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BACKGROUND HARMONIC SUPERFIELDS IN $N=2$ SUPERGRAVITY

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1. Introduction

Interactions of physical fields of the extended supergravity (SG_4^2): graviton g_{mn}, gravitino $\psi_m^{\alpha i}$ and the abelian gauge field A_m have firstly been investigated in the framework of the formalism that takes into account equations of motion for the proof of local supersymmetry [1, 2]. The problem of auxiliary fields of SG_4^2 has been solved in refs. [3, 4]. We shall consider a version of the Einstein $N = 2$ supergravity with 40 boson and 40 fermion field components. It should be noted that the component tensor calculus of SG_4^2 leads to very tedious calculations in constructing interactions with matter supermultiplets and analyzing quantum properties of the theory. So various superfield approaches to the study of the extended supergravity have been developed intensively in parallel with the component analysis [5, 6, 7, 8, 9].

A geometric approach to the superfield description of SG_4^2 has been proposed in the method of harmonic superspace (HSS) [10]. *Analytic prepotentials of SG_4^2 appear in* a decomposition of the invariant harmonic derivative Δ^{++} in terms of the operators of partial derivatives in HSS. We have shown in [12] that harmonic superfield constraints of zero dimension in SG_4^2 can be solved in the special *flat* coordinates by analogy with the solution of the corresponding harmonic constraints of the nonabelian supergauge theory [11, 13, 14]. Using this solution the authors of ref. [15] have solved all superfield constraints of SG_4^2 and constructed the superfield action nonlinear in prepotentials. The conformal $N = 2$ supergravity and alternative versions of the extended Einstein SG have also been studied in HSS [16].

Effectiveness of superfield formulations in supersymmetric theories is mainly connected with simplification of quantum calculations. It should be underlined that a procedure of HSS-quantization of the extended supergravity seems us technically more complicated than the quantization of Yang-Mills theory in HSS [11]. First of all, SG_4^2 has several analytic prepotentials analogous to one matrix prepotential of the gauge theory. Besides, in refs. [10, 15], the gauge symmetry of the classical formalism of the extended supergravity has been considered, and the additional background supersymmetry of perturbative expansion of the SG_4^2 action in degrees of the gravitational constant κ has not been discussed there. We know from the formalism of quantization of the superfield $N = 1$ supergravity [18, 19] that the background supersymmetry is very important in constructing superfield Feynman rules. This work develops the investigation of the background-supersymmetry problem in the harmonic formalism of SG_4^2 started in [17].

In sect. 2, *we discuss* a connection between different bases of differential operators in the background HSS and show that gravitational superfields depending linearly on the spinor nonanalytic coordinates θ^- (linear harmonic superfields) naturally arise in the covariant basis. Analytic prepotentials of a holonomic basis [10] can be treated in this approach as coefficients of the θ^--decomposition of background linear superfields; so they have nonstandard transformations with respect to the background supersymmetry.

Note that the use of an analytic compensator [15] guarantees only the background supersymmetry without the central charge in the SG_4^2 action, but the formal covariance is hidden after the shift of this compensator superfield on the flat part manifestly depending on spinor coordinates.

In sect. 3, we consider a solution of the analyticity condition in SG_4^2 which guarantees the covariance with respect to the background supersymmetry with the central charge (B-covariance). Alternative possibilities of choosing unconstrained superfield variables in the harmonic superspace are studied. In particular, we discuss the solution of the SG_4^2 constraints in HSS corresponding to the representation of linear gravitational superfields through the unconstrained harmonic spinor superfield of dimension $d = 5/2$. In a special gauge this solution can be expressed in terms of the harmonic-independent spinor prepotential, which has been considered earlier while describing the linearized supergravity in the ordinary superspace [9]. B-covariant solutions of the SG_4^2 constraints are discussed in sects. 4 and 5.

Sect.6 is devoted to a discussion of terms in the harmonic SG_4^2 action quadratic in the vector and scalar superfields. The structure of these terms is similar to the structure of the action in $N = 2$ gauge theory [17]. Note that the quadratic action has an additional global symmetry. B-covariant decomposition of the SG_4^2 action in κ can be constructed by an iteration method taking into account the gauge invariance.

In ref.[15], the nonlinear action of SG_4^2 has a form of the action for an analytic compensator in which the flat part manifestly depending on spinor coordinates is separated. This representation allows one to prove the gauge invariance; however, it does not possess the manifest B-covariance.

In sect.7, we discuss the alternative harmonic formalism of the linearized SG_4^2, in which spinor prepotentials of dimension $d = 3/2$ determine the dual invariant harmonic derivative Δ^{--}, a spinor component of the torsion with $d = -1/2$ vanishes identically, and equations of motion are equivalent to the dynamical analyticity condition (zero-curvature representation). This formalism is constructed by analogy with a dual formulation of the $N = 2$ Yang-Mills theory [21]. We hope that superfield methods will help us to study the quantum structure of $N = 2$ supergravity.

In Appendix, the definitions and notation of the basic derivatives in the flat HSS and some other useful formulae from refs. [10, 11] are written down.

2 FLAT BACKGROUND HARMONIC SUPERSPACE

It is convenient to consider superfields with a real central charge in the harmonic superspace $HSS(Z)$ [10] with the coordinates $u_i^\pm$ and

$$z_A^M = (x_A^5,\ x_A^m,\ \theta^{\mu+},\ \bar\theta^{\dot\mu+},\ \theta^{\mu-},\ \bar\theta^{\dot\mu-})\,, \tag{2.1}$$

where $u_i^\pm$ are the $SU(2)/U(1)$ harmonics , x_A^5 is a special coordinate associated with the central charge Z, and $m, \mu, \dot\mu$ are vector and spinor indices of the Lorentz group $SL(2, C)$. The analytic subspace $ASS(Z)$ is defined by the coordinates

$$\zeta(Z) = (x_A^5,\ \zeta) = (x_A^{\widehat m},\ \theta^{\widehat\mu+})\,, \tag{2.2}$$

where ζ describes 4-dimensional analytic coordinates, and the notation $\widehat m = (5,\ m)$ and $\widehat\mu = (\mu,\ \dot\mu)$ is introduced.

An introduction of the fifth coordinate is connected with a geometric interpretation of the central charge, and superfields can only have the cyclic dependence on x_A^5

$$\frac{\partial}{\partial x_A^5}\Phi = iZ\Phi\ . \tag{2.3}$$

The gauge supergroup of the extended supergravity ASG_4^2 is defined quite naturally using the transformations of the analytic coordinates z_A^M [10].

The basic infinitesimal parameters of the gauge transformations do not depend on $x_A^5,\ \theta^-,\ \bar\theta^-$

$$\delta x_A^{\widehat m} = \lambda^{\widehat m}(\zeta, u)\,, \tag{2.4}$$
$$\delta\theta^{\widehat\mu+} = \lambda^{\widehat\mu+}(\zeta, u)\,. \tag{2.5}$$

Harmonics do not transform in ASG_4^2 , and transformations of spinor coordinates with the charge -1 depend on all $HSS(Z)$ coordinates besides x_A^5

$$\delta\theta^{\widehat\mu-} = \lambda^{\widehat\mu-}(x_A^m, \theta^\pm, \bar\theta^\pm, u)\,. \tag{2.6}$$

A local gauge transformation of the general scalar superfield $\Phi(z_A, u)$ in ASG_4^2

$$\hat\delta\Phi = -\Lambda\Phi \tag{2.7}$$

is defined by the transformation operator Λ that includes the analytic operator λ

$$\Lambda = \lambda + \lambda^{\widehat\mu-}\partial^+_{\widehat\mu}\ , \quad \lambda = \lambda^{\widehat m}\partial^A_{\widehat m} + \lambda^{\widehat\mu+}\partial^-_{\widehat\mu}\ . \tag{2.8}$$

Gauge transformations of superfields preserve the Grassmann analyticity and the cyclicity condition (2.3)

$$[\partial^+_{\widehat\mu}, \lambda] = 0\ , \quad [\partial^+_{\widehat\mu}, \Lambda] = (\partial^+_{\widehat\mu}\lambda^{\widehat\nu-})\partial^+_{\widehat\nu}, \quad [\partial_5, \Lambda] = 0\ . \tag{2.9}$$

The local gauge transformations of analytic superfields have the form $\hat\delta\omega = -\lambda\omega$.

The differential operator Λ in eq.(2.8) is given in the holonomic basis, i.e., in a form of decomposition in terms of partial derivatives ∂^A_M. Gravitational prepotentials of SG_4^2 have also been defined in this holonomic basis as coefficients of the Λ-invariant harmonic differential operator

$$\Delta^{++} = \partial^{++} + H^{\widehat m++}\partial^A_{\widehat m} + H^{\widehat\mu(+3)}\partial^-_{\widehat\mu} + H^{\widehat\mu+}\partial^+_{\widehat\mu}\ . \tag{2.10}$$

Full gauge variations of these prepotentials have a very simple form

$$\delta_\lambda H^{\widehat m++} = \Delta^{++}\lambda^{\widehat m}\ , \quad \delta_\lambda H^{\widehat\mu(\pm1+2)} = \Delta^{++}\lambda^{\widehat\mu\pm}\ . \tag{2.11}$$

Commutator of two full variations can be calculated very simply in virtue of the invariance of the operator Δ^{++}

$$[\delta_2, \delta_1] H^{M++} = \Delta^{++} \lambda^{M}(1,2) , \qquad (2.12)$$

$$\lambda^{\widehat{M}}(1,2) = \lambda_2 \lambda_1^{\widehat{M}} - \lambda_1 \lambda_2^{\widehat{M}} , \qquad (2.13)$$

$$\lambda^{\widehat{\mu}-}(1,2) = \Lambda_2 \lambda_1^{\widehat{\mu}-} - \Lambda_1 \lambda_2^{\widehat{\mu}-} , \qquad (2.14)$$

where $\widehat{M} = (\widehat{m}, \widehat{\mu})$ and the differential operators Λ and λ (2.8) are used.

We shall use the following gauge for the nonanalytic gauge transformations [10]:

$$\Delta^{++} \lambda^{\widehat{\mu}-} = \lambda^{\widehat{\mu}+} . \qquad (2.15)$$

The matrix of induced tangent transformations is covariantly independent of harmonic variables in this gauge $\Delta^{++} \partial^{+}_{\mu} \lambda^{\widehat{\nu}-} = 0$.

The condition (2.15) corresponds to the following gauge of the nonanalytic prepotential:

$$H^{\widehat{\mu}+} = \theta^{\widehat{\mu}+} . \qquad (2.16)$$

In ref. [10], a flat limit of gravitational superfields is defined and a possibility of expansion in terms of the gravitational constant κ with respect to this limit is discussed. We shall consider the flat superspace $HSS(Z)$ as the background classical superspace of $N = 2$ supergravity. The background supersymmetry with the real central charge will be denoted by a symbol $B^2_4(Z)$, and the background superfields will be called B-superfields. Gravitational and matter superfields and the interaction in any degree in κ should be covariant under $B^2_4(Z)$ in this approach.

It should be underlined that the holonomic basis of $HSS(Z)$ is noncovariant with respect to $B^2_4(Z)$, and the corresponding prepotentials $H^{\widehat{m}++}$ (2.10) cannot be treated as B-superfields.

In ref. [12], the real coordinates have been used in $HSS(Z)$

$$z^{M} = (x^{5},\ x^{m},\ \theta^{\mu}_{i},\ \bar{\theta}^{i\dot{\mu}}) \qquad (2.17)$$

and the B-covariant decomposition of the invariant analytic operator has been considered

$$\Delta^{++} = \partial^{++} + G^{++} , \quad [D^{+}_{\alpha}, G^{++}] = 0 , \quad G^{++} = \hat{h}^{M++} \partial_{M} \equiv h^{M++} \partial^{A}_{M} . \qquad (2.18)$$

This representation is useful for studying an iterative solution to the harmonic equations of SG^2_4; however, it also uses the holonomic bases ∂_M and ∂^{A}_{M} noncovariant with respect to $B^2_4(Z)$. Gauge transformations of the prepotentials h^{M++} and $\hat{h}^{M++}$ can readily be obtained from the transformations (2.11) by using the relations

$$\hat{h}^{m++} = h^{m++} = H^{m++} + 2i\theta^{+}\sigma^{m}\bar{\theta}^{+} , \quad \hat{h}^{\widehat{\mu}(+3)} = H^{\widehat{\mu}(+3)} , \quad \hat{h}^{\widehat{\mu}+} = 0 , \qquad (2.19)$$

$$\hat{h}^{5++} = h^{5++} = H^{5++} + i(\bar{\theta}^{+})^2 - i(\theta^{+})^2 , \quad \hat{h}^{\widehat{\mu}k++} = u^{k-} h^{\widehat{\mu}(+3)} . \qquad (2.20)$$

Note that these prepotentials have a dimension $d = 1$ and $1/2$, they do not contain flat parts and are proportional to the constant κ.

It is convenient to study component stuff of the analytic prepotentials in the physical WZ-gauge [10]

$$h^{m++}_{WZ} = \kappa[-2i\theta^{+}\sigma^{a}\bar{\theta}^{+} h^{m}_{a}(x_A) + (\bar{\theta}^{+})^2 \theta^{\alpha+} u^{-}_{k} \psi^{mk}_{\alpha}(x_A) +$$
$$+(\theta^{+})^2 \bar{\theta}^{\dot{\alpha}+} u^{-}_{k} \bar{\psi}^{mk}_{\dot{\alpha}}(x_A) + (\theta^{+})^2 (\bar{\theta}^{+})^2 u^{k-} u^{l-} V^{m}_{(kl)}(x_A)] , \qquad (2.21)$$

$$h^{5++}_{WZ} = \kappa[i\theta^{+}\sigma^{a}\bar{\theta}^{+} A_{a}(x_A) + (\bar{\theta}^{+})^2 \theta^{\alpha+} u^{-}_{k} \rho^{k}_{\alpha}(x_A) +$$
$$+(\theta^{+})^2 \bar{\theta}^{\dot{\alpha}+} u^{-}_{k} \bar{\rho}^{k}_{\dot{\alpha}}(x_A) + (\theta^{+})^2 (\bar{\theta}^{+})^2 u^{-}_{k} u^{-}_{l} S^{(kl)}(x_A)] , \qquad (2.22)$$

$$h^{\mu(+3)}_{WZ} = \kappa\{(\theta^{+})^2 \bar{\theta}^{+}_{\dot{\alpha}} [B^{\mu\dot{\alpha}}(x_A) + iC^{\mu\dot{\alpha}}(x_A)] + (\bar{\theta}^{+})^2 \theta^{+}_{\alpha} [\varepsilon^{\mu\alpha}(M + iN)(x_A) +$$
$$+T^{(\mu\alpha)}(x_A)] + (\theta^{+})^2 (\bar{\theta}^{+})^2 u^{-}_{k} \xi^{\mu k}(x_A)\} . \qquad (2.23)$$

Here h^{m}_{a}, ψ^{mk}_{α} and A_a are physical fields, and other components play a role of auxiliary fields.

In Appendix, we consider two bases of B-covariant differential operators: 1) $D = (\partial^{++},\ \partial^{--},\ D_M)$ in the coordinates $u^{\pm}_{i}$, z^{M} and 2) $D^{A} = (D^{++},\ D^{--},\ D^{A}_{M})$ in the coordinates (2.1). Corresponding to these bases, decompositions of G^{++} and other objects of differential geometry will automatically be covariant with respect to the background supersymmetry.

Define now gravitational B-superfields G^{M++} in the basis D^{A}_{M}

$$\Delta^{++} = D^{++} + G^{\widehat{m}++} \partial^{A}_{\widehat{m}} - G^{\widehat{\mu}(+3)} D^{-}_{\widehat{\mu}} . \qquad (2.24)$$

Superfields $G^{\widehat{m}++}$ can be written in terms of the analytic prepotentials of the holonomic basis

$$G^{m++} = h^{m++} + 2i\theta^{-}\sigma^{m}\bar{h}^{(+3)} + 2ih^{(+3)}\sigma^{m}\bar{\theta}^{-} , \qquad (2.25)$$

$$G^{5++} = h^{5++} - 2i(\theta^{-}h^{(+3)}) + 2i(\bar{\theta}^{-}\bar{h}^{(+3)}) , \qquad (2.26)$$

$$G^{\widehat{\mu}(+3)} = h^{\widehat{\mu}(+3)} . \qquad (2.27)$$

By definition, B-superfields have trivial full background ϵ variations $\delta_B G^{M++} = 0$, and B-transformations of the analytic prepotentials have a noncovariant form

$$\delta_B h^{m++} = 2i(\sigma^{m})_{\mu\dot{\nu}} (\bar{\epsilon}^{k\dot{\nu}} h^{\mu(+3)} - \epsilon^{k\mu} \bar{h}^{\dot{\nu}(+3)}) u^{-}_{k} , \qquad (2.28)$$

$$\delta_B h^{5++} = 2i(\epsilon^{k\mu} h^{(+3)}_{\mu} + \bar{\epsilon}^{k\dot{\nu}} \bar{h}^{(+3)}_{\dot{\nu}}) u^{-}_{k} , \quad \delta_B h^{\widehat{\mu}(+3)} = 0 . \qquad (2.29)$$

The operator condition of analyticity $[D^{+}_{\widehat{\mu}}, G^{++}] = 0$ is equivalent to the following relations for the gravitational superfields:

$$D^{+}_{\widehat{\mu}} G^{\widehat{\nu}(+3)} = 0 , \quad D^{+}_{\widehat{\mu}} G^{5++} = -2iG^{(+3)}_{\widehat{\mu}} , \qquad (2.30)$$

$$D^{+}_{\alpha} G^{m++} = 2i(\sigma^{m})_{\alpha\dot{\alpha}} \bar{G}^{\dot{\alpha}(+3)} = -(\sigma^{m})_{\alpha\dot{\alpha}} \bar{D}^{\dot{\alpha}+} G^{5++} , \qquad (2.31)$$

$$\bar{D}^{+}_{\dot{\alpha}} G^{m++} = -2i(\sigma^{m})_{\alpha\dot{\alpha}} G^{\alpha(+3)} = (\sigma^{m})_{\alpha\dot{\alpha}} D^{\alpha+} G^{5++} . \qquad (2.32)$$

We shall consider the following decomposition of the transformation operator Λ in covariant bases:

$$\Lambda = \Lambda^{M} D^{A}_{M} = \hat{\Lambda}^{M} D_{M} , \qquad (2.33)$$

$$\Lambda^m = \hat{\Lambda}^m = \lambda^m + 2i\theta^- \sigma^m \bar{\lambda}^+ + 2i\lambda^+ \sigma^m \bar{\theta}^- , \tag{2.34}$$

$$\Lambda^5 = \hat{\Lambda}^5 = \lambda^5 - 2i(\theta^- \lambda^+) + 2i(\bar{\theta}^- \bar{\lambda}^+) , \quad \Lambda^{\widehat{\mu}\pm} = \lambda^{\widehat{\mu}\pm} , \tag{2.35}$$

$$\hat{\Lambda}^{\mu}_k = \lambda^{\mu+} u^-_k - \lambda^{\mu-} u^+_k . \tag{2.36}$$

It is evident that these B-superfield parameters satisfy the constraints analogous to eqs.(2.30-2.32), in particular, the parameters $\Lambda^{\widehat{m}}$ are linear in $\theta^{\widehat{\mu}-}$.

It should be stressed that the use of background supersymmetry in the formalism of $N = 1$ supergravity leads to appearance of linear vector superfield parameters which contain chiral parameters in the zero and first orders of the $\bar{\theta}$-decomposition [19].

Transformations of the differential operators of the covariant basis are

$$\delta D^{\pm\pm} = [\Lambda, D^{\pm\pm}] = -(D^{\pm\pm}\Lambda^M) D^A_M , \quad \delta D^A_M = [\Lambda, D^A_M] \equiv -\Lambda^N_M D^A_N , \tag{2.37}$$

$$\Lambda^N_5 = 0 , \quad \Lambda^n_m = \partial^A_m \Lambda^n , \quad \Lambda^5_m = \partial^A_m \Lambda^5 , \quad \Lambda^{\widehat{\nu}-+}_{\mu} = D^-_{\mu} \lambda^{\widehat{\nu}+} , \tag{2.38}$$

$$\Lambda^{m-}_{\mu} = D^-_{\mu} \Lambda^m - 2i(\sigma^m)_{\mu\dot{\nu}} \bar{\lambda}^{\dot{\nu}-} , \quad \Lambda^{5-}_{\mu} = D^-_{\mu}\Lambda^5 + 2i\lambda^-_{\mu} . \tag{2.39}$$

The operator Δ^{++} is invariant by definition, and this allows us to derive a transformation law of G^{++} in ΛSG^2_4

$$\delta G^{++} = -\delta D^{++} = [D^{++}, \Lambda] . \tag{2.40}$$

The full variations of the covariant gravitational superfields have the following form:

$$\delta G^{m++} = \Delta^{++}\Lambda^m + 2i(\sigma^m)_{\mu\dot{\nu}} (G^{\mu(+3)} \bar{\lambda}^{\dot{\nu}-} - \bar{G}^{\dot{\nu}(+3)} \lambda^{\mu-}) , \tag{2.41}$$

$$\delta G^{5++} = \Delta^{++}\Lambda^5 - 2iG^{\mu(+3)}\lambda^-_{\mu} - 2i\bar{G}^{\dot{\mu}(+3)}\bar{\lambda}^-_{\dot{\mu}} . \tag{2.42}$$

Corresponding local gauge transformations $\hat{\delta} G^{M++} = \delta G^{M++} - \Lambda G^{M++}$ are

$$\hat{\delta} G^{M++} = (D^{++} + G^{++})\Lambda^M - \lambda G^{M++} \equiv R^{M++}_N (G) \Lambda^N , \tag{2.43}$$

where the components of the local transformation operator $R^{M++}_N(G)$ are defined. It is evident that these transformations are consistent with the constraints (2.30-2.32).

The local variation of the operator G^{++} is determined by the transformations $\hat{\delta} G^{M++}$

$$\hat{\delta} G^{++} = [(D^{++} + G^{++}), \lambda] , \tag{2.44}$$

since the operators D^A_M are not change with respect to local transformations. This relation is useful for calculation of linear bracket parameters in the commutator of transformations (2.43)

$$\Lambda^{\widehat{m}}(1,2) = \lambda_2 \Lambda^{\widehat{m}}_1 - \lambda_1 \Lambda^{\widehat{m}}_2 , \quad \lambda = \Lambda^n \partial_n - \lambda^{\widehat{\nu}+} D^-_{\widehat{\nu}} , \tag{2.45}$$

which are constructed by analogy with eq.(2.13) for analytic parameters.

The functions $\lambda^{\widehat{\mu}-}$ in the gauge (2.15) are the series in terms of the superfields G

$$\lambda^{\widehat{\mu}-}(z,u) = \int \frac{du_1 du_2}{(u^+ u^+_1)} [\lambda^{\widehat{\mu}+}(z,u_1) - \frac{G^{++}(z,u_1)\lambda^{\widehat{\mu}+}(z,u_2)}{(u^+_1 u^+_2)}] + O(G^2) . \tag{2.46}$$

Note that the local variation of gravitational superfields (in distinction with the full variation $\delta G^{\widehat{m}++}$ and the operator Λ) does not contain $\lambda^{\widehat{\mu}-}$; so the operator $R^{M++}_N(G)$ in the perturbation theory can be decomposed into the sum of terms of the first and zero degree in the gravitational constant (superfields G are linear in κ).

3. ALTERNATIVE REPRESENTATIONS OF GRAVITATIONAL SUPERFIELDS

Let us remind that in the harmonic representation the nonlinear superfield constraints of SG^2_4 are reduced to the analyticity conditions for the prepotentials H^{M++} or to the equivalent linear constraints (2.30-2.32) for the gravitational background superfields. Consider the manifestly covariant representation for $G^{\widehat{m}++}$[17]

$$G^{m++}(\Psi) = (\sigma^m)_{\alpha\dot{\beta}} [(D^+)^2 \bar{D}^{\dot{\beta}+} \Psi^{\alpha-} - (\bar{D}^+)^2 D^{\alpha+} \bar{\Psi}^{\dot{\beta}-}] . \tag{3.1}$$

$$G^{5++}(\Psi) = (\bar{D}^+)^2 D^+_{\alpha} \Psi^{\alpha-} + (D^+)^2 \bar{D}^+_{\dot{\alpha}} \bar{\Psi}^{\dot{\alpha}-} . \tag{3.2}$$

which allows one to solve all constraints and to rewrite all geometric quantities through the unconstrained spinor superfields $\Psi^-_{\alpha}(z,u)$ and $\bar{\Psi}^-_{\dot{\alpha}}(z,u)$ of dimension $d = 5/2$. In a special gauge the spinor prepotentials are linear in u^-_k and proportional to an ordinary superfield $\Psi^{\alpha}_k(z)$ analogous to the gauge superfield of the linearized SG^2_4 [9].

A solution of constraints for the superfield parameters $\Lambda^{\widehat{m}}$ (2.34,2.35) can also be written in terms of unconstrained parameters $K^{(-3)}_{\alpha}(z,u)$

$$\Lambda^m(K) = (\sigma^m)_{\alpha\dot{\beta}} [(D^+)^2 \bar{D}^{\dot{\beta}+} K^{\alpha(-3)} - (\bar{D}^+)^2 D^{\alpha+} \bar{K}^{\dot{\beta}(-3)}] , \tag{3.3}$$

$$\Lambda^5(K) = (\bar{D}^+)^2 D^+_{\alpha} K^{\alpha(-3)} + (D^+)^2 \bar{D}^+_{\dot{\alpha}} \bar{K}^{\dot{\alpha}(-3)} , \tag{3.4}$$

$$\lambda^+_{\alpha}(K) = \frac{i}{4}(D^+)^2(\bar{D}^+)^2 K^{(-3)}_{\alpha} , \quad \lambda(K) = \Lambda^m(K)\partial^A_m - \lambda^{\widehat{\mu}+}(K) D^-_{\widehat{\mu}} . \tag{3.5}$$

Local transformations of the superfields Ψ^-_{α} contain the fermionic parameters $K^{(-3)}_{\alpha}$ as well as the additional bosonic parameters of transformations

$$\begin{aligned} \hat{\delta}\Psi^-_{\alpha} = {} & [D^{++} + G^{++}(\Psi)] K^{(-3)}_{\alpha} - \lambda(K)\Psi^-_{\alpha} + D^+_{\alpha} B^{--} + \\ & + D^{\beta+} B^{--}_{(\alpha\beta)} + \bar{D}^{\dot{\beta}+} B^{--}_{\alpha\dot{\beta}} , \end{aligned} \tag{3.6}$$

where B^{--} and $B^{--}_{\alpha\dot{\beta}}$ are real parameters, and $B^{--}_{(\alpha\beta)}$ are symmetrical in spinor indices. It is easy to show that these transformations produce the local transformations (2.43) for $G^{\widehat{m}++}(\Psi)$ dependent on the fermionic parameters only.

Commutators of additional transformations vanish, and the bracket parameters of nontrivial commutators have the following form:

$$K^{(-3)}_{\alpha}(1,2) = \lambda(K_2) K^{(-3)}_{\alpha 1} - (1 \leftrightarrow 2) , \tag{3.7}$$

$$B^{--}_{\widetilde{\mu\nu}}(1,2) = \lambda(K_2) B^{--}_{\widetilde{\mu\nu}1} - (1 \leftrightarrow 2) . \tag{3.8}$$

A derivation of these relations is given by using eq. (2.44).

By analogy with eqs.(2.25,2.26), we now consider the B-covariant nonlocal representation for the solution of the SG^2_4 constraints (2.30-2.32), which is useful in the analysis of the linearized supergravity

$$G^{m++} = g^{m++} + 2i(\sigma^m)_{\mu\dot{\nu}} [\Theta^{\mu-} \bar{h}^{\dot{\nu}(+3)} - \Theta^{\dot{\nu}-} h^{\mu(+3)}] , \tag{3.9}$$

$$G^{5++} = g^{5++} - 2i\Theta^{\mu-} h^{(+3)}_{\mu} - 2i\Theta^{\dot{\mu}-} h^{(+3)}_{\dot{\mu}} . \tag{3.10}$$

where the following operators are introduced:

$$\Theta^{\mu-} = \frac{i}{2\Box}\partial^{\mu\dot{\nu}}\bar{D}^{-}_{\dot{\nu}} \ , \quad \bar{\Theta}^{\dot{\mu}-} = -\frac{i}{2\Box}\partial^{\nu\dot{\mu}}D^{-}_{\nu} \ . \tag{3.11}$$

It is evident that one can restore locality of the representation using variables $g^{(+3)}_{\hat{\mu}} = \Box^{-1}h^{(+3)}_{\hat{\mu}}$ of dimension 3/2; however, this changes dimensions of the corresponding component fields.

It is not difficult to establish relations between the auxiliary B-superfields g and the local prepotentials h

$$g^{m++} = h^{m++} - \frac{1}{\Box}(\sigma^{m})_{\mu\dot{\nu}}(\partial^{\mu\dot{\rho}}\bar{\partial}^{-}_{\dot{\rho}}\bar{h}^{\dot{\nu}(+3)} + \partial^{\rho\dot{\nu}}\partial^{-}_{\rho}h^{\mu(+3)}) \ , \tag{3.12}$$

$$g^{5++} = h^{5++} + \frac{1}{\Box}(\partial^{\mu\dot{\rho}}\bar{\partial}^{-}_{\dot{\rho}}h^{(+3)}_{\mu} - \partial^{\rho\dot{\mu}}\partial^{-}_{\rho}\bar{h}^{(+3)}_{\dot{\mu}}) \ . \tag{3.13}$$

Due to the nonlocality of these relations, one should carefully study connections between field components of these superfield representations. In various treatments of the gauge symmetry, components of the superfields g^{M++} could differ from the standard set of components of the prepotentials h^{M++} (2.21-2.23).

Consider the additive gauge transformations of the superfields $g^{\hat{m}++}$ induced by linearized transformations of $h^{\hat{m}++}$

$$\delta_0 g^{m++} = D^{++}l^{m} = D^{++}[\lambda^{m} - \frac{1}{\Box}(\sigma^{m})_{\mu\dot{\nu}}(\partial^{\mu\dot{\rho}}\bar{\partial}^{-}_{\dot{\rho}}\bar{\lambda}^{\dot{\nu}+} + \partial^{\rho\dot{\nu}}\partial^{-}_{\rho}\lambda^{\mu+})] \ , \tag{3.14}$$

$$\delta_0 g^{5++} = D^{++}l^{5} = D^{++}[\lambda^{5} - \frac{1}{\Box}(\partial^{\mu\dot{\rho}}\bar{\partial}^{-}_{\dot{\rho}}\lambda^{+}_{\mu} - \partial^{\rho\dot{\mu}}\partial^{-}_{\rho}\bar{\lambda}^{+}_{\dot{\mu}})] \ . \tag{3.15}$$

The transformations are nonlocal in this treatment, and components of $g^{\hat{m}++}$ are nonlocal combinations of the standard components. If one would treat $g^{\hat{m}++}$ and the parameters $l^{\hat{m}}$ as independent analytic superfields, a stuff of auxiliary components could change essentially; so we shall not develop here such a treatment.

4. SOLUTION OF HARMONIC CONSTRAINTS IN SG^2_4

Solution of the basic condition of the Grassmann analyticity allows one to construct all geometric quantities of SG^2_4: supervielbein, connection and tensors of torsion and curvature.

In the harmonic formalism of SG^2_4, one introduces the 2 nd invariant harmonic operator in addition to the basic operator (2.24)

$$\Delta^{--} = D^{--} + G^{--} \ , \quad G^{--} = h^{M--}\partial^{A}_{M} = G^{M--}D^{A}_{M} \ , \tag{4.1}$$

where the corresponding coefficients are considered in different bases. The B-superfields $G^{\hat{m}--}, G^{\hat{\mu}-}$ and $G^{\hat{\mu}(-3)}$ play an important role in the geometry of supergravity; they can be written through the basic superfields $G^{\hat{m}++}$ in perturbation theory.

Gauge transformations of the B-superfields G^{M--} are determined by the invariance condition of the operator Δ^{--}

$$\delta G^{m--} = \Delta^{--}\Lambda^{m} + 2i(\sigma^{m})_{\mu\dot{\nu}}(G^{\mu-}\bar{\lambda}^{\dot{\nu}-} - \bar{G}^{\dot{\nu}-}\lambda^{\mu-}) \ , \tag{4.2}$$

$$\delta G^{5--} = \Delta^{--}\Lambda^{5} - 2iG^{\mu-}\lambda^{-}_{\mu} - 2i\bar{G}^{\dot{\mu}-}\bar{\lambda}^{-}_{\dot{\mu}} \ , \tag{4.3}$$

$$\delta G^{\hat{\mu}-} = \Delta^{--}\lambda^{\hat{\mu}+} - \lambda^{\hat{\mu}-} \ , \quad \delta G^{\hat{\mu}(-3)} = \Delta^{--}\lambda^{\hat{\mu}-} \ . \tag{4.4}$$

Commutation relation between the invariant harmonic derivatives is the fundamental constraint of the harmonic formalism of SG^2_4 [10, 12]

$$[\Delta^{++},\Delta^{--}] = [(D^{++} + G^{++}),(D^{--} + G^{--})] = D^{0} \ . \tag{4.5}$$

This harmonic equation has the manifest iterative solution [12]

$$G^{--} = \sum_{n=1}^{\infty}(-1)^{n}\int du_1 \dots du_n \frac{G^{++}(z,u_1)\dots G^{++}(z,u_n)}{(u^{+}u^{+}_1)\dots(u^{+}_n u^{+})} \ . \tag{4.6}$$

A solution to the functions h^{M--} has an analogous structure in all degrees of perturbation theory.

B-covariant equations for the coefficients in expansion of this operator have the following form:

$$\begin{aligned} &D^{++}G^{m--} - D^{--}G^{m++} + G^{++}G^{m--} - G^{--}G^{m++} + \\ &+2i(\sigma^{m})_{\mu\dot{\nu}}(G^{\mu(+3)}\bar{G}^{\dot{\nu}(-3)} + G^{\mu(-3)}\bar{G}^{\dot{\nu}(+3)}) = 0 \ , \end{aligned} \tag{4.7}$$

$$\begin{aligned} &D^{++}G^{5--} - D^{--}G^{5++} + G^{++}G^{5--} - G^{--}G^{5++} - \\ &-2iG^{\mu(+3)}G^{(-3)}_{\mu} + 2i\bar{G}^{(+3)}_{\dot{\mu}}\bar{G}^{\dot{\mu}(-3)} = 0 \ , \end{aligned} \tag{4.8}$$

$$D^{++}G^{\hat{\mu}-} - D^{--}G^{\hat{\mu}(+3)} + G^{++}G^{\hat{\mu}-} - G^{--}G^{\hat{\mu}(+3)}_{k} = 0 \ , \tag{4.9}$$

$$G^{\hat{\mu}-} = (D^{++} + G^{++})G^{\hat{\mu}(-3)} \ . \tag{4.10}$$

These equations can readily be solved in perturbation theory; for instance, in the first two orders we have

$$G^{\hat{m}--}_{(1)}(z,u) = \int \frac{du_1}{(u^{+}u^{+}_1)^2}G^{\hat{m}++}(z,u_1) \ , \tag{4.11}$$

$$G^{\hat{\mu}(-2\pm1)}_{(1)}(z,u) = \int \frac{du_1(u^{\pm}u^{-}_1)}{(u^{+}u^{+}_1)^2}G^{\hat{\mu}(+3)}(z,u_1) \ , \tag{4.12}$$

$$\begin{aligned} G^{m--}_{(2)}(z,u) = &\int \frac{du_1 du_2}{(u^{+}u^{+}_1)(u^{+}_1u^{+}_2)(u^{+}_2u^{+})}\{G^{++}(z,u_1)G^{m++}(z,u_2) + \\ &+2i(u^{-}_1u^{-}_2)(\sigma^{m})_{\mu\dot{\nu}}G^{\mu(+3)}(z,u_1)\bar{G}^{\dot{\nu}(+3)}(z,u_2)\} \ , \end{aligned} \tag{4.13}$$

$$\begin{aligned} G^{5--}_{(2)}(z,u) = &\int \frac{du_1 du_2}{(u^{+}u^{+}_1)(u^{+}_1u^{+}_2)(u^{+}_2u^{+})}\{G^{++}(z,u_1)G^{5++}(z,u_2) - \\ &-i(u^{-}_1u^{-}_2)[G^{\mu(+3)}(z,u_1)G^{(+3)}_{\mu}(z,u_2) - \bar{G}^{(+3)}_{\dot{\mu}}(z,u_1)\bar{G}^{\dot{\mu}(+3)}(z,u_2)]\} \ , \end{aligned} \tag{4.14}$$

where the harmonic distributions $(u^{k\pm}_1 u^{\pm}_{k2})$ and $(u^{k+}_1 u^{+}_{k2})^{-n}$ [11] are considered.

It is reasonable to discuss the 1 st order solution only in the nonlocal representation (3.9-3.10)

$$G^{m--}_{(1)} = g^{m--}_{(1)} + 2i(\sigma^m)_{\mu\dot\nu}[\Theta^{\mu-}\bar h^{\dot\nu-}_{(1)} - \bar\Theta^{\dot\nu-}h^{\mu-}_{(1)} - \Theta^{\mu+}\bar h^{\dot\nu(-3)}_{(1)} + \bar\Theta^{\dot\nu+}h^{\mu(-3)}_{(1)}] \,, \quad (4.15)$$

$$G^{5--} = g^{5--}_{(1)} + 2i[\Theta^-_\mu h^{\mu-}_{(1)} + \bar\Theta^-_{\dot\mu}\bar h^{\dot\mu-}_{(1)} - \Theta^+_\mu h^{\mu(-3)}_{(1)} - \bar\Theta^+_{\dot\mu}\bar h^{\dot\mu(-3)}_{(1)}] \,, \quad (4.16)$$

where $\Theta^{\mu+} = \frac{i}{2\Box}\partial^{\mu\dot\nu}D^+_{\dot\nu}$, and also there are defined quantities $g^{\widehat{m}--}_{(1)}$ and $h^{\widehat{\mu}(-2\pm1)}_{(1)}$ which can be written via the corresponding prepotentials by analogy with (4.11,4.12).

5. SUPERFIELD DECOMPOSITIONS OF SUPERVIELBEIN AND CONNECTION

Differential geometry of SG^2_4 has been considered in the holonomic basis [15]. We shall study the background supersymmetry of basic geometric objects in the B-covariant basis. By analogy with the formalism of $D = 4, N = 1$ conformal supergravity [20] one can introduce in HSS a so-called *almost covariant* basis of differential operators E_B, which helps to define B-superfield blocks necessary for construction of supervielbeins and connections of the theory. The initial step of this construction is connected with the following spinor operator:

$$E^-_{\widehat\alpha} \equiv [D^+_{\widehat\alpha}, \Delta^{--}] = -D^-_{\widehat\alpha} + [D^+_{\widehat\alpha}, G^{--}] \,. \quad (5.1)$$

Define also vector and scalar operators

$$E_a = -\frac{i}{4}(\bar\sigma_a)^{\dot\beta\alpha}\{D^+_\alpha, E^-_{\dot\beta}\} = \partial_a + \frac{i}{4}(\bar\sigma_a)^{\dot\beta\alpha}\{D^+_\alpha, [G^{--}, \bar D^+_{\dot\beta}]\} \,, \quad (5.2)$$

$$E_5 = \frac{i}{2}\{D^{\alpha+}, E^-_\alpha\} = \partial_5 - \frac{i}{2}\{D^{\alpha+}, [G^{--}, D^+_\alpha]\} \,, \quad (5.3)$$

$$\bar E_5 = \frac{i}{2}\{\bar D^{\dot\alpha+}, E^-_{\dot\alpha}\} \,. \quad (5.4)$$

By definition, components of the basis E_B satisfy the relations

$$[\Delta^{++}, E^-_{\widehat\alpha}] = D^+_{\widehat\alpha} \,, \quad [\Delta^{--}, E^-_{\widehat\alpha}] = 0 \,, \quad [\Delta^{\pm\pm}, E_a] = [\Delta^{\pm\pm}, E_5] = [\Delta^{\pm\pm}, \bar E_5] = 0 \,. \quad (5.5)$$

Decomposition of the almost covariant operator $E_B = G^M_B D^A_M$ determines matrix elements of supervielbein. The corresponding density $E = \mathrm{Ber}G^M_B$ has the correct transformation law

$$\delta E = (\partial_m\Lambda^m + D^-_{\widehat\mu}\lambda^{\widehat\mu+} - D^+_{\widehat\mu}\lambda^{\widehat\mu-})E \,. \quad (5.6)$$

Note that the density E of this theory is defined uniquely and does not depend on a choice of basis; however, we prefer to use its expression in terms of the B-superfields.

By definition, this quantity is B-covariant and does not depend on the scalar superfield G^{5++}. The linear approximation for E is

$$E_{(1)} = D^+_{\widehat\mu}G^{\widehat\mu-}_{(1)} - \frac{i}{4}D^{\alpha+}\bar D^{\dot\beta+}G^{--}_{\alpha\dot\beta(1)} \,. \quad (5.7)$$

Higher-order terms in E are calculated straightforwardly from decompositions of the supervielbein or by using the equation

$$[D^{++} + G^{++} - \partial^A_m G^{m++} - D^-_{\widehat\mu}G^{\widehat\mu(+3)}]E = 0 \,. \quad (5.8)$$

A gauge-covariant $SL(2,C)$-basis in the harmonic superspace of SG^2_4 has been constructed in ref.[15]. One defines the $SL(2,C)$-covariant spinor operator instead of the flat operator D^+_α

$$\Delta^+_\alpha \equiv u^+_i\Delta^i_\alpha = D^+_\alpha + F^{\dot\mu}_\alpha\bar D^+_{\dot\mu} \,, \quad (5.9)$$

where

$$F^{\dot\mu}_\alpha = \Delta^+_\alpha\bar\psi^{\dot\mu-} = D^+_\alpha\bar\psi^{\dot\nu-}(1 - \bar D^+\bar\psi^-)^{-1\dot\mu}_{\dot\nu} \quad (5.10)$$

is the matrix which is expressed in terms of the transition function $\bar\psi^{\dot\mu-}$ for the so-called hybrid basis.

We consider the terms in this matrix linear in κ

$$F_{\alpha\dot\mu(1)} = u^+_k D^+_\alpha\bar G^{k-}_{\dot\mu(1)} + \frac{i}{8}(\sigma_n)_{\alpha\dot\mu}(D^+)^2G^{n--}_{(1)} = D^+_\alpha\bar\psi^-_{\dot\mu(1)} \,. \quad (5.11)$$

The $SL(2,C)$-basis also contains a function F which depends on the superfield G^{5++} [15]. The linear approximation for this function is

$$F_{(1)} = -\frac{1}{2}D^+_\mu G^{\mu-}_{(1)} - \frac{i}{8}(D^+)^2G^{5--}_{(1)} \,, \quad (5.12)$$

$$\hat\delta_{(0)}F_{(1)} = \frac{1}{2}D^+_\mu\lambda^{\mu-} \,. \quad (5.13)$$

The functions $F, \bar F, F^{\dot\mu}_\alpha$ and $\bar F^{\mu}_{\dot\alpha}$ determine components of superfield connection.

6. B-SUPERFIELD ACTION OF SG^2_4

In the harmonic SG^2_4 formalism we can consider iterative construction of the B-superfield action $S^{cl} = \sum S^{cl}_{(n)}$ which begins with the quadratic action of linearized theory $S^{cl}_{(2)}$, and terms of higher order in superfields G should be restored with the help of the gauge symmetry

$$\hat\delta_{(1)}S^{cl}_{(n)} + \hat\delta_{(0)}S^{cl}_{(n+1)} = 0 \,. \quad (6.1)$$

The linearized harmonic action has been constructed in our work [17]

$$S^{cl}_{(2)} = \frac{1}{2\kappa^2}\int d^{12}z du[G^{5++}G^{5--}_{(1)} + \frac{1}{2}G^{m++}G^{--}_{m(1)}] \,, \quad (6.2)$$

where eq. (4.11) is used. This expression is a quadratic form, each term being similar to the action of the abelian gauge superfield V^{++} in the full harmonic superspace. It should be noted that this quadratic form possesses additional $SO(3,2)$ symmetry, and the components of the 5-vector $G^{\widehat m++}$ have a nonstandard normalization with respect to this symmetry.

One can easily verify, using WZ-gauge (2.21-2.23), that this action is equivalent to the component action of linearized SG^2_4. This can be checked quite simply for the terms quadratic in auxiliary components . The corresponding terms in $G^{\widehat{m}--}_{(1)WZ}$ can be calculated in the coordinates z^M_A using relations of the following type:

$$D^{--}[(\theta^+)^2\bar{\theta}^{\dot{\alpha}+}u^-_k] = D^{++}[(\theta^+\theta^-)\bar{\theta}^{\dot{\alpha}-}u^-_k + \frac{1}{2}(\theta^-)^2\bar{\theta}^{\dot{\alpha}+}u^-_k - \frac{1}{2}(\theta^-)^2\bar{\theta}^{\dot{\alpha}-}u^+_k] , \qquad (6.3)$$

$$D^{--}[(\theta^+)^2\theta^{\alpha-}\bar{\theta}^{\dot{\alpha}+}] = -D^{++}[(\theta^-)^2\theta^{\alpha+}\bar{\theta}^{\dot{\alpha}-}] . \qquad (6.4)$$

It is useful to consider the equivalent analytic representation of $S^{cl}_{(2)}$

$$\frac{1}{2\kappa^2}\int d\zeta^{(-4)}du[G^{5++}T^{5++}_{(1)} + \frac{1}{2}G^{m++}T^{++}_{m(1)} + G^{\widehat{\mu}(+3)}T^{+}_{\widehat{\mu}(1)}] , \qquad (6.5)$$

where we introduce the linearized components of torsion which can easily be written via spinor derivative of harmonic superfields $G^{\widehat{m}--}_{(1)}$ comparing various integral representations of $S_{(2)}$ and using eqs.(8.15).

$$T^+_{\mu(1)} = -\frac{i}{4}(\bar{D}^+)^2D^+_\mu G^{5--}_{(1)} - \frac{i}{8}(D^+)^2\bar{D}^{\dot{\nu}+}G^{--}_{\mu\dot{\nu}(1)} , \qquad (6.6)$$

$$D^+_\nu T^+_{\mu(1)} = -2i\varepsilon_{\nu\mu}T^{5++}_{(1)} , \quad \bar{D}^+_{\dot{\nu}}T^+_{\mu(1)} = -iT^{++}_{\mu\dot{\nu}(1)} . \qquad (6.7)$$

Note that the linearized components of torsion satisfy the following conditions:

$$D^{++}T^+_{\widehat{\mu}(1)} = 0 , \quad D^+_{\widehat{\alpha}}D^+_{\widehat{\beta}}T^+_{\widehat{\mu}(1)} = 0 . \qquad (6.8)$$

The action $S_{(2)}$ is invariant under the linearized transformations $\hat{\delta}_{(0)}G^{++}_{\widehat{m}} = D^{++}\Lambda_{\widehat{m}}$.

Using the representation (3.1,3.2) for the superfields $G^{\widehat{m}++}(\Psi)$ in the action (6.2) one can obtain the linearized equation of motion for SG^2_4 varying in the spinor prepotential $\Psi^{\mu-}$

$$T^+_{\mu(1)}(\Psi) = 0 . \qquad (6.9)$$

In the nonlocal superfield representation (3.9,3.10), one can construct the invariant of linearized transformations for the analytic B-superfield g^{5++}

$$\int\frac{d^{12}zdu_1du_2}{(u^+_1u^+_2)^2}g^{5++}(u_1)g^{5++}(u_2) \qquad (6.10)$$

and the analogous independent invariant for g^{m++}. The only local invariant of local transformations and appropriate dimension exists in the initial superfield variables (6.2).

The nonlocal invariant can also be constructed in terms of the gauge spinor superfield

$$\int\frac{d^{12}zdu_1du_2}{\Box(u^+_1u^+_2)^3}h^{\mu(+3)}[\partial_{\mu\dot{\nu}} - \frac{i}{2}(u^+_1u^+_2)D^-_{\mu1}\bar{D}^-_{\dot{\nu}2}]\bar{h}^{\dot{\nu}(+3)} . \qquad (6.11)$$

In principle, nonlocal invariants can be used in the quantum effective action of this theory.

Note that it is possible to use an alternative approach in which the background superspace is connected with a flat supersymmetry without the central charge $B^2_4 \equiv B'$

$$\Delta^{++} = D^{++}_4 + G^{m++}\partial^A_m + H^{5++}\partial^A_5 - G^{\hat{\mu}(+3)}(D^-_4)_{\hat{\mu}} , \qquad (6.12)$$

where the B'-covariant harmonic and spinor derivatives D^{++}_4 and D^-_4 (independent of derivative ∂^A_5) are introduced. Transformations of the superfields G^{m++} and $G^{\hat{\mu}(+3)}$ are identical in different background bases, but the analytic compensator transforms covariantly with respect to the B' supersymmetry only, and its transformations in $B^2_4(Z)$ contain inhomogeneous θ-dependent terms. Note that connection of different harmonic superfield representations and background bases has been discussed, for the gauge theory, in refs. [13, 22, 23].

A nonlinear action of SG^2_4 has been constructed in ref. [15] as the gauge-invariant action of analytic compensator (6.12)

$$S(H^5) = \int d^{12}zduE^{-1}H^{5++}H^{5--} , \qquad (6.13)$$

which possesses independent B'-invariance by definition. The use of the expression of $H^{5\pm\pm}$ via the B-superfields $G^{\widehat{m}++}$ in this formula leads to the appearance of terms manifestly depending on spinor coordinates, in particular, the corresponding quadratic action contains similar terms. Apparently, it is possible to reconstruct this action as an expansion in terms of superfields G and see a transmutation of the background supersymmetry, but we prefer simple iterative constructions with the manifest B-covariance.

7. ON DEVELOPMENT OF THE HARMONIC FORMALISM OF SG^2_4

Our investigation can be used for the superfield quantization of $N = 2$ supergravity in the flat background superspace by analogy with the superfield formalism of quantization in $N = 1$ supergravity [18, 19]. The use of the representation of gravitational superfields via the spinor harmonic superfield $\Psi^-_\alpha(z,u)$ seems to be most adequate for the solution of this problem. A more interesting and difficult problem is the study of nonperturbative structure of theory taking into account results that can be obtained by perturbative superfield methods and by the dual transformations, insufficiently analyzed in supergravity. We shall discuss one possibility of a dual description of $N = 2$ supergravity in terms of alternative superfield variables in the harmonic approach.

Superfield constraints of the nonlinear SG^2_4 theory are reduced to the kinematic analyticity condition in the standard formalism , and nonlinear equation of motion for the superfields G^{M++} are additional dynamical restrictions on the components of torsion.

By analogy with a dual formulation of the $N = 2$ supersymmetric gauge theory [21] we can consider the dual harmonic formalism of the linearized SG^2_4, in which the equation for the spinor ($d = -1/2$) component of torsion is solved, and then the dynamical analyticity condition arises. The basic operator of the dual formalism is $\Delta^{--} = D^{--} + G^{--}$, and the linearized equation of motion of the standard approach with the analytic G^{++} transforms into a solvable linear constraint $T^+_{\mu(1)} = 0$ for the dual superfields G^{M--}. A solution of this constraint can be written in terms of the new nonanalytic spinor prepotentials $A^{\widehat{\mu}(-3)}$ of dimension 3/2

$$G^{5--}(A) = D^+_\mu A^{\mu(-3)} + \bar{D}^+_{\dot\mu} \bar{A}^{\dot\mu(-3)} , \quad (7.1)$$

$$G^{--}_{\mu\dot\nu}(A) = \bar{D}^+_{\dot\nu} A^{(-3)}_\mu - D^+_\mu \bar{A}^{(-3)}_{\dot\nu} , \quad (7.2)$$

$$T^+_{\mu(1)}(A) \sim (\bar{D}^+)^2 D^+_\mu G^{5--}(A) + \frac{1}{2}(D^+)^2 \bar{D}^{\dot\nu +} G^{--}_{\mu\dot\nu}(A) \equiv 0 . \quad (7.3)$$

The operator G^{++} in this formalism can be calculated with the help of eq.(4.5) and does not satisfy the analyticity condition off-shell. The dynamical zero-curvature condition becomes a dual equation of motion

$$[D^+_{\hat\alpha}, \Delta^{++}] = 0 , \quad (7.4)$$

which is equivalent to the linearized equations of motion for the physical fields of SG^2_4 and to the disappearance of all auxiliary components.

Equations of the dual formalism of SG^2_4 for the prepotentials $A^{(-3)}_{\hat\mu}$ are equivalent to standard equations for the linear superfields G; however, off-shell structures are essentially different in alternative formulations. In particular, the dual formalism has an infinite number of auxiliary components in the gravitational multiplets which vanish on the mass shell only.

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8. APPENDIX

In this appendix we shall consider some useful definitions and relations connected with the flat harmonic superspace [10]. The harmonic derivatives ∂^{++}, ∂^{--} and ∂^0 satisfy the relations of the Lie algebra $SU(2)$ and are defined by their action on the harmonics

$$[\partial^{++}, \partial^{--}] = \partial^0 , \quad [\partial^0, \partial^{\pm\pm}] = \pm 2\partial^{\pm\pm} , \quad (8.1)$$

$$\partial^{++} u^+_i = 0, \quad \partial^{++} u^-_i = u^+_i, \quad \partial^0 u^\pm_i = \pm u^\pm_i , \quad (8.2)$$

$$\partial^{--} u^-_i = 0, \quad \partial^{--} u^+_i = u^-_i . \quad (8.3)$$

The holonomic basis in the set of differential operators ∂^A contains partial derivatives on the coordinates z^M_A (2.1)

$$\partial^A = (\partial^{++},\ \partial^{--},\ \partial^A_m,\ \partial^A_5,\ \partial^-_\mu,\ \bar\partial^-_{\dot\mu},\ \partial^+_\mu,\ \bar\partial^+_{\dot\mu}) . \quad (8.4)$$

The holonomic basis in the flat real coordinates z^M consists of the operators $\partial/\partial z^M$.

The $B^2_4(Z)$-covariant harmonic derivatives have the following form:

$$D^{++} = \partial^{++} - 2i\theta^{\mu+}\bar\theta^{\dot\nu+}\partial^A_{\mu\dot\nu} + i[(\theta^+)^2 - (\bar\theta^+)^2]\partial^A_5 + \theta^{\mu+}\partial^+_\mu + \bar\theta^{\dot\mu+}\bar\partial^+_{\dot\mu} , \quad (8.5)$$

$$D^{--} = \partial^{--} - 2i\theta^{\mu-}\bar\theta^{\dot\nu-}\partial^A_{\mu\dot\nu} + i[(\theta^-)^2 - (\bar\theta^-)^2]\partial^A_5 + \theta^{\mu-}\partial^-_\mu + \bar\theta^{\dot\mu-}\bar\partial^-_{\dot\mu} , \quad (8.6)$$

where $(\theta^\pm)^2 = \theta^{\alpha\pm}\theta^\pm_\alpha$ and $(\bar\theta^\pm)^2 = \bar\theta^\pm_{\dot\beta}\bar\theta^{\dot\beta\pm}$.

Write also known expressions for B-covariant spinor derivatives

$$D^-_\mu = -\partial/\partial\theta^{\mu+} + 2i\bar\theta^{\dot\nu-}\partial^A_{\mu\dot\nu} - 2i\theta^-_\mu\partial^A_5 , \quad (8.7)$$

$$\bar{D}^-_{\dot\nu} = -\partial/\partial\bar\theta^{\dot\nu+} - 2i\theta^{\mu-}\partial^A_{\mu\dot\nu} - 2i\bar\theta^-_{\dot\nu}\partial^A_5 , \quad (8.8)$$

$$D^+_\mu = \partial/\partial\theta^{\mu+} \equiv \partial^+_\mu , \quad \bar{D}^+_{\dot\mu} = \partial/\partial\bar\theta^{\dot\mu+} \equiv \bar\partial^+_{\dot\mu} . \quad (8.9)$$

These operators form the B-covariant basis

$$D^A = (D^{++},\ D^{--},\ \partial^A_m,\ \partial^A_5,\ -D^-_{\hat\mu},\ D^+_{\hat\mu}) . \quad (8.10)$$

The alternative B-covariant basis is independent of harmonics

$$D_M = (\partial_m,\ \partial_5,\ D^k_{\hat\mu}) , \quad [D_M, D^{\pm\pm}] = 0 \quad (8.11)$$

and contains covariant spinor derivatives in the coordinates z^M

$$D^k_\mu = \partial^k_\mu + i\bar\theta^{k\dot\nu}\partial_{\mu\dot\nu} - i\theta^k_\mu\partial_5 , \quad \bar{D}^k_{\dot\mu} = \bar\partial^k_{\dot\mu} - i\theta^{\nu k}\partial_{\nu\dot\mu} - i\bar\theta^k_{\dot\mu}\partial_5 . \quad (8.12)$$

We shall use the following condensed notation for scalar elements in the algebra of spinor derivatives:

$$(D^\pm)^2 = D^{\alpha\pm}D^\pm_\alpha , \quad (\bar{D}^\pm)^2 = \bar{D}^\pm_{\dot\beta}\bar{D}^{\dot\beta\pm} , \quad (8.13)$$

$$(D^\pm)^4 = \frac{1}{16}(D^\pm)^2(\bar{D}^\pm)^2 , \quad (D)^4 = \frac{1}{16}(D^+)^2(D^-)^2 . \quad (8.14)$$

These elements are included in the definition of integration measure of the full harmonic superspace and the analytic measure

$$d^{12}z_A = d^4x_A(D^-)^4(D^+)^4 , \quad d^{12}z = d^4x(D)^4(\bar{D})^4 , \quad d\zeta^{(-4)} = d^4x(D^-)^4 . \quad (8.15)$$

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