages: cooling, expensive detectors, polarization implies significant loss in $\mathcal{L}$, only annihilation channel works ($J = 1$) hence $\sigma \sim s^{-1}, s \to \infty$.

But apparently, the main positive feature of the $\mu^+\mu^-$ collider is the possibility to study the s-channel Higgs boson production (*Higgs factory*), because the cross section of the latter is proportional to the lepton mass. However, when $m_H \geq 2M_W$, it is better to study Higgs boson production in association with the Z^0-boson or photon [4]. In the latter case at the Born level the cross section for the process

$$\mu^-(p_1) \ + \ \mu^+(p_2) \ \longrightarrow \ H(q) \ + \ \gamma(k_1) \tag{1}$$

has, at high energies, the following form

$$\frac{d\sigma_0^{\gamma H}}{dc} = \frac{\pi\alpha^2(s^2 + M_H^4)}{2\sin\theta_W^2(1 - c^2)s^2(s - M_H^2)}\left(\frac{m_\mu}{M_W}\right)^2 , \tag{2}$$

$$c = \cos(\widehat{p_1 k_1}), \qquad 1 - |c| \gg m_\mu^2/M_H^2.$$

Sharp dependence on photon energy $\omega = (s - M_H^2)/(2\sqrt{s})$ in eq.(2) makes manifest a reason why the calculations of the radiative corrections (RC) to this process are urgently desirable, so we proceed with it. In paper [5] it was shown that in the region $2\omega/\sqrt{s} \sim 1$ the main contribution to the cross-section arises from 1–loop electroweak corrections.

2 Radiative corrections

The result of the first order RC to the differential cross-section of the process $\mu^+\mu^- \to H\gamma$ due to virtual (Fig. 1) and soft real photons in the case when photon is hitting a non-forward detector looks as follows:

$$\frac{d\sigma^{\gamma H}_{B+S+V}}{dc} = \frac{d\sigma^{\gamma H}_0}{dc}\left(1 + \frac{2\alpha}{\pi}\ln\frac{s}{m_\mu^2}\ln\frac{\Delta\varepsilon}{\varepsilon}\right)\left(1 - \frac{\alpha}{2\pi}K\right), \qquad (3)$$

$$K = -4 + 2\mathrm{Li}_2\left(1 + \frac{\chi_1}{M_H^2}\right) + 2\mathrm{Li}_2\left(1 + \frac{\chi_2}{M_H^2}\right) - 4\mathrm{Li}_2\left(1 - \frac{M_H^2}{s}\right) -$$

$$- \frac{8\pi^2}{3} - \ln^2\left(\frac{s}{\chi_1}\right) - \ln^2\left(\frac{s}{\chi_2}\right) + \ln^2\left(\frac{M_H^2}{\chi_1}\right) + \ln^2\left(\frac{M_H^2}{\chi_2}\right) -$$

$$- \frac{M_H^2(s - M_H^2) + 12\chi_1\chi_2\zeta_2}{s^2 + M_H^4}, \qquad (4)$$

$$\mathrm{Li}_2(z) = -\int_0^1\frac{\ln(1-zx)}{x}dx, \qquad \chi_{1,2} = 2p_{1,2}k_1.$$

For hard (with energy more than $\Delta\varepsilon$) collinear photon emission we obtain

$$\frac{d\sigma^{hard}}{dc} = \frac{\alpha}{\pi}\int_{\Delta\varepsilon/\varepsilon}^1\frac{dx}{x}\left[\left(1 - x + \frac{x^2}{2}\right)\ln\frac{s}{m_\mu^2} - (1-x)\right]\times$$

$$\times \left[\frac{d\tilde\sigma_0(p_1(1-x),p_2)}{dc} + \frac{d\tilde\sigma_0(p_1,p_2(1-x))}{dc}\right], \qquad (5)$$

where $d\tilde\sigma_0((1-x)p_1,p_2)/dc$ and $d\tilde\sigma_0(p_1,(1-x)p_2)/dc$ are the so-called *shifted* (or *boosted*) cross section. In general case, when photon emission is allowed from both initial leptons, we have

$$\frac{d\tilde\sigma_0(z_1p_1,z_2p_2)}{dc} = \frac{\pi\alpha^2}{2\sin^2\theta_W}\left(\frac{m_\mu}{M_W}\right)^2\frac{s^2z_1^2z_2^2 + M_H^4}{s^2z_1z_2(1-c^2)[sz_1z_2 - M_H^2]}. \qquad (6)$$

One can see the cancellation of $\Delta\varepsilon/\varepsilon$ when integrate over x from $\Delta\varepsilon/\varepsilon$ up to 1.

Summing up the leading terms from Eqs. (3,5), we get the first order leading logarithmic correction in the form

$$\frac{d\sigma^{hard+S+V}}{dc} = \frac{\alpha}{2\pi}\ln\frac{s}{m_\mu^2}\int_0^1 dx\left(\frac{1}{x}\right)_+(1 + (1-x)^2)\left[\frac{d\tilde\sigma_0((1-x)p_1,p_2)}{dc} + \right.$$

$$\left. + \frac{d\tilde\sigma_0(p_1,(1-x)p_2)}{dc}\right]. \qquad (7)$$

The *plus operation* acts as usually:

$$\left(\frac{1}{x}\right)_+ f(x) = \frac{f(x) - f(0)}{x}. \qquad (8)$$

Our result may be compared with that obtained by B. Kniehl [6] on RC to the fermionic width of Higgs in the case $M_H^2 \gg m_\mu^2$. Indeed, looking at the leading logarithms we see an agreement.

The contribution of higher orders of perturbation theory can be taken into account in the leading logarithmic approximation. Really, as opposite to the case of electron-positron annihilation into one virtual photon (or Z-boson) [7] we have to use here another kernel for evolution equations. Considering only the non-singlet of relevant structure functions, we write down:

$$\widetilde{D}(z,L) = \delta(1-z) +$$

$$+ \frac{\alpha}{2\pi}(L-1)\tilde{P}^{(1)}(z) + \frac{1}{2!}\left(\frac{\alpha}{2\pi}(L-1)\right)^2\tilde{P}^{(2)}(z) + \dots. \qquad (9)$$

$$L = \ln\frac{s}{m_\mu^2}, \quad \tilde{P}^{(1)}(z) = \lim_{\Delta\to 0}\left\{\frac{1+z^2}{1-z}\Theta(1 - z - \Delta) + \right.$$

$$\left. + 2\delta(1-z)\ln\Delta\right\}, \qquad \int_0^1 dz\tilde{P}^{(1)}(z) = -\frac{3}{2},$$

$$\tilde{P}^{(2)}(z) = \int_z^1\frac{dt}{t}\tilde{P}^{(1)}(t)\tilde{P}^{(1)}\left(\frac{z}{t}\right).$$

A *smoothed* representation for the modified D-function inferred looks as follows:

$$\widetilde{D}(z,L) = -\frac{d}{dz}\int_z^1 dx\,\overline{D}(x,L) = \frac{1}{2}\beta(1-z)^{\beta/2-1}(1+z^2)(1+\mathcal{O}(\beta^2)). \qquad (10)$$

$$\beta = \frac{2\alpha}{\pi}(L-1).$$

The master formula for radiatively corrected cross section has the form of the Drell-Yan cross section. So, we suggest to write the result as a convolution of the modified lepton structure functions with the shifted cross section of the hard subprocess. It reads

$$\frac{d\sigma}{dc} = \int_{z_1^{min}}^1 dz_1\widetilde{D}(z_1)\int_{z_2^{min}}^1 dz_2\widetilde{D}(z_2)\frac{d\tilde\sigma_0(z_1p_1,z_2p_2)}{dc}\left(1 - \frac{\alpha}{2\pi}K\right)\Theta(\omega - \omega_{th}). \qquad (11)$$

where a part of non-leading terms is taken into account by the K-factor and ω_{th} is the experimental energy threshold of the photon registration. The energy conservation law gives us the energy of the detected photon

$$\omega = \frac{sz_1z_2 - M_H^2}{2\varepsilon(z_1 + z_2 - c(z_1 - z_2))}. \qquad (12)$$
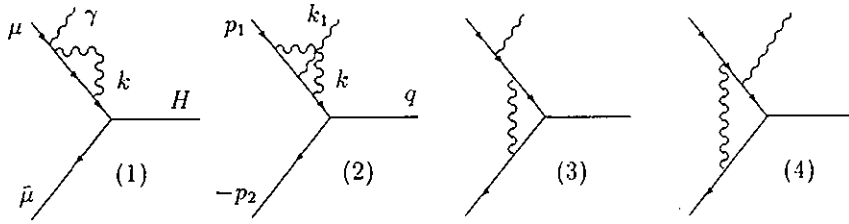
Figure 1: The subset of Feynman diagrams for the process $\mu\bar{\mu} \to H\gamma$.

The lower limits for integration over $z_{1,2}$ are to be defined also just from the above expression by imposing the condition $\omega > \omega_{th}$:

$$z_1^{min} = \frac{M_H^2 + \sqrt{s}\omega_{th}(1+c)}{s - \sqrt{s}\omega_{th}(1-c)}, \qquad z_2^{min} = \frac{M_H^2 + \sqrt{s}z_1\omega_{th}(1-c)}{sz_1 - \sqrt{s}\omega_{th}(1+c)}. \qquad (13)$$

3 Conclusions

We use the normalization for $e\bar{e}H(q)$ vertex function $\Gamma^{(2)}(q^2)|_{q^2=0} = 0$ as is akin to one exploited for $e\bar{e}Z$ vertex in the paper of Berends et al. [8]. The choice of subtraction point $q^2 = 0$ is common to the Standard Model parameters $\sin\theta_W$, M_Z, M_H normalization.

The kernel of the evolution equation for $\tilde{D}(z)$, $\tilde{P}^{(1)}(z)$ differs from the one, that appears in the evolution equation for $D^{NS}(z)$ [9]. The kernel $P^{(1)}(z)$ is responsible for a, say, single photon annihilation of $\mu^+\mu^-$ into hadrons or the leading twist contribution in deep inelastic scattering. This fact is natural, since the kernels $\tilde{P}^{(1)}(z)$ and $P^{(1)}(z)$ describe evolution of matrix elements of twist-3 operators $\bar{\psi}\psi$ and twist-2 operators $\bar{\psi}\gamma_\mu\psi$, respectively.

Due to the fact that the cross section of $\mu^+\mu^- \to H\gamma$ is proportional to muon mass squared, the validity of the Kinoshita–Lee–Nauenberg theorem [10] proving the absence of singularities in the limit $m_\mu \to 0$ is restored. The problem was first noted and discussed in calculations of radiative corrections to the Higgs decay width into fermions [11].

In the Fig. 2 we presented the values of radiative corrections as functions of the center-of-mass energy

$$\delta(\sqrt{s}) = \frac{\int_{c_{min}}^{c_{max}} dc(d\sigma/dc)}{\int_{c_{min}}^{c_{max}} dc(d\sigma_0^{\gamma H}/dc)} 100\%. \qquad (14)$$
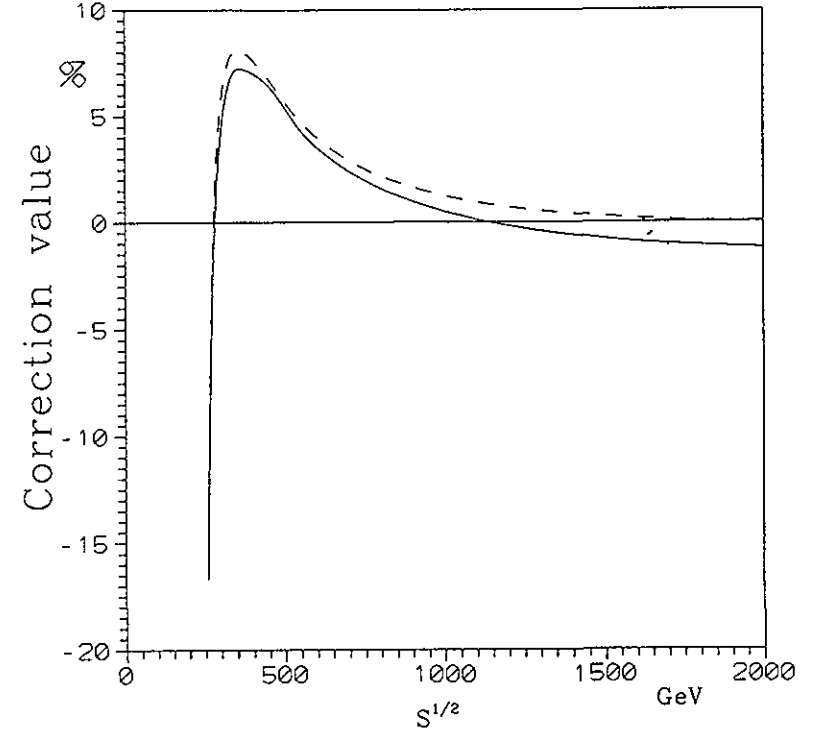

Figure 2: Radiative corrections to the process $\mu\bar{\mu} \to H\gamma$.

We took $M_H = 250$ GeV, the value of photon energy threshold $\omega_{th}=5$ GeV, and the angular range for photon detection $-0.999 < c < 0.999$. The dashed line represents the first order leading logarithmic correction, calculated according to Eq. (7) with K-factor included. The solid line shows the values of the complete RC according to Eq. (11).

Acknowledgments

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