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ИНСТИТУТА
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ИССЛЕДОВАНИЙ

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TABLE OF INTEGRALS. ASYMPTOTICAL EXPRESSIONS FOR NON-COLLINEAR KINEMATICS

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1 Introduction

We give below a list of 4-dimensional integrals in momentum space which were used in calculation of 1-loop radiative corrections to the inelastic processes of kind $e^+e^- \to e^+e^-\gamma$, $\mu^-e^- \to \mu^-e^-\gamma$ at high energies. For definiteness we consider the case when all the scalar products of external 4-momenta are large compared to the mass of external real particles squared $p_ip_j \gg p_i^2 = m_i^2$. We omit systematically terms of order m_i^2/p_ip_j compared to ones of order unity and restrict ourselves to consideration of scattering type Feynman diagrams only. Remaining ones can be obtained by application of crossing and analytical continuation transformations. We introduce the ultraviolet, infrared cut-offs and do not use the dimensional regularization. In paper of one of authors [3] 1-loop integrals, related to the $2 \to 2$ type Feynman diagrams, were considered. Here we give a set of integrals for description of inelastic $2 \to 3$ type processes. Our paper is organized as follows. In the first part we consider the 4-momentum integrals, appearing in one-loop vertex and five-point diagrams. The cases when fermions have different and equal masses are considered. They may be helpful in constructing the five-point diagrams with one or two external off-shell mass particles . Scalar, vector and tensor integrals are considered up to the case of four denominators. Tensor integrals are calculated up to a third rank.

2 Five-point Feynman diagrams

The case of equal fermion masses.

Typical process: $e^+e^- \to e^+e^-\gamma$

2.1 Notations

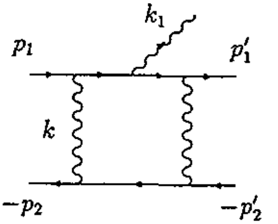

Figure 1: **Five-point Feynman diagram**

$$I_{ijklm}^{1,\mu,\mu\nu} \equiv \int \frac{d^4k}{i\pi^2} \frac{1, k^\mu, k^\mu k^\nu}{(i)(j)(k)(l)(m)} \tag{1}$$

$$(1) = (p_1 - k)^2 - m^2,\ (2) = (p_1 - k_1 - k)^2 - m^2,\ (3) = (p_2 + k)^2 - m^2, \quad (2)$$
$$(4) = (q - k)^2 - \lambda^2, \quad (5) = k^2 - \lambda^2.$$

Invariants

$$\begin{aligned}
&\chi_1 = 2p_1k_1,\ \chi_1' = 2p_1'k_1,\ \chi_2 = 2p_2k_1 = s - s_1 - \chi_1,\\
&\chi_2' = 2p_2'k_1 = s - s_1 - \chi_1',\ s = (p_1 + p_2)^2,\ s_1 = (p_1' + p_2')^2,\\
&t = q^2,\ q = p_2' - p_2,\ t_1 = q'^2 = t + \chi_1 - \chi_1',\ q' = p_1' - p_1,\\
&\chi_1 + \chi_2 = s - s_1,\ p_1^2 = p_2^2 = p_1'^2 = p_2'^2 = m^2,\ k_1^2 = 0. \qquad (3)
\end{aligned}$$

$$L_t = \ln\left(\tfrac{-t}{m^2}\right),\ L_s = \ln\left(\tfrac{s}{m^2}\right),\ L_\Lambda = \ln\left(\tfrac{\Lambda^2}{m^2}\right),\ L_\lambda = \ln\left(\tfrac{\lambda^2}{m^2}\right), \quad (4)$$
$$\mathrm{Li}_2(z) = -\int_0^z \tfrac{dx}{x}\ln(1-x),$$

$$\mathcal{P}^2 = m^2 - x\bar{x}s - i0,\quad \mathcal{P}_1^2 = m^2 - x\bar{x}s_1 - i0,\quad \text{where}\quad \bar{x} = 1 - x \quad (5)$$

Throughout the paper Λ is an UV cut-off parameter and λ is a fictitious *photon mass*.

2.2 Two-propagator integrals

2.2.1 Scalar integrals

$$\begin{aligned}
I_{12} &= -1 + L_\Lambda, \quad I_{13} = -1 - \int_0^1 dx \ln\left(\frac{\mathcal{P}^2}{\Lambda^2}\right) = 1 + L_\Lambda - L_s + i\pi,\\
I_{14} &= -\int_0^1 dx \ln\left(\frac{xm^2 - \bar{x}\chi_1'}{\Lambda^2}\right) = 1 + L_\Lambda - L_{\chi_1'} + i\pi,\\
I_{15} &= I_{24} = I_{34} = I_{35} = 1 + L_\Lambda,\ I_{23} = -1 - \int_0^1 dx \ln\left(\frac{\mathcal{P}_1^2}{\Lambda^2}\right) = 1 + L_\Lambda - L_{s_1} + i\pi,\\
I_{25} &= -\int_0^1 dx \ln\left(\frac{xm^2 + \bar{x}\chi_1}{\Lambda^2}\right) = 1 + L_\Lambda - L_{\chi_1}, \quad I_{45} = 1 + L_\Lambda - L_t. \qquad (6)
\end{aligned}$$

2.2.2 Vector integrals

$$\begin{aligned}
I_{12}^\mu &= \left(p_1 - \frac{k_1}{2}\right)^\mu \left(L_\Lambda - \frac{3}{2}\right),\ I_{13}^\mu = (p_1 - p_2)^\mu \left(\frac{1}{4} + \frac{1}{2}L_\Lambda - \frac{1}{2}L_s + \frac{i\pi}{2}\right),\\
I_{14}^\mu &= (p_1 + q)^\mu \left(\frac{1}{4} + \frac{1}{2}L_\Lambda - \frac{1}{2}L_{\chi_1'} + \frac{i\pi}{2}\right),\ I_{15}^\mu = \frac{p_1^\mu}{2}\left(L_\Lambda - \frac{1}{2}\right),
\end{aligned}$$

$$\begin{aligned}
I_{24}^\mu &= (p_1 - k_1)^\mu \left(\frac{1}{2} + L_\Lambda\right) - \frac{p_1'^\mu}{2}\left(\frac{3}{2} + L_\Lambda\right),\\
I_{34}^\mu &= \frac{p_2'^\mu}{2}\left(\frac{3}{2} + L_\Lambda\right) - p_2^\mu\left(\frac{1}{2} + L_\Lambda\right),\\
I_{35}^\mu &= \frac{p_2^\mu}{2}\left(\frac{1}{2} - L_\Lambda\right),\ I_{23}^\mu = (p_1 - k_1 - p_2)^\mu\left(\frac{1}{4} + \frac{1}{2}L_\Lambda - \frac{1}{2}L_{s_1} + \frac{i\pi}{2}\right),\\
I_{25}^\mu &= (p_1 - k_1)^\mu\left(\frac{1}{4} + \frac{1}{2}L_\Lambda - \frac{1}{2}L_{\chi_1}\right),\ I_{45}^\mu = q^\mu\left(\frac{1}{4} + \frac{1}{2}L_\Lambda - \frac{1}{2}L_t\right). \qquad (7)
\end{aligned}$$

2.3 Three-propagator integrals

2.3.1 Scalar integrals

$$\begin{aligned}
I_{123} &= \frac{1}{s - s_1}\int_0^1 \frac{dx}{x}\ln\left(\frac{\mathcal{P}^2}{\mathcal{P}_1^2}\right) = \frac{1}{s - s_1}\left[\frac{1}{2}L_s^2 - \frac{1}{2}L_{s_1}^2 + i\pi\left(L_{s_1} - L_s\right)\right].\\
I_{345} &= -\int_0^1 \frac{dx}{\bar{x}^2m^2 + xt}\ln\left(\frac{m^2\bar{x}^2}{-tx}\right) = \frac{1}{t}\left[\frac{1}{2}L_t^2 + 4\zeta(2)\right].\\
I_{124} &= \int_0^1 \frac{dxdy}{x\bar{y}\chi_1' - ym^2} = \frac{1}{\chi_1'}\left[\frac{1}{2}L_{\chi_1'}^2 - \zeta(2) - i\pi L_{\chi_1'}\right].\\
I_{125} &= -\int_0^1 \frac{dxdy}{x\bar{y}\chi_1 + ym^2} = \frac{1}{\chi_1}\left[-\frac{1}{2}L_{\chi_1}^2 - 2\zeta(2)\right].\\
I_{134} &= -\int_0^1 \frac{dxdy}{y\mathcal{P}^2 - x\bar{y}\chi_1'} = \frac{1}{s - \chi_1'}\left[\frac{3}{2}L_s^2 + \frac{1}{2}L_{\chi_1'}^2 - 2L_sL_{\chi_1'} + 2\mathrm{Li}_2\left(1 - \frac{\chi_1'}{s}\right)\right.\\
&+ \left. i\pi\left(L_{\chi_1'} - L_s\right)\right],\\
I_{235} &= -\int_0^1 \frac{dxdy}{y\mathcal{P}_1^2 + x\bar{y}\chi_1} = \frac{1}{s_1 + \chi_1}\left[\frac{3}{2}L_{s_1}^2 + \frac{1}{2}L_{\chi_1}^2 - 2L_{s_1}L_{\chi_1} - 9\zeta(2)\right.\\
&+ \left. 2\mathrm{Li}_2\left(1 + \frac{\chi_1}{s_1}\right) + i\pi\left(2L_{\chi_1} - 3L_{s_1}\right)\right],\\
I_{135} &= -\frac{1}{2}\int_0^1 \frac{dx}{\mathcal{P}^2}\ln\left(\frac{\mathcal{P}^2}{\lambda^2}\right) = \frac{1}{s}\left[\frac{1}{2}L_s^2 - L_sL_\lambda - 4\zeta(2) + i\pi\left(L_\lambda - L_s\right)\right].\\
I_{234} &= -\frac{1}{2}\int_0^1 \frac{dx}{\mathcal{P}_1^2}\ln\left(\frac{\mathcal{P}_1^2}{\lambda^2}\right) = \frac{1}{s_1}\left[\frac{1}{2}L_{s_1}^2 - L_{s_1}L_\lambda - 4\zeta(2) + i\pi\left(L_\lambda - L_{s_1}\right)\right].\\
I_{245} &= \int_0^1 \frac{dx}{x\chi_1 - xt - x^2m^2}\ln\left(\frac{x^2m^2}{x\chi_1 - xt}\right) = \frac{1}{\chi_1 + t}\left[\frac{1}{2}L_t^2 - \frac{1}{2}L_{\chi_1}^2\right.\\
&+ \left. 2\mathrm{Li}_2\left(1 - \frac{\chi_1}{-t}\right)\right],\\
I_{145} &= -\int_0^1 \frac{dxdy}{yx^2m^2 - \bar{y}xt - yx\chi_1'} = \frac{1}{\chi_1' - t}\left[\frac{1}{2}L_{\chi_1'}^2 - \frac{1}{2}L_t^2 - 3\zeta(2)\right.
\end{aligned}$$

$$- \; 2\mathrm{Li}_2\left(1+\frac{\chi_1'}{-t}\right) - i\pi L_{\chi_1'}\Big]. \tag{8}$$

2.3.2 Vector integrals

The parametrization reads

$$I^{\mu}_{ijk} = a_{ijk}p_1^{\mu} + b_{ijk}p_2^{\mu} + c_{ijk}k_1^{\mu} + d_{ijk}p_1'^{\mu}. \tag{9}$$

$$\begin{aligned}
a_{245} &= -c_{245} = \frac{1}{t+\chi_1}\left[\frac{1}{2}L_t^2 - \frac{1}{2}L_{\chi_1}^2 + L_{\chi_1} - L_t + 2\mathrm{Li}_2\left(1-\frac{\chi_1}{-t}\right)\right],\ b_{245}=0,\\
d_{245} &= \frac{1}{t+\chi_1}\left[-L_t - \frac{\chi_1}{t+\chi_1}\left[\frac{1}{2}L_t^2 - \frac{1}{2}L_{\chi_1}^2 + 2L_{\chi_1} - 2L_t + 2\mathrm{Li}_2\left(1-\frac{\chi_1}{-t}\right)\right]\right].
\end{aligned} \tag{10}$$

$$\begin{aligned}
a_{145} &= \frac{1}{\chi_1'-t}\Big[2L_{\chi_1'} - L_t - 2i\pi + \frac{t}{\chi_1'-t}\Big[\frac{1}{2}L_t^2 - \frac{1}{2}L_{\chi_1'}^2\\
&+ 2L_{\chi_1'} - 2L_t + 3\zeta(2) + 2\mathrm{Li}_2\left(1+\frac{\chi_1'}{-t}\right) + i\pi\left(L_{\chi_1'}-2\right)\Big]\Big],\\
b_{145} &= 0,\ c_{145} = d_{145} = \frac{1}{\chi_1'-t}\left[L_t - L_{\chi_1'} + i\pi\right].
\end{aligned} \tag{11}$$

$$a_{345} = -c_{345} = -d_{345} = \frac{1}{t}L_t,\ b_{345} = \frac{1}{t}\left[-\frac{1}{2}L_t^2 + 2L_t - 4\zeta(2)\right]. \tag{12}$$

$$a_{125} = \frac{1}{\chi_1}\left[-\frac{1}{2}L_{\chi_1}^2 + L_{\chi_1} - 2\zeta(2)\right],\ b_{125} = d_{125} = 0,\ c_{125} = \frac{1}{\chi_1}\left[L_{\chi_1} - 2\right]. \tag{13}$$

$$\begin{aligned}
a_{235} &= -c_{235} = \frac{1}{s_1+\chi_1}\left[L_{s_1} - L_{\chi_1} - i\pi\right],\ d_{235} = 0,\\
b_{235} &= \frac{1}{s_1+\chi_1}\Big[-L_{s_1} + i\pi\\
&+ \frac{\chi_1}{s_1+\chi_1}\Big[2L_{s_1} - 2L_{\chi_1} - \frac{3}{2}L_{s_1}^2 - \frac{1}{2}L_{\chi_1}^2 + 2L_{s_1}L_{\chi_1} + 9\zeta(2) - 2\mathrm{Li}_2\left(1+\frac{\chi_1}{s_1}\right)\\
&+ i\pi\left(-2 - 2L_{\chi_1} + 3L_{s_1}\right)\Big]\Big].
\end{aligned} \tag{14}$$

$$a_{135} = -b_{135} = \frac{1}{s}\left[L_s - i\pi\right],\ c_{135} = d_{135} = 0. \tag{15}$$

$$\begin{aligned}
a_{234} &= -c_{234} = -b_{234} - d_{234} = \frac{1}{s_1}\Big[\frac{1}{2}L_{s_1}^2 - L_{s_1}L_\lambda - L_{s_1} - 4\zeta(2)\\
&+ i\pi\left(1 + L_\lambda - L_{s_1}\right)\Big],\ b_{234} = a_{234} - I_{234} = \frac{1}{s_1}\left[-L_{s_1} + i\pi\right],\\
d_{234} &= \frac{1}{s_1}\left[-\frac{1}{2}L_{s_1}^2 + L_{s_1}L_\lambda + 2L_{s_1} + 4\zeta(2) + i\pi\left(-2 + L_{s_1} - L_\lambda\right)\right].
\end{aligned} \tag{16}$$

$$\begin{aligned}
a_{134} &= \frac{1}{s-\chi_1'}\left[L_s - 2L_{\chi_1'} + i\pi - \frac{2s}{s-\chi_1'}\left[L_s - L_{\chi_1'}\right] + sI_{134}\right],\\
b_{134} &= a_{134} - I_{134} = \frac{1}{s-\chi_1'}\left[-L_s + i\pi - \frac{2\chi_1'}{s-\chi_1'}\left[L_s - L_{\chi_1'}\right] + \chi_1' I_{134}\right],\\
c_{134} &= d_{134} = \frac{1}{s-\chi_1'}\left[L_{\chi_1'} - i\pi + \frac{2s}{s-\chi_1'}\left[L_s - L_{\chi_1'}\right] - sI_{134}\right].
\end{aligned} \tag{17}$$

$$\begin{aligned}
a_{124} &= I_{124} = \frac{1}{\chi_1'}\left[\frac{1}{2}L_{\chi_1'}^2 - \zeta(2) - i\pi L_{\chi_1'}\right],\ d_{124} = \frac{1}{\chi_1'}\left[-L_{\chi_1'} + i\pi\right],\\
c_{124} &= \frac{1}{\chi_1'}\left[-\frac{1}{2}L_{\chi_1'}^2 + L_{\chi_1'} + \zeta(2) - 2 + i\pi\left(L_{\chi_1'} - 1\right)\right],\ b_{124} = 0.
\end{aligned} \tag{18}$$

$$\begin{aligned}
a_{123} &= b_{123} - I_{123} = \frac{1}{s-s_1}\left[\frac{1}{2}L_s^2 - \frac{1}{2}L_{s_1}^2 - L_s + L_{s_1} + i\pi\left(L_{s_1} - L_s\right)\right],\\
b_{123} &= -\frac{1}{s-s_1}\left[L_s - L_{s_1}\right],\ d_{123} = 0,\ c_{123} = \frac{1}{s-s_1}\Big[L_{s_1} - 2 - i\pi\\
&+ \frac{s}{s-s_1}\left[-\frac{1}{2}L_s^2 + \frac{1}{2}L_{s_1}^2 + 2L_s - 2L_{s_1} + i\pi\left(L_s - L_{s_1}\right)\right]\Big].
\end{aligned} \tag{19}$$

2.4 Four-propagator integrals

2.4.1 Scalar integrals

$$\begin{aligned}
I_{1245} &= \int_0^1 \frac{\mathrm{d}x\mathrm{d}y}{\left[xy\chi_1 - \bar{y}t\right]\left[ym^2 - \bar{x}\bar{y}\chi_1'\right]} = \frac{1}{\chi_1\chi_1'}\Big[-L_{\chi_1}^2 - L_{\chi_1'}^2 - L_t^2 - 2L_{\chi_1}L_{\chi_1'}\\
&+ 2L_{\chi_1}L_t + 2L_{\chi_1'}L_t + 4\zeta(2) + i\pi\left(2L_{\chi_1} + 2L_{\chi_1'} - 2L_t\right)\Big],\\
I_{2345} &= \frac{1}{t}\int_0^1 \frac{\mathrm{d}x}{\mathcal{P}_1^2}\left[-\frac{1}{2}\ln\left(\frac{\mathcal{P}_1^2}{\lambda^2}\right) + \ln\left(\frac{x\chi_1}{-t}\right)\right] = \frac{1}{s_1 t}\Big[L_{s_1}^2 - L_{s_1}L_\lambda - 2L_{s_1}L_{\chi_1}\\
&+ 2L_{s_1}L_t - 5\zeta(2) + i\pi\left(2L_{\chi_1} - 2L_t - 2L_{s_1} + L_\lambda\right)\Big],\\
I_{1345} &= \frac{1}{t}\int_0^1 \frac{\mathrm{d}x}{\mathcal{P}^2}\left[-\frac{1}{2}\ln\left(\frac{\mathcal{P}^2}{\lambda^2}\right) + \ln\left(\frac{-x\chi_1'}{-t}\right)\right]\\
&= \frac{1}{st}\left[L_s^2 - L_sL_\lambda - 2L_sL_{\chi_1'} + 2L_sL_t + 7\zeta(2) + i\pi\left(2L_{\chi_1'} - 2L_t + L_\lambda\right)\right],\\
I_{1235} &= \frac{1}{\chi_1}\int_0^1 \frac{\mathrm{d}x}{\mathcal{P}^2}\left[\frac{1}{2}\ln\left(\frac{\mathcal{P}^2}{\lambda^2}\right) - \ln\left(\frac{\mathcal{P}_1^2}{x\chi_1}\right)\right] = \frac{1}{s\chi_1}\Big[L_{s_1}^2 - 2L_sL_{\chi_1} + L_sL_\lambda
\end{aligned}$$

$$\begin{aligned}
&- 5\zeta(2) + 2\mathrm{Li}_2\left(1-\frac{s_1}{s}\right) + i\pi\left(-2L_{s_1} + 2L_{\chi_1} - L_\lambda\right)\Big], \\
I_{1234} &= \frac{1}{\chi'_1}\int_0^1 \frac{\mathrm{d}x}{\mathcal{P}_1^2}\left[-\frac{1}{2}\ln\left(\frac{\mathcal{P}_1^2}{\lambda^2}\right) + \ln\left(\frac{\mathcal{P}^2}{-x\chi'_1}\right)\right] = \frac{1}{s_1\chi'_1}\Big[-L_s^2 + 2L_{s_1}L_{\chi'_1} \\
&- L_{s_1}L_\lambda - 7\zeta(2) - 2\mathrm{Li}_2\left(1-\frac{s}{s_1}\right) + i\pi\left(2L_s - 2L_{s_1} - 2L_{\chi'_1} + L_\lambda\right)\Big]. \qquad (20)
\end{aligned}$$

Useful integrals

$$\begin{aligned}
\int_0^1 \frac{\mathrm{d}x}{\mathcal{P}^2} &= \frac{2}{s}\left[-L_s + i\pi\right], \quad \int_0^1 \frac{\mathrm{d}x}{\mathcal{P}^2}\ln x = \frac{1}{s}\left[\frac{1}{2}L_s^2 - \zeta(2) - i\pi L_s\right], \\
\int_0^1 \frac{\mathrm{d}x}{\mathcal{P}^2}\ln\left(\frac{\mathcal{P}^2}{m^2}\right) &= \frac{1}{s}\left[-L_s^2 + 8\zeta(2) + 2i\pi L_s\right], \\
\int_0^1 \frac{\mathrm{d}x}{\mathcal{P}^2}\ln\left(\frac{\mathcal{P}_1^2}{m^2}\right) &= \frac{1}{s}\left[-L_{s_1}^2 + 8\zeta(2) - 2\mathrm{Li}_2\left(1-\frac{s_1}{s}\right) + 2i\pi L_{s_1}\right]. \qquad (21)
\end{aligned}$$

2.4.2 Vector integrals

Parametrization

$$I^\mu_{ijkl} = a_{ijkl}p_1^\mu + b_{ijkl}p_2^\mu + c_{ijkl}k_1^\mu + d_{ijkl}p_1'^\mu \qquad (22)$$

$$a_{1245} = \frac{\Delta^{(1)}}{\Delta}, \quad b_{1245} = 0, \quad c_{1245} = \frac{\Delta^{(2)}}{\Delta}, \quad d_{1245} = \frac{\Delta^{(3)}}{\Delta}, \qquad (23)$$

$$\begin{aligned}
\Delta &= -2t_1\chi_1\chi'_1, \quad \Delta^{(1)} = \chi'_1\Big[-\chi'_1 I_{124} + \chi_1 I_{125} + (\chi_1 + t)I_{245} \\
&+ (\chi'_1 - t - 2\chi_1)I_{145} + \chi_1(\chi'_1 - 2(t+\chi_1))I_{1245}\Big], \\
\Delta^{(2)} &= t_1\left[-\chi'_1 I_{124} - \chi_1 I_{125} + (\chi_1 + t)I_{245} + (\chi'_1 - t)I_{145} + \chi_1\chi'_1 I_{1245}\right], \\
\Delta^{(3)} &= \chi_1\left[\chi'_1 I_{124} - \chi_1 I_{125} + (\chi_1 + t - 2\chi'_1)I_{245} + (\chi'_1 - t)I_{145} + \chi_1\chi'_1 I_{1245}\right] \qquad (24)
\end{aligned}$$

$$a_{1235} = \frac{\Delta^{(1)}}{\Delta}, \quad b_{1235} = \frac{\Delta^{(2)}}{\Delta}, \quad c_{1235} = \frac{\Delta^{(3)}}{\Delta}, \quad d_{1235} = 0, \qquad (25)$$

$$\begin{aligned}
\Delta &= -2s\chi_1\chi_2, \\
\Delta^{(1)} &= \chi_2\left[(\chi_1 + \chi_2)I_{123} - \chi_1 I_{125} - sI_{135} + (s - \chi_2)I_{235} - s\chi_1 I_{1235}\right], \\
\Delta^{(2)} &= \chi_1\left[-(\chi_1 + \chi_2)I_{123} + \chi_1 I_{125} - sI_{135} + (s + \chi_2)I_{235} - s\chi_1 I_{1235}\right], \\
\Delta^{(3)} &= s\left[(\chi_1 - \chi_2)I_{123} - \chi_1 I_{125} + sI_{135} - (s - \chi_2)I_{235} + s\chi_1 I_{1235}\right]. \qquad (26)
\end{aligned}$$

$$a_{1345} = \frac{\Delta^{(1)}}{\Delta}, \quad b_{1345} = \frac{\Delta^{(2)}}{\Delta}, \quad c_{1345} = d_{1345} = \frac{\Delta^{(3)}}{\Delta}, \qquad (27)$$

$$\begin{aligned}
u &= -s - t + \chi'_1, \ \Delta = 2stu, \ \Delta^{(1)} = -(-su + t\chi'_1)I_{134} + s(s+t)I_{135} \\
&+ t(s+t)I_{345} - (-tu + s\chi'_1)I_{145} - st(s+t)I_{1345}, \ \Delta^{(2)} = -(-\chi'_1 u + st)I_{134} \\
&- s(t - \chi'_1)I_{135} + t(2s + t - \chi'_1)I_{345} - (t - \chi'_1)^2 I_{145} + st(t - \chi'_1)I_{1345}, \\
\Delta^{(3)} &= s\left[(s + 2t - \chi'_1)I_{134} - sI_{135} - tI_{345} - (t - \chi'_1)I_{145} + stI_{1345}\right]. \qquad (28)
\end{aligned}$$

$$a_{2345} = -c_{2345} = \frac{\Delta^{(1)}}{\Delta}, \quad b_{2345} = \frac{\Delta^{(2)}}{\Delta}, \quad d_{2345} = \frac{\Delta^{(3)}}{\Delta}. \qquad (29)$$

$$\begin{aligned}
u_1 &= -s_1 - t - \chi_1, \ \Delta = 2s_1 t u_1, \\
\Delta^{(1)} &= -u_1\left[-(t + \chi_1)I_{245} - s_1 I_{234} + (s_1 + \chi_1)I_{235} + tI_{345} - s_1 t I_{2345}\right], \\
\Delta^{(2)} &= -(t + \chi_1)^2 I_{245} - s_1(t + \chi_1)I_{234} + (-\chi_1 u_1 - s_1 t)I_{235} \\
&+ t(2s_1 + t + \chi_1)I_{345} + s_1 t(t + \chi_1)I_{2345}, \\
\Delta^{(3)} &= (-\chi_1 u_1 - s_1 t)I_{245} + s_1(s_1 + 2t + \chi_1)I_{234} - (s_1 + \chi_1)^2 I_{235} \\
&- t(s_1 + \chi_1)I_{345} + s_1 t(s_1 + \chi_1)I_{2345}. \qquad (30)
\end{aligned}$$

$$\begin{aligned}
a_{1234} &= I_{1234} + \frac{\Delta^{(2)}}{\Delta}, \ b_{1234} = \frac{\Delta^{(2)}}{\Delta}, \ c_{1234} = -I_{1234} - \frac{\Delta^{(2)}}{\Delta} + \frac{\Delta^{(3)}}{\Delta}, \\
d_{1234} &= -I_{1234} + \frac{\Delta^{(1)}}{\Delta} - \frac{\Delta^{(2)}}{\Delta}, \qquad (31)
\end{aligned}$$

$$\begin{aligned}
\Delta &= 2s_1\chi'_1\chi'_2, \quad \chi'_2 = s - s_1 - \chi'_1, \\
\Delta^{(1)} &= \chi'_2\left[-(s - s_1)I_{123} + (s - \chi'_1)I_{134} + \chi'_1 I_{124} - s_1 I_{234} + s_1\chi'_1 I_{1234}\right], \\
\Delta^{(2)} &= \chi'_1\left[(s - s_1)I_{123} + (2s_1 - s + \chi'_1)I_{134} - \chi'_1 I_{124} - s_1 I_{234} + s_1\chi'_1 I_{1234}\right], \\
\Delta^{(3)} &= s_1\left[(\chi'_2 - \chi'_1)I_{123} - (s - \chi'_1)I_{134} + \chi'_1 I_{124} + s_1 I_{234} - s_1\chi'_1 I_{1234}\right]. \qquad (32)
\end{aligned}$$

2.4.3 Tensor

Parametrization

$$\begin{aligned}
I^{\mu\nu}_{ijkl} &= g^T_{ijkl}g^{\mu\nu} + a^T_{ijkl}p_1^\mu p_1^\nu + b^T_{ijkl}p_2^\mu p_2^\nu + c^T_{ijkl}k_1^\mu k_1^\nu + d^T_{ijkl}p_1'^\mu p_1'^\nu \\
&+ \alpha^T_{ijkl}\{p_1^\mu p_2^\nu\} + \beta^T_{ijkl}\{p_1^\mu k_1^\nu\} + \gamma^T_{ijkl}\{p_1^\mu p_1'^\nu\} \\
&+ \rho^T_{ijkl}\{p_2^\mu p_1'^\nu\} + \sigma^T_{ijkl}\{p_2^\mu k_1^\nu\} + \tau^T_{ijkl}\{p_1'^\mu k_1^\nu\}, \qquad (33)
\end{aligned}$$

where $\{\cdots\}$ means symmetrization with respect to Lorentz indices: $\{v_\mu u_\nu\} = v_\mu u_\nu + v_\nu u_\mu$.

$$
\begin{aligned}
g^T_{1245} &= \frac{1}{2}\left[2I_{124} - a_{124} - \chi_1 c_{1245} + (t+\chi_1) d_{1245}\right], \\
a^T_{1245} &= \frac{1}{t_1\chi_1}\Big[\chi_1'(-I_{124} + a_{124} - c_{145}) + t_1 a_{145} - (t+\chi_1) a_{245} \\
&+ t_1\chi_1 a_{1245} - \chi_1'(t+\chi_1) d_{1245}\Big], \\
c^T_{1245} &= \frac{1}{\chi_1\chi_1'}\left[t_1(-I_{124} + a_{124}) + \chi_1 c_{125} + (t_1 - \chi_1) c_{145} - \chi_1\chi_1' c_{1245}\right], \\
d^T_{1245} &= \frac{1}{t_1\chi_1'}\left[\chi_1(-I_{124} + a_{124} - a_{245}) + (t_1 - \chi_1) c_{145} - t_1 d_{245} - \chi_1\chi_1' d_{1245}\right], \\
\beta^T_{1245} &= \frac{1}{\chi_1}\left[-I_{124} + a_{124} + c_{145} + \chi_1 c_{1245}\right],\ b^T_{1245} = \alpha^T_{1245} = \rho^T_{1245} = \sigma^T_{1245} = 0, \\
\gamma^T_{1245} &= \frac{1}{t_1}\left[I_{124} - a_{124} + a_{245} + c_{145} + (t+\chi_1) d_{1245}\right], \\
\tau^T_{1245} &= \frac{1}{\chi_1'}\left[-I_{124} + a_{245} + \chi_1 c_{1245} - (t+\chi_1) d_{1245}\right]. \qquad (34)
\end{aligned}
$$

$$
\begin{aligned}
g^T_{1235} &= \frac{1}{2}\left[2I_{123} - a_{123} + b_{123} - \chi_1 c_{1235}\right], \\
a^T_{1235} &= \frac{1}{s\chi_1}\left[\chi_2 I_{123} - (\chi_1 + \chi_2) a_{123} + \chi_1 a_{125} - \chi_1\chi_2 c_{1235}\right], \\
b^T_{1235} &= \frac{1}{s\chi_2}\left[\chi_1(I_{123} - a_{235}) + (\chi_1 + \chi_2) b_{123} - \chi_2 b_{235} - \chi_1^2 c_{1235}\right], \\
c^T_{1235} &= \frac{1}{\chi_1\chi_2}\left[s(I_{123} + b_{123}) - (s - \chi_2) a_{235} + \chi_2 c_{123} - s\chi_1 c_{1235}\right],\ d^T_{1235} = 0, \\
\alpha^T_{1235} &= \frac{1}{s}\left[-I_{123} + a_{123} - a_{235} - b_{123}\right],\ \beta^T_{1235} = \frac{1}{\chi_1}\left[-I_{123} + a_{123} + \chi_1 c_{1235}\right], \\
\sigma^T_{1235} &= \frac{1}{\chi_2}\left[-I_{123} + a_{235} - b_{123} + \chi_1 c_{1235}\right],\ \gamma^T_{1235} = \rho^T_{1235} = \tau^T_{1235} = 0. \qquad (35)
\end{aligned}
$$

$$
\begin{aligned}
g^T_{1345} &= \frac{1}{2}\left[I_{134} + t c_{1345}\right],\ b^T_{1345} = \frac{1}{s}\left[b_{134} - b_{345} - (\chi_1' - t)\rho^T_{1345}\right], \\
a^T_{1345} &= \frac{1}{st(\chi_1' - s - t)}\big[(s+t)^2 I_{134} + t(\chi_1' - s - t) a_{145} - (s(s+t) + t\chi_1') a_{134} \\
&\qquad + \chi_1'(s+t)(c_{145} - c_{134}) + t(s+t)^2 c_{1345}\big], \\
c^T_{1345} &= d^T_{1345} = \tau^T_{1345} = \frac{1}{t(\chi_1' - s - t)}\left[(\chi_1' - t)(c_{145} - c_{134}) - s(b_{134} - t c_{1345})\right], \\
\alpha^T_{1345} &= \frac{1}{st(\chi_1' - s - t)}\big[-t(\chi_1' - s - t) a_{345} + \chi_1'(\chi_1' - t)(c_{145} - c_{134})
\end{aligned}
$$

$$
\begin{aligned}
&- s\chi_1'(a_{134} - I_{134}) + st\chi_1' c_{1345}\big], \\
\beta^T_{1345} &= \gamma^T_{1345} = \frac{1}{t(\chi_1' - s - t)}\left[(s+t)(b_{134} - t c_{1345}) - \chi_1'(c_{145} - c_{134})\right], \\
\rho^T_{1345} &= \sigma^T_{1345} = \frac{1}{st(\chi_1' - s - t)}\big[(\chi_1'(\chi_1' - t) - st) c_{134} - (\chi_1' - t)^2 c_{145} \\
&+ t(\chi_1' - s - t) a_{345} + s(\chi_1' - t) b_{134} - st(\chi_1' - t) c_{1345}\big]. \qquad (36)
\end{aligned}
$$

$$
\begin{aligned}
g^T_{2345} &= \frac{1}{2}\left[I_{234} + \chi_1 a_{2345} + (t+\chi_1) d_{2345}\right],\ \alpha^T_{2345} = -\sigma^T_{2345} = \frac{1}{s_1 t}\big[-\chi_1 a_{235} \\
&- t a_{345}\big],\ a^T_{2345} = c^T_{2345} = -\beta^T_{2345} = \frac{1}{s_1 t}\left[-t a_{345} - (s_1 + \chi_1) a_{235} + s_1 t a_{2345}\right], \\
b^T_{2345} &= \frac{1}{s_1 t(\chi_1 + s_1 + t)}\big[s_1 t(b_{235} - b_{345}) - \chi_1(t+\chi_1) a_{235} \\
&- t(t+\chi_1) a_{345} - s_1 t(t+\chi_1) b_{2345}\big], \\
d^T_{2345} &= \frac{1}{(\chi_1 + s_1 + t)}\Big[d_{245} - d_{234} - \frac{(\chi_1 + s_1)}{s_1 t(\chi_1 + s_1 + t)}\Big[s_1 t(a_{245} - a_{234}) \\
&+ t(\chi_1 + s_1) a_{345} + (\chi_1 + s_1)^2 a_{235} - s_1 t(\chi_1 + s_1) a_{2345}\Big]\Big], \\
\gamma^T_{2345} &= -\tau^T_{2345} = \frac{1}{s_1 t(\chi_1 + s_1 + t)}\big[s_1 t(a_{245} - a_{234}) + t(\chi_1 + s_1) a_{345} \\
&+ (\chi_1 + s_1)^2 a_{235} - s_1 t(\chi_1 + s_1) a_{2345}\big], \\
\rho^T_{2345} &= \frac{1}{s_1 t(\chi_1 + s_1 + t)}\big[-s_1 t a_{234} + \chi_1(\chi_1 + s_1) a_{235} + t(\chi_1 + s_1) a_{345} \\
&- s_1 t\chi_1 a_{2345} - s_1 t(\chi_1 + t) d_{2345}\big]. \qquad (37)
\end{aligned}
$$

$$
\begin{aligned}
g^T_{1234} &= \frac{1}{2}\left[I_{123} - \chi_1'\frac{\Delta^{(3)}}{\Delta}\right],\ a^T_{1234} = 2\frac{\Delta^{(2)}}{\Delta} + I_{1234} + \tilde{b}_{1234},\ b^T_{1234} = \tilde{b}_{1234}, \\
c^T_{1234} &= 2\frac{\Delta^{(2)}}{\Delta} - 2\frac{\Delta^{(3)}}{\Delta} + I_{1234} + \tilde{b}_{1234} + \tilde{c}_{1234} - 2\tilde{\gamma}_{1234}, \\
d^T_{1234} &= 2\frac{\Delta^{(2)}}{\Delta} - 2\frac{\Delta^{(1)}}{\Delta} + I_{1234} + \tilde{a}_{1234} + \tilde{b}_{1234} - 2\tilde{\alpha}_{1234}, \\
\alpha^T_{1234} &= \frac{\Delta^{(2)}}{\Delta} + \tilde{b}_{1234},\ \beta^T_{1234} = \frac{\Delta^{(3)}}{\Delta} - 2\frac{\Delta^{(2)}}{\Delta} - I_{1234} - \tilde{b}_{1234} + \tilde{\gamma}_{1234}, \\
\gamma^T_{1234} &= \frac{\Delta^{(1)}}{\Delta} - 2\frac{\Delta^{(2)}}{\Delta} - I_{1234} - \tilde{b}_{1234} + \tilde{\alpha}_{1234}, \\
\rho^T_{1234} &= -\frac{\Delta^{(2)}}{\Delta} - \tilde{b}_{1234} + \tilde{\alpha}_{1234},\ \sigma^T_{1234} = -\frac{\Delta^{(2)}}{\Delta} - \tilde{b}_{1234} + \tilde{\gamma}_{1234}, \\
\tau^T_{1234} &= 2\frac{\Delta^{(2)}}{\Delta} - \frac{\Delta^{(1)}}{\Delta} - \frac{\Delta^{(3)}}{\Delta} + I_{1234} + \tilde{b}_{1234} - \tilde{\alpha}_{1234} + \tilde{\beta}_{1234} - \tilde{\gamma}_{1234}, \qquad (38)
\end{aligned}
$$

where

$$
\begin{aligned}
\tilde{a}_{1234} &= \frac{1}{s_1\chi_1'}\left[\chi_1'(I_{124}-I_{123}+d_{124})-(\chi_1'+\chi_2')b_{123}-\chi_1'\chi_2'\frac{\Delta^{(3)}}{\Delta}\right],\\
\tilde{b}_{1234} &= \frac{1}{s_1\chi_2'}\left[-\chi_1'(I_{134}-I_{123}+c_{134})+(\chi_1'+\chi_2')(b_{123}-b_{134})-\chi_1'^2\frac{\Delta^{(3)}}{\Delta}\right],\\
\tilde{c}_{1234} &= \frac{1}{\chi_1'\chi_2'}\Big[(s_1+\chi_2')(I_{123}-I_{134}+b_{123}-b_{134}-c_{134})+\chi_2'c_{123}\\
&- s_1\chi_1'\frac{\Delta^{(3)}}{\Delta}\Big],\\
\tilde{\alpha}_{1234} &= \frac{1}{s_1}\left[-b_{134}-c_{134}-I_{134}\right],\ \tilde{\beta}_{1234}=\frac{1}{\chi_1'}\left[b_{123}+\chi_1'\frac{\Delta^{(3)}}{\Delta}\right],\\
\tilde{\gamma}_{1234} &= \frac{1}{\chi_2'}\left[-b_{123}+b_{134}+c_{134}+I_{134}-I_{123}+\chi_1'\frac{\Delta^{(3)}}{\Delta}\right].
\end{aligned}
\tag{39}
$$

2.4.4 Pentagon

Following ref. [1] we express the pentagon diagram in terms of box graphs.

$$
I_{12345}=-\frac{1}{\Delta}\left[\Delta^{(1)}I_{2345}+\Delta^{(2)}I_{1345}+\Delta^{(3)}I_{1245}+\Delta^{(4)}I_{1235}+\Delta^{(5)}I_{1234}\right],\tag{40}
$$

where

$$
\begin{aligned}
&\Delta=2ss_1t\chi_1\chi_1',\ \Delta^{(1)}=s_1t\left[-(s-s_1)t-s\chi_1-s_1\chi_1'-\chi_1\chi_1'\right],\\
&\Delta^{(2)}=st\left[(s-s_1)t+s\chi_1+s_1\chi_1'-\chi_1\chi_1'\right],\\
&\Delta^{(3)}=\chi_1\chi_1'\left[-(s+s_1)t-s\chi_1+s_1\chi_1'+\chi_1\chi_1'\right],\\
&\Delta^{(4)}=s\chi_1\left[(s-s_1)t+s\chi_1-s_1\chi_1'-\chi_1\chi_1'\right],\\
&\Delta^{(5)}=s_1\chi_1'\left[(s-s_1)t-s\chi_1+s_1\chi_1'+\chi_1\chi_1'\right].
\end{aligned}
\tag{41}
$$

The case of different fermion masses.

Typical process: $\mu^-e^-\to\mu^-e^-\gamma$

2.5 Notations (see fig.1)

$$
I^{1,\mu,\mu\nu}_{ijklm}\equiv\int\frac{d^4k}{i\pi^2}\frac{1,k^\mu,k^\mu k^\nu}{(i)(j)(k)(l)(m)}\tag{42}
$$

$$
\begin{aligned}
&(1)=(p_1-k)^2-m^2,\quad(2)=(p_1-k_1-k)^2-m^2,\quad(3)=(p_2+k)^2-\mu^2,\\
&(4)=(q-k)^2-\lambda^2,\qquad(5)=k^2-\lambda^2.
\end{aligned}
\tag{43}
$$

Invariants

$$
\begin{aligned}
&\chi_1=2p_1k_1,\ \chi_1'=2p_1'k_1,\ \chi_2=2p_2k_1=s-s_1-\chi_1,\\
&\chi_2'=2p_2'k_1=s-s_1-\chi_1',\ s=(p_1+p_2)^2,\ s_1=(p_1'+p_2')^2,\\
&t=q^2,\ q=p_2'-p_2,\ t_1=q'^2=t+\chi_1-\chi_1',\ q'=p_1'-p_1,\\
&\chi_1+\chi_2=s-s_1,\ p_1^2=p_1'^2=m^2,\ p_2^2=p_2'^2=\mu^2,\ k_1^2=0.
\end{aligned}
\tag{44}
$$

$$
\begin{aligned}
&L_{\Lambda_m}=\ln\left(\frac{\Lambda^2}{m^2}\right),\ L_{\Lambda_\mu}=\ln\left(\frac{\Lambda^2}{\mu^2}\right),\ L_{\lambda_m}=\ln\left(\frac{\lambda^2}{m^2}\right),\ L_{\lambda_\mu}=\ln\left(\frac{\lambda^2}{\mu^2}\right),\\
&L_{t_m}=\ln\left(\frac{-t}{m^2}\right),\ L_{t_\mu}=\ln\left(\frac{-t}{\mu^2}\right),\ L_{s_m}=\ln\left(\frac{s}{m^2}\right),\ L_{s_\mu}=\ln\left(\frac{s}{\mu^2}\right),\\
&\mathrm{Li}_2(z)=-\int_0^z\frac{dx}{x}\ln(1-x).
\end{aligned}
\tag{45}
$$

$$
\mathcal{P}^2=m^2x+\mu^2(1-x)-xxs-i0,\quad\mathcal{P}_1^2=m^2x+\mu^2(1-x)-xxs_1-i0,\quad\text{where}\quad x=1-x\tag{46}
$$

2.6 Two-propagator integrals

2.6.1 Scalar

$$
\begin{aligned}
I_{12} &= -1+L_{\Lambda_m},\ I_{13}=1+L_{\Lambda_\mu}-L_{s_\mu}+i\pi,\\
I_{14} &= 1+L_{\Lambda_m}-L_{\chi'_{1m}}+i\pi,\ I_{15}=I_{24}=1+L_{\Lambda_m},\\
I_{34} &= I_{35}=1+L_{\Lambda_\mu},\ I_{23}=1+L_{\Lambda_\mu}-L_{s_{1\mu}}+i\pi,\\
I_{25} &= 1+L_{\Lambda_m}-L_{\chi_{1m}},\ I_{45}=1+L_{\Lambda_m}-L_{t_m}.
\end{aligned}
\tag{47}
$$

2.6.2 Vector

$$
\begin{aligned}
I^\mu_{12} &= \left(p_1-\frac{k_1}{2}\right)^\mu\left(L_{\Lambda_m}-\frac{3}{2}\right),\ I^\mu_{13}=(p_1-p_2)^\mu\left(\frac{1}{4}+\frac{1}{2}L_{\Lambda_m}-\frac{1}{2}L_{s_m}+\frac{i\pi}{2}\right),\\
I^\mu_{14} &= (p_1+q)^\mu\left(\frac{1}{4}+\frac{1}{2}L_{\Lambda_m}-\frac{1}{2}L_{\chi'_{1m}}+\frac{i\pi}{2}\right),\ I^\mu_{15}=\frac{p_1^\mu}{2}\left(L_{\Lambda_m}-\frac{1}{2}\right),\\
I^\mu_{24} &= (p_1-k_1)^\mu\left(\frac{1}{2}+L_{\Lambda_m}\right)-\frac{p_1'^\mu}{2}\left(\frac{3}{2}+L_{\Lambda_m}\right).
\end{aligned}
$$

$$
\begin{aligned}
I^{\mu}_{34} &= \frac{p'^{\mu}_2}{2}\left(\frac{3}{2}+L_{\Lambda_\mu}\right)-p^{\mu}_2\left(\frac{1}{2}+L_{\Lambda_\mu}\right),\ I^{\mu}_{35}=\frac{p^{\mu}_2}{2}\left(\frac{1}{2}-L_{\Lambda_\mu}\right),\\
I^{\mu}_{23} &= (p_1-k_1-p_2)^{\mu}\left(\frac{1}{4}+\frac{1}{2}L_{\Lambda_m}-\frac{1}{2}L_{s_{1m}}+\frac{i\pi}{2}\right),\\
I^{\mu}_{25} &= (p_1-k_1)^{\mu}\left(\frac{1}{4}+\frac{1}{2}L_{\Lambda_m}-\frac{1}{2}L_{\chi_{1m}}\right),\ I^{\mu}_{45}=q^{\mu}\left(\frac{1}{4}+\frac{1}{2}L_{\Lambda_m}-\frac{1}{2}L_{t_m}\right).
\end{aligned}
\tag{48}
$$

2.7 Three-propagator integrals

2.7.1 Scalar

$$
\begin{aligned}
I_{123} &= \frac{1}{s-s_1}\left[\frac{1}{2}L^2_{s_m}-\frac{1}{2}L^2_{s_{1m}}+i\pi\left(L_{s_{1m}}-L_{s_m}\right)\right],\ I_{345}=\frac{1}{t}\left[\frac{1}{2}L^2_{t_\mu}+4\zeta(2)\right],\\
I_{124} &= \frac{1}{\chi'_1}\left[\frac{1}{2}L^2_{\chi'_{1m}}-\zeta(2)-i\pi L_{\chi'_{1m}}\right],\ I_{125}=\frac{1}{\chi_1}\left[-\frac{1}{2}L^2_{\chi_{1m}}-2\zeta(2)\right],\\
I_{134} &= \frac{1}{s-\chi'_1}\left[\frac{3}{2}L^2_{s_\mu}+\frac{1}{2}L^2_{\chi'_{1\mu}}-2L_{s_\mu}L_{\chi'_{1\mu}}+2\mathrm{Li}_2\left(1-\frac{\chi'_1}{s}\right)+i\pi\left(L_{\chi'_{1\mu}}-L_{s_\mu}\right)\right],\\
I_{235} &= \frac{1}{s_1+\chi_1}\left[\frac{3}{2}L^2_{s_{1\mu}}+\frac{1}{2}L^2_{\chi_{1\mu}}-2L_{s_{1\mu}}L_{\chi_{1\mu}}-9\zeta(2)\right.\\
&+ \left.2\mathrm{Li}_2\left(1+\frac{\chi_1}{s_1}\right)+i\pi\left(2L_{\chi_{1\mu}}-3L_{s_{1\mu}}\right)\right],\\
I_{135} &= \frac{1}{s}\left[\frac{1}{2}L^2_{s_m}-L_{\lambda_m}L_{s_m}+i\pi\left(L_{\lambda_m}-L_{s_m}\right)-4\zeta(2)+2\ln^2\frac{\mu}{m}+2\ln\frac{\mu}{m}L_{\lambda_\mu}\right],\\
I_{234} &= \frac{1}{s_1}\left[\frac{1}{2}L^2_{s_{1m}}-L_{\lambda_m}L_{s_{1m}}+i\pi\left(L_{\lambda_m}-L_{s_{1m}}\right)-4\zeta(2)+2\ln^2\frac{\mu}{m}\right.\\
&+ \left.2\ln\frac{\mu}{m}L_{\lambda_\mu}\right],\\
I_{245} &= \frac{1}{\chi_1+t}\left[\frac{1}{2}L^2_{t_m}-\frac{1}{2}L^2_{\chi_{1m}}+2\mathrm{Li}_2\left(1-\frac{\chi_1}{-t}\right)\right],\\
I_{145} &= \frac{1}{\chi'_1-t}\left[\frac{1}{2}L^2_{\chi'_{1m}}-\frac{1}{2}L^2_{t_m}-3\zeta(2)-2\mathrm{Li}_2\left(1+\frac{\chi'_1}{-t}\right)-i\pi L_{\chi'_{1m}}\right].
\end{aligned}
\tag{49}
$$

2.7.2 Vector

Parametrization

$$
I^{\mu}_{ijk}=a_{ijk}p^{\mu}_1+b_{ijk}p^{\mu}_2+c_{ijk}k^{\mu}_1+d_{ijk}p'^{\mu}_1 \tag{50}
$$

$$
\begin{aligned}
a_{245} &= -c_{245}=\frac{1}{t+\chi_1}\left[\frac{1}{2}L^2_{t_m}-\frac{1}{2}L^2_{\chi_{1m}}+L_{\chi_{1m}}-L_{t_m}+2\mathrm{Li}_2\left(1-\frac{\chi_1}{-t}\right)\right],\\
b_{245} &= 0,\ d_{245}=\frac{1}{t+\chi_1}\left[-L_{t_m}-\frac{\chi_1}{t+\chi_1}\left[\frac{1}{2}L^2_{t_m}-\frac{1}{2}L^2_{\chi_{1m}}+2L_{\chi_{1m}}-2L_{t_m}\right.\right.
\end{aligned}
$$

$$
+\ \left.\left.2\mathrm{Li}_2\left(1-\frac{\chi_1}{-t}\right)\right]\right]. \tag{51}
$$

$$
\begin{aligned}
a_{145} &= \frac{1}{\chi'_1-t}\left[2L_{\chi'_{1m}}-L_{t_m}-2i\pi+\frac{t}{\chi'_1-t}\left[\frac{1}{2}L^2_{t_m}-\frac{1}{2}L^2_{\chi'_{1m}}+2L_{\chi'_{1m}}\right.\right.\\
&- \left.\left.2L_{t_m}+3\zeta(2)+2\mathrm{Li}_2\left(1+\frac{\chi'_1}{-t}\right)+i\pi\left(L_{\chi'_{1m}}-2\right)\right]\right],\\
b_{145} &= 0,\ c_{145}=d_{145}=\frac{1}{\chi'_1-t}\left[L_{t_m}-L_{\chi'_{1m}}+i\pi\right].
\end{aligned}
\tag{52}
$$

$$
a_{345} = -c_{345}=-d_{345}=\frac{1}{t}L_{t_\mu},\ b_{345}=\frac{1}{t}\left[-\frac{1}{2}L^2_{t_\mu}+2L_{t_\mu}-4\zeta(2)\right]. \tag{53}
$$

$$
\begin{aligned}
a_{125} &= \frac{1}{\chi_1}\left[-\frac{1}{2}L^2_{\chi_{1m}}+L_{\chi_{1m}}-2\zeta(2)\right],\ b_{125}=d_{125}=0,\\
c_{125} &= \frac{1}{\chi_1}\left[L_{\chi_{1m}}-2\right].
\end{aligned}
\tag{54}
$$

$$
\begin{aligned}
a_{235} &= -c_{235}=\frac{1}{s_1+\chi_1}\left[L_{s_{1\mu}}-L_{\chi_{1\mu}}-i\pi\right],\ d_{235}=0,\\
b_{235} &= \frac{1}{s_1+\chi_1}\left[-L_{s_{1\mu}}+i\pi+\frac{\chi_1}{s_1+\chi_1}\left[2L_{s_{1\mu}}-2L_{\chi_{1\mu}}-\frac{3}{2}L^2_{s_{1\mu}}-\frac{1}{2}L^2_{\chi_{1\mu}}\right.\right.\\
&+ \left.\left.2L_{s_{1\mu}}L_{\chi_{1\mu}}+9\zeta(2)-2\mathrm{Li}_2\left(1+\frac{\chi_1}{s_1}\right)+i\pi\left(-2-2L_{\chi_{1\mu}}+3L_{s_{1\mu}}\right)\right]\right].
\end{aligned}
\tag{55}
$$

$$
a_{135} = -b_{135}=\frac{1}{s}\left[L_{s_m}-i\pi\right],\ c_{135}=d_{135}=0. \tag{56}
$$

$$
\begin{aligned}
a_{234} &= -c_{234}=-b_{234}-d_{234}=\frac{1}{s_1}\left[-L_{s_{1\mu}}+i\pi+s_1I_{234}\right],\\
b_{234} &= a_{234}-I_{234}=\frac{1}{s_1}\left[-L_{s_{1\mu}}+i\pi\right],\\
d_{234} &= \frac{1}{s_1}\left[L_{s_{1m}}+L_{s_{1\mu}}-2i\pi-s_1I_{234}\right].
\end{aligned}
\tag{57}
$$

$$
\begin{aligned}
a_{134} &= \frac{1}{s-\chi'_1}\left[L_{s\mu}-2L_{\chi'_{1\mu}}+i\pi-\frac{2s}{s-\chi'_1}\left[L_{s\mu}-L_{\chi'_{1\mu}}\right]+sI_{134}\right],\\
b_{134} &= a_{134}-I_{134}=\frac{1}{s-\chi'_1}\left[-L_{s\mu}+i\pi-\frac{2\chi'_1}{s-\chi'_1}\left[L_{s\mu}-L_{\chi'_{1\mu}}\right]+\chi'_1I_{134}\right],\\
c_{134} &= d_{134}=\frac{1}{s-\chi'_1}\left[L_{\chi'_{1\mu}}-i\pi+\frac{2s}{s-\chi'_1}\left[L_{s\mu}-L_{\chi'_{1\mu}}\right]-sI_{134}\right].
\end{aligned}
\tag{58}
$$

$$
a_{124} = I_{124}=\frac{1}{\chi'_1}\left[\frac{1}{2}L^2_{\chi'_{1m}}-\zeta(2)-i\pi L_{\chi'_{1m}}\right],\ d_{124}=\frac{1}{\chi'_{1m}}\left[-L_{\chi'_{1m}}+i\pi\right],
$$

$$c_{124} = \frac{1}{\chi_1'}\left[-\frac{1}{2}L^2_{\chi'_{1m}} + L_{\chi'_{1m}} + \zeta(2) - 2 + i\pi\left(L_{\chi'_{1m}} - 1\right)\right],\ b_{124} = 0. \tag{59}$$

$$\begin{aligned}
a_{123} &= b_{123} - I_{123} = \frac{1}{s-s_1}\Big[\frac{1}{2}L^2_{s_m} - \frac{1}{2}L^2_{s_{1m}} - L_{s_m} + L_{s_{1m}} \\
&+ i\pi\left(L_{s_{1m}} - L_{s_m}\right)\Big],\ b_{123} = -\frac{1}{s-s_1}\left[L_{s_m} - L_{s_{1m}}\right],\ d_{123} = 0, \\
c_{123} &= \frac{1}{s-s_1}\Big[L_{s_{1m}} - 2 - i\pi + \frac{s}{s-s_1}\Big[-\frac{1}{2}L^2_{s_m} + \frac{1}{2}L^2_{s_{1m}} \\
&+ 2L_{s_m} - 2L_{s_{1m}} + i\pi\left(L_{s_m} - L_{s_{1m}}\right)\Big]\Big].
\end{aligned} \tag{60}$$

2.8 Four-propagator integrals

2.8.1 Scalar

$$\begin{aligned}
I_{1245} &= \int_0^1 \frac{\mathrm{d}x\mathrm{d}y}{[xy\chi_1 - \bar{y}t]\,[ym^2 - \bar{x}\bar{y}\chi_1']} = \frac{1}{\chi_1\chi_1'}\Big[-L^2_{\chi_{1m}} - L^2_{\chi'_{1m}} - L^2_{t_m} \\
&- 2L_{\chi_{1m}}L_{\chi'_{1m}} + 2L_{\chi_{1m}}L_{t_m} + 2L_{\chi'_{1m}}L_t + 4\zeta(2) + 2i\pi\left(L_{\chi_{1m}} + L_{\chi'_{1m}} - L_{t_m}\right)\Big], \\
I_{2345} &= \frac{1}{t}\int_0^1 \frac{\mathrm{d}x}{\mathcal{P}_1^2}\left[-\frac{1}{2}\ln\left(\frac{\mathcal{P}_1^2}{\lambda^2}\right) + \ln\left(\frac{x\chi_1}{-t}\right)\right] \\
&= \frac{1}{s_1 t}\Big[L^2_{s_{1\mu}} - L_{s_{1\mu}}L_{\lambda_\mu} - 2L_{s_{1\mu}}L_{\chi_{1\mu}} + 2L_{s_{1\mu}}L_{t_\mu} - 5\zeta(2) \\
&+ i\pi\left(2L_{\chi_{1\mu}} - 2L_{t_\mu} - 2L_{s_{1\mu}} + L_{\lambda_\mu}\right) + \ln\frac{\mu}{m}\left(-2L_{\chi_{1\mu}} + 2L_{t_\mu} - L_{\lambda_\mu}\right) - \ln^2\frac{\mu}{m}\Big], \\
I_{1345} &= \frac{1}{t}\int_0^1 \frac{\mathrm{d}x}{\mathcal{P}^2}\left[-\frac{1}{2}\ln\left(\frac{\mathcal{P}^2}{\lambda^2}\right) + \ln\left(\frac{-x\chi_1'}{-t}\right)\right] \\
&= \frac{1}{st}\Big[\frac{1}{2}L^2_{s_m} + \frac{1}{2}L^2_{s_\mu} - L_{s_m}L_{\lambda_m} - 2L_{s_m}L_{\chi'_{1m}} + 2L_{s_m}L_{t_m} + 7\zeta(2) \\
&+ i\pi\left(2L_{\chi'_{1m}} - 2L_{t_m} + L_{\lambda_m}\right) - \ln^2\frac{\mu}{m} - \ln\frac{\mu}{m}\left(L_{\lambda_m} + 2L_{\chi'_{1m}} - 2L_{t_m} - 4i\pi\right)\Big], \\
I_{1235} &= \frac{1}{\chi_1}\int_0^1 \frac{\mathrm{d}x}{\mathcal{P}^2}\left[\frac{1}{2}\ln\left(\frac{\mathcal{P}^2}{\lambda^2}\right) - \ln\left(\frac{\mathcal{P}_1^2}{x\chi_1}\right)\right] = \frac{1}{s\chi_1}\Big[\frac{1}{2}L^2_{s_\mu} - \frac{1}{2}L^2_{s_m} + \frac{1}{2}L^2_{s_{1m}} \\
&+ \frac{1}{2}L^2_{s_{1\mu}} - 2L_{s_m}L_{\chi_{1m}} + L_{s_m}L_{\lambda_m} - 5\zeta(2) + 2\mathrm{Li}_2\left(1 - \frac{s_1}{s}\right) + i\pi(-2L_{s_{1\mu}} \\
&+ 2L_{\chi_{1m}} - L_{\lambda_m}) + \ln^2\frac{\mu}{m} + \ln\frac{\mu}{m}\left(2L_{s_\mu} + 2L_{\chi_{1m}} - L_{\lambda_m} - 2i\pi\right)\Big], \\
I_{1234} &= \frac{1}{\chi_1'}\int_0^1 \frac{\mathrm{d}x}{\mathcal{P}_1^2}\left[-\frac{1}{2}\ln\left(\frac{\mathcal{P}_1^2}{\lambda^2}\right) + \ln\left(\frac{\mathcal{P}^2}{-x\chi_1'}\right)\right] = \frac{1}{s_1\chi_1'}\Big[-\frac{1}{2}L^2_{s_\mu} - \frac{1}{2}L^2_{s_m} + \frac{1}{2}L^2_{s_{1m}} \\
&- \frac{1}{2}L^2_{s_{1\mu}} + 2L_{s_{1m}}L_{\chi'_{1m}} - L_{s_{1m}}L_{\lambda_m} - 7\zeta(2) - 2\mathrm{Li}_2\left(1 - \frac{s}{s_1}\right) + i\pi(2L_{s_\mu} - 2L_{s_{1m}}
\end{aligned}$$

$$- 2L_{\chi'_{1m}} + L_{\lambda_m}) - \ln^2\frac{\mu}{m} + \ln\frac{\mu}{m}\left(-2L_{s_{1\mu}} - 2L_{\chi'_{1m}} + L_{\lambda_m} + 4i\pi\right)\Big]. \tag{61}$$

Useful integrals

$$\begin{aligned}
\int_0^1 \frac{\mathrm{d}x}{\mathcal{P}^2} &= \frac{2}{s}\left[-L_{s_m} + \ln\frac{\mu}{m} + i\pi\right]. \\
\int_0^1 \frac{\mathrm{d}x}{\mathcal{P}^2}\ln x &= \frac{1}{s}\left[\frac{1}{2}L^2_{s_\mu} - \zeta(2) - i\pi L_{s_\mu}\right]. \\
\int_0^1 \frac{\mathrm{d}x}{\mathcal{P}^2}\ln\left(\frac{\mathcal{P}^2}{m^2}\right) &= \frac{1}{s}\left[-L^2_{s_m} + 8\zeta(2) + 2i\pi L_{s_m} + 2\ln^2\frac{\mu}{m}\right]. \\
\int_0^1 \frac{\mathrm{d}x}{\mathcal{P}^2}\ln\left(\frac{\mathcal{P}_1^2}{m^2}\right) &= \frac{1}{s}\Big[-\frac{1}{2}L^2_{s_{1m}} - \frac{1}{2}L^2_{s_{1\mu}} + 8\zeta(2) - 2\mathrm{Li}_2\left(1 - \frac{s_1}{s}\right) \\
&- 2\ln\frac{\mu}{m}L_{s_\mu} + i\pi\left(L_{s_{1m}} + L_{s_{1\mu}} + 2\ln\frac{\mu}{m}\right)\Big].
\end{aligned} \tag{62}$$

The vector, tensor four propagator and pentagon integrals are the same as in the case of equal fermion masses.

3 Self–energy and real–photon vertex corrections

In this section the masses are taken into account.

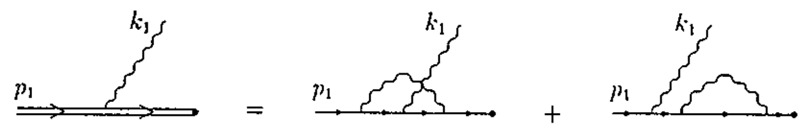

Figure 2: **Self–energy and real–photon vertex corrections to the incoming fermion**

$$\mathcal{L}_1 = \left[\frac{\not{p}_1 - \not{k}_1 + m}{-\chi_1}\Gamma_\mu e^\mu + \Sigma(\not{p}_1 - \not{k}_1)\frac{1}{(\not{p}_1 - \not{k}_1 - m)^2}\,\not{e}\right]u(p_1) \tag{63}$$

Using well known expressions for the off-shell vertex function Γ and mass operator Σ, we obtain

$$\mathcal{L}_1 = \frac{\alpha}{2\pi}\left[\mathcal{A}_1\left(\not{e} - \not{k}_1\frac{ep_1}{k_1p_1}\right) + \mathcal{A}_2\,\not{k}_1\not{e}\right]u(p_1). \tag{64}$$

where

$$
\begin{aligned}
\mathcal{A}_1 &= -\frac{m}{2(\chi_1 - m^2)}\left[1 - \frac{\chi_1}{\chi_1 - m^2} L_{\chi_1}\right], \\
\mathcal{A}_2 &= -\frac{1}{2(\chi_1 - m^2)} + \frac{2\chi_1^2 - 3m^2\chi_1 + 2m^4}{2\chi_1(\chi_1 - m^2)^2} L_{\chi_1} + \frac{m^2}{\chi_1^2}\left[-\mathrm{Li}_2\left(1 - \frac{\chi_1}{m^2}\right)\right. \\
&+ \left.\zeta(2)\right].
\end{aligned} \tag{65}
$$

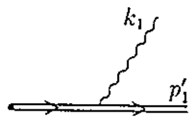

Figure 3: **Self–energy and real–photon vertex corrections to the outgoing fermion**

$$
\mathcal{L}'_1 = \frac{\alpha}{2\pi}\bar{u}(p'_1)\left[\mathcal{B}_1\left(\not\epsilon - \not k_1 \frac{ep'_1}{k_1 p'_1}\right) + \mathcal{B}_2 \not k_1 \not\epsilon\right], \tag{66}
$$

where

$$
\begin{aligned}
\mathcal{B}_1 &= \frac{m}{2(\chi'_1 + m^2)}\left[1 - \frac{\chi'_1}{\chi'_1 + m^2}\left(L_{\chi'_1} - i\pi\right)\right], \\
\mathcal{B}_2 &= \frac{1}{2(\chi'_1 + m^2)} - \frac{2\chi'^2_1 + 3m^2\chi'_1 + 2m^4}{2\chi'_1(\chi'_1 + m^2)^2}\left(L_{\chi'_1} - i\pi\right) + \frac{m^2}{\chi'^2_1}\left[-\mathrm{Li}_2\left(1 + \frac{\chi'_1}{m^2}\right)\right. \\
&+ \left.\zeta(2)\right].
\end{aligned} \tag{67}
$$

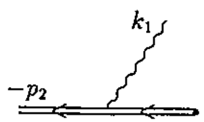

Figure 4: **Self–energy and real–photon vertex corrections to the incoming antifermion**

$$
\mathcal{L}_2 = \frac{\alpha}{2\pi}\bar{u}(-p_2)\left[\mathcal{C}_1\left(\not\epsilon - \not k_1 \frac{ep_2}{k_1 p_2}\right) + \mathcal{C}_2 \not k_1 \not\epsilon\right], \tag{68}
$$

where

$$
\begin{aligned}
\mathcal{C}_1 &= -\frac{m}{2(\chi_2 - m^2)}\left[1 - \frac{\chi_2}{\chi_2 - m^2} L_{\chi_2}\right], \\
\mathcal{C}_2 &= -\frac{1}{2(\chi_2 - m^2)} + \frac{2\chi_2^2 - 3m^2\chi_2 + 2m^4}{2\chi_2(\chi_2 - m^2)^2} L_{\chi_2} + \frac{m^2}{\chi_2^2}\left[-\mathrm{Li}_2\left(1 - \frac{\chi_2}{m^2}\right)\right. \\
&+ \left.\zeta(2)\right].
\end{aligned} \tag{69}
$$

Figure 5: **Self–energy and real–photon vertex corrections to the outgoing antifermion**

$$
\mathcal{L}'_2 = \frac{\alpha}{2\pi}\left[\mathcal{D}_1\left(\not\epsilon - \not k_1 \frac{ep'_2}{k_1 p'_2}\right) + \mathcal{D}_2 \not k_1 \not\epsilon\right] u(-p'_2), \tag{70}
$$

where

$$
\begin{aligned}
\mathcal{D}_1 &= \frac{m}{2(\chi'_2 + m^2)}\left[1 - \frac{\chi'_2}{\chi'_2 + m^2}\left(L_{\chi'_2} - i\pi\right)\right], \\
\mathcal{D}_2 &= \frac{1}{2(\chi'_2 + m^2)} - \frac{2\chi'^2_2 + 3m^2\chi'_2 + 2m^4}{2\chi'_2(\chi'_2 + m^2)^2}\left(L_{\chi'_2} - i\pi\right) \\
&+ \frac{m^2}{\chi'^2_2}\left[-\mathrm{Li}_2\left(1 + \frac{\chi'_2}{m^2}\right) + \zeta(2)\right].
\end{aligned} \tag{71}
$$

4 Heavy–photon vertex diagrams

4.1 Notations

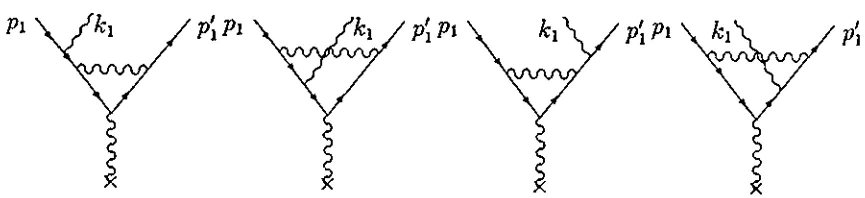

Figure 6: Heavy–photon vertex diagrams with real–photon emission

$$J^{1,\mu,\mu\nu}_{ijkl} \equiv \int \frac{d^4k}{i\pi^2} \frac{1, k^\mu, k^\mu k^\nu}{(i)(j)(k)(l)} \tag{72}$$

$$\begin{aligned} (0) &= k^2 - \lambda^2,\ (1) = (p_1 - k)^2 - m^2,\ (2) = (p_1' - k)^2 - m^2, \\ (q) &= (p_1 - k_1 - k)^2 - m^2. \end{aligned} \tag{73}$$

4.2 Two-propagator integrals

4.2.1 Scalar

$$\begin{aligned} J_{01} &= J_{02} = L_\Lambda + 1,\ J_{12} = L_\Lambda - L_{t_1} + 1,\ J_{0q} = L_\Lambda + 1 - L_{\chi_1}, \\ J_{1q} &= L_\Lambda - 1,\ J_{2q} = L_\Lambda - L_t + 1. \end{aligned} \tag{74}$$

4.2.2 Vector

$$\begin{aligned} J^\mu_{01} &= p_1^\mu \left[\frac{1}{2}L_\Lambda - \frac{1}{4}\right],\ J^\mu_{02} = p_1'^\mu \left[\frac{1}{2}L_\Lambda - \frac{1}{4}\right],\ J^\mu_{1q} = \left(p_1 - \frac{1}{2}k_1\right)^\mu \left[L_\Lambda - \frac{3}{2}\right], \\ J^\mu_{12} &= (p_1 + p_1')^\mu \left[\frac{1}{2}L_\Lambda - \frac{1}{2}L_{t_1} + \frac{1}{4}\right],\ J^\mu_{0q} = (p_1 - k_1)^\mu \left[\frac{1}{2}L_\Lambda + \frac{1}{4} - \frac{1}{2}L_{\chi_1}\right], \\ J^\mu_{2q} &= (p_1' + p_1 - k_1)^\mu \left[\frac{1}{2}L_\Lambda + \frac{1}{4} - \frac{1}{2}L_t\right]. \end{aligned} \tag{75}$$

4.3 Three-propagator integrals

4.3.1 Scalar

$$\begin{aligned} J_{012} &= \frac{1}{2t_1}\left[-2L_\lambda L_{t_1} + L_{t_1}^2 - 2\zeta(2)\right],\ J_{12q} = \frac{1}{2(\chi_1' - \chi_1)}\left[L_t^2 - L_{t_1}^2\right], \\ J_{01q} &= -\frac{1}{\chi_1}\left[-\mathrm{Li}_2\left(1 - \frac{\chi_1}{m^2}\right) + \zeta(2)\right], \\ J_{02q} &= \frac{1}{\chi_1' + t_1}\left[L_t\left(L_t - L_{\chi_1}\right) + \frac{1}{2}\left(L_t - L_{\chi_1}\right)^2 + 2\mathrm{Li}_2\left(1 + \frac{\chi_1}{t}\right)\right]. \end{aligned} \tag{76}$$

4.3.2 Vector

Parametrization

$$J^\mu_{ijk} = a_{ijk}p_1^\mu + b_{ijk}p_1'^\mu + c_{ijk}q^\mu. \tag{77}$$

$$\begin{aligned} a_{012} &= b_{012} = \frac{1}{t_1}L_{t_1},\ c_{012} = 0, \\ a_{01q} &= \frac{1}{\chi_1}\left[\chi_1 J_{01q} + 2L_{\chi_1} - 2\right],\ b_{01q} = -c_{01q} = \frac{1}{\chi_1}\left[-L_{\chi_1} + 2\right], \\ a_{02q} &= 0,\ b_{02q} = \frac{\chi_1}{\chi_1' + t_1}J_{02q} + 2\frac{t}{(\chi_1' + t_1)^2}L_t - \frac{t - \chi_1}{(\chi_1' + t_1)^2}L_{\chi_1}, \\ c_{02q} &= -\frac{1}{\chi_1' + t_1}L_t + \frac{1}{\chi_1' + t_1}L_{\chi_1}, \\ a_{12q} &= \frac{t}{\chi_1' - \chi_1}J_{12q} + \frac{t + t_1}{(\chi_1' - \chi_1)^2}L_{t_1} - 2\frac{t}{(\chi_1' - \chi_1)^2}L_t + \frac{2}{\chi_1' - \chi_1}, \\ b_{12q} &= J_{12q} - a_{12q}, \\ c_{12q} &= \frac{t_1}{\chi_1' - \chi_1}J_{12q} + 2\frac{t_1}{(\chi_1' - \chi_1)^2}L_{t_1} - \frac{t + t_1}{(\chi_1' - \chi_1)^2}L_t + \frac{2}{\chi_1' - \chi_1}. \end{aligned} \tag{78}$$

4.3.3 Tensor

$$\begin{aligned} J^{\mu\nu}_{ijk} &= g^T_{ijk}g^{\mu\nu} + a^T_{ijk}p_1^\mu p_1^\nu + b^T_{ijk}p_1'^\mu p_1'^\nu + c^T_{ijk}q^\mu q^\nu \\ &+ \alpha^T_{ijk}\{p_1^\mu p_1'^\nu\} + \beta^T_{ijk}\{p_1^\mu q^\nu\} + \gamma^T_{ijk}\{p_1'^\mu q^\nu\}. \end{aligned} \tag{79}$$

$$\begin{aligned} g^T_{012} &= \frac{1}{4}L_\Lambda - \frac{1}{4}L_{t_1} + \frac{3}{8},\ a^T_{012} = b^T_{012} = \frac{1}{2t_1}L_{t_1} - \frac{1}{2t_1}, \\ \alpha^T_{012} &= \frac{1}{2t_1},\ c^T_{012} = \beta^T_{012} = \gamma^T_{012} = 0. \end{aligned} \tag{80}$$

$$
\begin{aligned}
g^T_{01q} &= -\frac{1}{4}L_{\chi_1} + \frac{1}{4}L_\Lambda + \frac{3}{8},\ a^T_{01q} = J_{01q} + \frac{3}{\chi_1}L_{\chi_1} - \frac{9}{2\chi_1},\\
b^T_{01q} &= c^T_{01q} = -\gamma^T_{01q} = -\frac{1}{2\chi_1}L_{\chi_1} + \frac{1}{\chi_1},\ \beta^T_{01q} = -\alpha^T_{01q} = \frac{1}{2\chi_1}L_{\chi_1} - \frac{3}{2\chi_1}.
\end{aligned} \tag{81}
$$

$$
\begin{aligned}
g^T_{02q} &= -\frac{1}{4}\frac{\chi_1}{\chi'_1+t_1}L_{\chi_1} - \frac{1}{4}\frac{t}{(\chi'_1+t_1)}L_t + \frac{1}{4}L_\Lambda + \frac{3}{8},\\
b^T_{02q} &= \left[-\frac{\chi_1(t-\chi_1)}{(\chi'_1+t_1)^3} - \frac{1}{2}\frac{(t^2+2t\chi_1-\chi_1^2)}{(\chi'_1+t_1)^3}\right]L_{\chi_1} + \frac{t(t+4\chi_1)}{(\chi'_1+t_1)^3}L_t\\
&+ \frac{t-\chi_1}{2(\chi'_1+t_1)^2} + \frac{\chi_1^2}{(\chi'_1+t_1)^2}J_{02q},\\
c^T_{02q} &= -\frac{1}{2}\frac{1}{\chi'_1+t_1}L_{\chi_1} + \frac{1}{2(\chi'_1+t_1)}L_t,\ a^T_{02q} = \alpha^T_{02q} = \beta^T_{02q} = 0,\\
\gamma^T_{02q} &= \frac{t+2\chi_1}{2(\chi'_1+t_1)^2}L_{\chi_1} - \frac{t+2\chi_1}{2(\chi'_1+t_1)^2}L_t - \frac{1}{2(\chi'_1+t_1)}.
\end{aligned} \tag{82}
$$

$$
\begin{aligned}
g^T_{12q} &= \frac{1}{4}\frac{t_1}{\chi'_1-\chi_1}L_{t_1} - \frac{1}{4}\frac{t}{\chi'_1-\chi_1}L_t + \frac{1}{4}L_\Lambda + \frac{3}{8},\\
a^T_{12q} &= \frac{t^2}{(\chi'_1-\chi_1)^2}J_{12q} + \frac{(3t^2+4t_1t-t_1^2)}{2(\chi'_1-\chi_1)^3}L_{t_1} - \frac{3t^2}{(\chi'_1-\chi_1)^3}L_t + \frac{4t-t_1}{(\chi'_1-\chi_1)^2},\\
b^T_{12q} &= \frac{t_1^2}{(\chi'_1-\chi_1)^2}J_{12q} + \frac{(-t^2+4t_1t+3t_1^2)}{2(\chi'_1-\chi_1)^3}L_{t_1} + \frac{t(t-4t_1)}{(\chi'_1-\chi_1)^3}L_t + \frac{3t_1}{(\chi'_1-\chi_1)^2},\\
c^T_{12q} &= \frac{t_1^2}{(\chi'_1-\chi_1)^2}J_{12q} + \frac{3t_1^2}{(\chi'_1-\chi_1)^3}L_{t_1} + \frac{(t^2-4t_1t-3t_1^2)}{2(\chi'_1-\chi_1)^3}L_t + \frac{(4t_1-t)}{(\chi'_1-\chi_1)^2},\\
\alpha^T_{12q} &= -\frac{t_1t}{(\chi'_1-\chi_1)^2}J_{12q} - \frac{(t^2+4t_1t+t_1^2)}{2(\chi'_1-\chi_1)^3}L_{t_1} + \frac{t(t+2t_1)}{(\chi'_1-\chi_1)^3}L_t - \frac{(2t+t_1)}{(\chi'_1-\chi_1)^2},\\
\beta^T_{12q} &= \frac{t_1t}{(\chi'_1-\chi_1)^2}J_{12q} + \frac{t_1(5t+t_1)}{2(\chi'_1-\chi_1)^3}L_{t_1} - \frac{t(t+5t_1)}{2(\chi'_1-\chi_1)^3}L_t + \frac{3}{2}\frac{t+t_1}{(\chi'_1-\chi_1)^2},\\
\gamma^T_{12q} &= -\frac{t_1^2}{(\chi'_1-\chi_1)^2}J_{12q} - \frac{t_1(t+5t_1)}{2(\chi'_1-\chi_1)^3}L_{t_1} + \frac{-t^2+5t_1t+2t_1^2}{2(\chi'_1-\chi_1)^3}L_t\\
&+ \frac{t-7t_1}{2(\chi'_1-\chi_1)^2}.
\end{aligned} \tag{83}
$$

4.4 Four-propagator integrals

4.4.1 Scalar

$$
J_{012q} = -\frac{1}{\chi_1 t_1}\left[-L_\lambda L_{t_1} + 2L_{t_1}L_{\chi_1} - L_t^2 - 2\mathrm{Li}_2\left(1-\frac{t}{t_1}\right) - \zeta(2)\right]. \tag{84}
$$

4.4.2 Vector

Parametrization

$$
J^\mu_{012q} = a_{012q}p_1^\mu + b_{012q}p_1'^\mu + c_{012q}q^\mu. \tag{85}
$$

$$
\begin{aligned}
a_{012q} &= \frac{1}{d}[-(t_1\chi_1+t\chi'_1)J_{12q} + (\chi'_1+t_1)^2J_{02q} - \chi_1(\chi'_1-t_1)J_{01q}\\
&- t_1(\chi'_1+t_1)(J_{012}+\chi_1J_{012q})],\\
b_{012q} &= \frac{1}{d}[(t_1\chi'_1+t\chi_1)J_{12q} - (\chi_1\chi'_1+t_1t)J_{02q} - \chi_1(t_1-\chi_1)J_{01q}\\
&+ t_1(t_1-\chi_1)(J_{012}+\chi_1J_{012q})],\\
c_{012q} &= \frac{1}{d}[-t_1(\chi_1+\chi'_1)J_{12q} + t_1(\chi'_1+t_1)J_{02q} + \chi_1t_1J_{01q}\\
&- t_1^2(J_{012}+\chi_1J_{012q})].
\end{aligned} \tag{86}
$$

where $d = -2t_1\chi_1\chi'_1$.

4.4.3 Tensor

Parametrization

$$
\begin{aligned}
J^{\mu\nu}_{012q} &= g^T_{012q}g^{\mu\nu} + a^T_{012q}p_1^\mu p_1^\nu + b^T_{012q}p_1'^\mu p_1'^\nu + c^T_{012q}q^\mu q^\nu\\
&+ \alpha^T_{012q}\{p_1^\mu p_1'^\nu\} + \beta^T_{012q}\{p_1^\mu q^\nu\} + \gamma^T_{012q}\{p_1'^\mu q^\nu\}.
\end{aligned} \tag{87}
$$

$$
\begin{aligned}
g^T_{012q} &= \frac{1}{2}[J_{12q} - \chi_1c_{012q}],\\
a^T_{012q} &= \frac{1}{d}[(\chi'_1+t_1)^2(J_{12q}-\chi_1c_{012q}) - (\chi_1t_1+\chi'_1t)a_{12q}\\
&- \chi_1(\chi'_1-t_1)a_{01q} - t_1(\chi'_1+t_1)(a_{012}+\chi_1a_{012q})],\\
b^T_{012q} &= \frac{1}{d}[(t_1-\chi_1)^2(J_{12q}-\chi_1c_{012q}) + (\chi'_1t_1+\chi_1t)b_{12q}\\
&- (t_1t+\chi_1\chi'_1)b_{02q} + \chi_1(\chi_1-t_1)b_{01q}\\
&+ t_1(t_1-\chi_1)(a_{012}+\chi_1b_{012q})],\\
c^T_{012q} &= \frac{1}{d}[t_1^2(J_{12q}-2\chi_1c_{012q}) - t_1(\chi'_1+\chi_1)c_{12q}\\
&- t_1\chi_1b_{01q} + t_1(t_1+\chi'_1)c_{02q}],\\
\alpha^T_{012q} &= \frac{1}{d}[-(t_1t+\chi_1\chi'_1)(J_{12q}-\chi_1c_{012q}) + (\chi'_1t_1+\chi_1t)a_{12q}\\
&+ \chi_1(\chi_1-t_1)a_{01q} + t_1(t_1-\chi_1)(a_{012}+\chi_1a_{012q})],\\
\beta^T_{012q} &= \frac{1}{d}[t_1(t_1+\chi'_1)(J_{12q}-2\chi_1c_{012q}) - (\chi_1t_1+\chi'_1t)c_{12q}
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad (t_1 + \chi_1')^2 c_{02_q} + \chi_1(\chi_1' - t_1) b_{01_q}], \\
\gamma^T_{012_q} \quad &= \quad \frac{1}{d}[t_1(\chi_1 - t_1)(J_{12_q} - 2\chi_1 c_{012_q}) + (\chi_1' t_1 + \chi_1 t) c_{12_q} \\
&- \quad (t_1 t + \chi_1' \chi_1) c_{02_q} - \chi_1(\chi_1 - t_1) b_{01_q}]. \qquad (88)
\end{aligned}
$$

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