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THE WEYL NON-ABELIAN GAUGE FIELD AND THE THOMAS PRECESSION

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1. In Ref. 1 it has been shown that the congruent transference introduced by Weyl [2] in 1921 defines a non-Abelian gauge field. In Ref. 3, a spinor representation of the Weyl gauge group has been constructed and the corresponding spinor current, the source of that gauge field, has been found. Existence of a spinor source of the Weyl non-Abelian gauge field is an important feature of this field that justifies the problem of observable manifestations of that kind of interactions on macroscopic scales. One possible manifestation will be dealt with in this note.

As it is known (see, e.g., [4]), the most general law of parallel transport of vectors along a given curve $x^i = x^i(\tau)$ is defined by a system of ordinary differential equations of the form

$$\frac{dS^i}{d\tau} = -\Gamma^i_{jk} u^j S^k,$$

where $u^i = dx^i/d\tau$. Since the length of a vector is conserved in the course of parallel transference, the coefficients of linear connection Γ^i_{jk} should obey the system of algebraic equations

$$\partial_j g_{ik} - \Gamma^l_{ji} g_{lk} - \Gamma^l_{jk} g_{il} = 0.$$

It is assumed that the metric tensor g_{ij} is given, and the scalar product is defined in the form $g(S,S) = g_{ij} S^i S^j$. The general solution to that system of equations can be written in the form

$$\Gamma^i_{jk} = \left\{{}^{\,i}_{jk}\right\} + G^i_{jk}, \tag{1}$$

where $G^i_{jk} = g^{il} G_{jkl}$ and G_{jkl} is a tensor field of third rank that is skew-symmetric in the last two indices $G_{jkl} + G_{jlk} = 0$. This is just the Weyl non-Abelian gauge field whose properties have been examined in the above papers. Symbols $\left\{{}^{\,i}_{jk}\right\}$ in formula (1) denote components of the Levi-Civita connection of the metric g_{ij}

$$\{^i_{jk}\} = \frac{1}{2}g^{il}(\partial_j g_{kl} + \partial_k g_{jl} - \partial_l g_{jk}).$$

Connection (1) defines the congruent transference introduced by Weyl according to which a vector field under the congruent transference changes by the law

$$\frac{dS^i}{d\tau} = -\{^i_{jk}\}u^j S^k - G^i_{jk}u^j S^k. \qquad (2)$$

The law (2) includes the displacement belonging to the Riemann geometry and the rotation given by the metric g_{ij} and bivector $G_{jkl}u^j$. The Weyl gauge theory is realization of the abstract theory of gauge fields in the framework of classical differential geometry which makes no difference between space-time and gauge space. The Weyl non-Abelian gauge theory produces a number of consequences; one of them is discussed in this note.

2. Consider a rotating body, for instance, a gyroscope, accelerated by forces applied to its center of masses. Forces of that type do not produce the torque, therefore, they do not change the vector S^i of the proper angular momentum of the body if one eliminates the rotation in the plane spanned over the vectors of velocity u^i and acceleration a^i

$$a^i = \frac{\delta u^i}{\delta\tau} = \frac{d^2x^i}{d\tau^2} + \{^i_{jk}\}\frac{dx^j}{d\tau}\frac{dx^k}{d\tau}.$$

A rotation like that preserves the orthogonality of the vector S^i and 4-velocity u^i. Geometrically, this means that the proper angular momentum of the body is transferred according to the Fermi-Walker transference rule [5,6,7]

$$\frac{\delta S^i}{\delta\tau} + (u^i a^j - u^j a^i)S_j = 0,$$

where

$$\frac{\delta S^i}{\delta\tau} = \frac{dS^i}{d\tau} + \{^i_{jk}\}u^j S^k.$$

It can be verified that the length of a vector and the angle between vectors are conserved in the Fermi-Walker transference. Since the Weyl connection given by (1) and (2) is characterized by the same properties, it is natural to consider the motion of a gyroscope when its rotation in the plane of 4-velocity and 4-acceleration is determined by the Weyl field, namely, by the term $G^i_{lj}u^l S^j$, of the Weyl connection. This signifies that the trajectory of motion of a gyroscope is a solution to a system of ordinary differential equations of the form

$$u_i a_j - u_j a_i + G_{lij}u^l = 0. \qquad (3)$$

As $u_i u^i = 1$, from eq. (3) we obtain by contraction with u^i, that any solution to eq. (3) is also a solution to equations

$$a^i + G^i_{jk}u^j u^k = 0. \qquad (4)$$

It is easy to see that relations (4) are equations of geodesics in the connection (1). We note that the Fermi-Walker transference along curves that are solutions to equations (3) coincides with the Weyl congruent transference (3).

If a rotating body that is not subjected to the action of rotational moments undergoes acceleration, the direction of its rotation with respect to the inertial system changes in accordance with the Fermi-Walker transport rule. This phenomenon is called the Thomas precession. As it follows from (3), the Thomas precession can be produced by the gauge field G_{ijk}. At a qualitative level, one can try to discover the precession by tracing the behaviour of a gyroscope on a cosmic station because gravity forces in this case are zero and do not influence the gyroscope precession. To make quantitative estimates, we should know solutions to equations of the gauge field G_{ijk}, which we will obtain and which are of independent interest. Once G_{ijk} are known, we can find solutions to equations (3).

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References

1. B.M.Barbashov and A.B.Pestov, *Mod. Phys. Lett.* **A10**, 193 (1995).
2. H. Weyl, *Space-Time-Matter* (Dover, 1922).
3. B.M.Barbashov and A.B.Pestov, *Mod. Phys. Lett.* **A12**, 1957 (1997).
4. K.Hayashi *Phys.Lett.* **B65**, 437 (1976).
5. J.L.Synge *Relativity: the General Theory* (Amsterdam, North-Holland Publishing Co,1960).
6. C.W.Misner, K.S.Thorne, J.A. Wheeler *Gravitation* (W.H.Freeman and Company, 1973).
7. S.Weinberg *Gravitation and Cosmology* (John Wiley and Sons, 1972).

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