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A POSSIBLE ESTIMATION OF ATMOSPHERIC CHERENKOV LIGHT PARAMETERS

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1.Introduction

The gamma ray astronomy based on Atmosphere Cherenkov Technique (ACT) has achieved important experimental information since 1990. The significance of the Extensive Air Shower (EAS) Cherenkov light increases due to the possibility to separate events initiated by different primary particles or gamma quanta. The energy range 10GeV-200GeV becomes very important in attempt to fill the gap of energy sensitivities between satellite and ground based gamma telescopes.

2.The Method

The lateral distribution function of Atmospheric Cherenkov Light (ACL) of EAS depends on the energy and the kind of primary particle(or gamma quantum), the height of the shower birth , etc. (see for example [3]) . Near to the shower axis the lateral distribution has more or less plate maximum, after some hundred meters it is strongly decreasing to zero. The Mathematical Model (m) of the atmospheric Cherenkov light distribution at has to be in a class of functions with such behavior

$$q^m\left(X;\overline{x},\overline{y},\overline{z};\theta,\varphi\right), \quad (1)$$

where

$$X=\left[x_0,y_0,a,b,c,...,d_N\right]^T\in R^N \qquad (2)$$

is a vector of N real unknowns, where x_0 and y_0 are the coordinates of the shower axis in the detector plane; a,b,c,...,d_N are the $N-2$ parameters of the mathematical model ; $\overline{x}$, $\overline{y}$ and $\overline{z}$ are the detector coordinates; θ and φ are the zenith and azimuth angles of the EAS respectively .

The number of the Cherenkov photons N_q in the shower is

$$N_q=\int\limits_0^{x_{max}}\int\limits_0^{y_{max}} q\left(\overline{X};x,y,0;\theta,\varphi\right)dxdy, \qquad (3)$$

where $x_{\max}, y_{\max}$ are parameters depending on the speed of light in the atmosphere, the height of the showers birth and the energy of primary particle.

The different triples $(\overline{x_i}, \overline{y_i}, \overline{z_i})$, $i = 1, ..., M$ respond to different detector coordinates. The unknown components of X were obtained by solution of the overdetermined system of nonlinear equations $(M > N)$

$$q^m\left(X; \overline{x_i}, \overline{y_i}, \overline{z_i}; \theta, \varphi\right) = q_i^{\exp t}, \qquad (4)$$

where $q_i^{\exp t}$ is the density measured by the i-th detector.

The system (4) was analyzed by means of the autoregularized Newton Type method [4]. The solution (the values of X) was obtained by minimization of the functional

$$\chi^2 = \sum_{i=1}^{M} w_i \left(q^m\left(X; \overline{x_i}, \overline{y_i}, \overline{z_i}; \theta, \varphi\right) - q_i^{\exp t}\right)^2, \qquad (5)$$

where $[w_1, w_2, ..., w_M]$ is the weight vector.

The REGN computer code [5] was used. χ^2 is one of the different criteria applied in REGN [6] for the choice of the appropriate mathematical model.

By examining different $q^m(X; \overline{x}, \overline{y}, \overline{z}; \theta, \varphi)$ in the left-hand side of (4) for about two thousands real experimental EAS events registered by the Cherenkov light telescope HOTOVO [1] $q_i^{\exp t}, i = 1, ..7$ (right-hand side of (4)) and solving [6] every time the nonlinear system (4) we arrived at:

$$q(X; \overline{x}, \overline{y}, \overline{z}; \theta, \varphi) = \frac{a}{\left(e^{b^2 R^2} - c\right)^2 + \frac{1}{a}}, \qquad (6)$$

where the distance R from detector $(\overline{x}, \overline{y}, \overline{z})$ to the shower axis with parameters $x_0, y_0, z_0 = 0, \theta$ and φ is given by [7]

$$R = \sqrt{[\xi\cos(\varphi) + \eta\sin(\varphi)]^2\cos^2\theta + [\xi\sin(\varphi) - \eta\cos(\varphi)]^2}, \qquad (7)$$
$$\xi = \overline{x} - x_0 - \overline{z}tg(\theta)\cos(\varphi),$$
$$\eta = \overline{y} - y_0 - \overline{z}tg(\theta)\sin(\varphi).$$

Replacing (6) in the integral (3) with $x_{\max} = y_{\max} = \infty$ we estimate the number of photons:

$$N_q = \pi a^2 \cos(\theta) \frac{\left\{\sqrt{ac}\left[\pi + 2arctg(\sqrt{a}(c-1))\right] + \ln\frac{1 + a(c-1)^2}{a}\right\}}{2b^2(ac^2 + 1)}. \qquad (8)$$

The energy of the primary particle is $E_q = \kappa N_q$, if we assume a linear dependence between E_q and N_q.

The solutions of the system (4) with the mathematical model (6) for data "Hotovo-expt396" is illustrated in the next figures:

o The x,y-distribution of shower axes at z=0 (the sensors plane)-Figure 1;

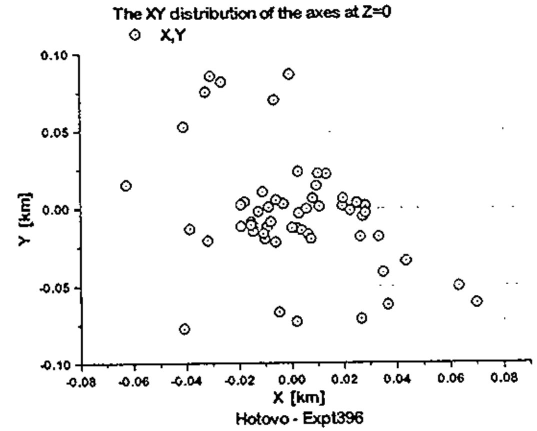


Figure 1

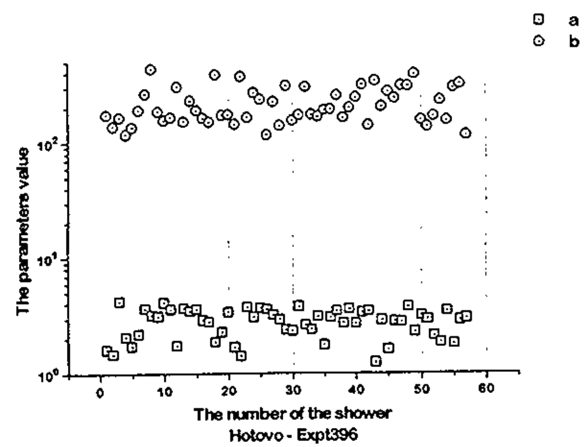


Figure 2

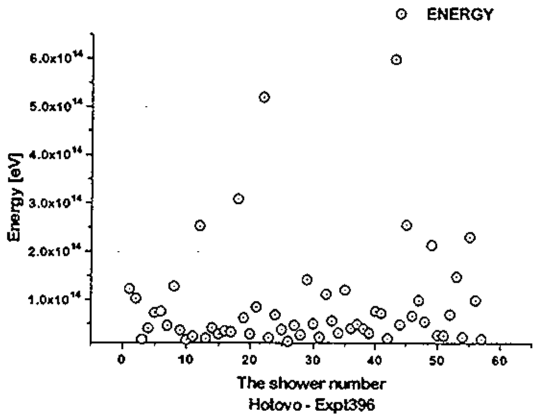


Figure 3

- ∘ The values of parameters a,b, (c=1) for every shower- Figure 2;
- ∘ The distribution of the showers energy is presented in Figure 3.

3. The r-distribution

The model (6) was tested with $R = r$ to describe the emulated data [8] at CELESTE observation depth for protons and gamma quanta, carried out with the CORSIKA code [9]. The system (4) was solved by REGN for $N = 3$ and $M = 30$ (the number of different distances r_i from the shower axis, $i = 1...M$) with left-hand side

$$q^m(r, A, B, C) = \frac{A}{\left(e^{B^2 r^2} - C\right)^2 + \frac{1}{A}}. \qquad (9)$$

The obtained solutions for unknown parameters A,B,C - "amplitude" , "breath" and "position" are given in *Tables 1a* and *1b*.

Table1a: Obtained parameters for gamma quanta

Energy	A	B	C
10GeV	0.94362	3.70945	0.54238
100GeV	3.88099	3.05848	0.79503
1TeV	29.70647	2.83039	0.68731
10TeV	161.2541	2.74761	0.83832

Table 1b: Obtained parameters for protons

100GeV	0.38769	2.57324	0.81317
1TeV	15.60083	2.98426	0.47566
10TeV	62.38982	2.31635	0.81412

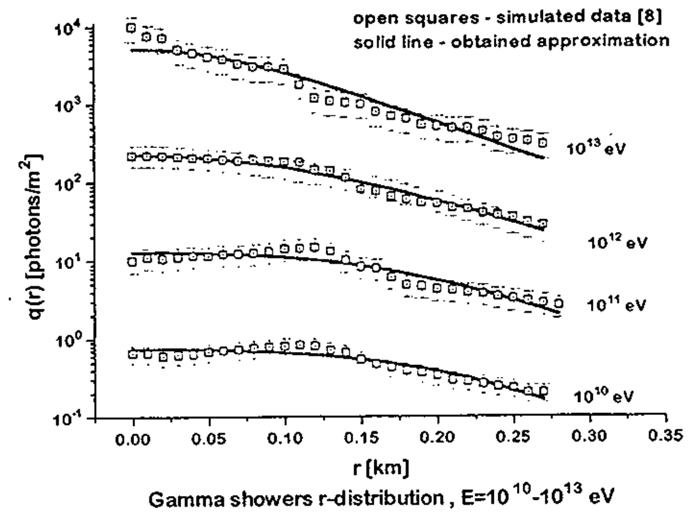


Figure 4

The Figures 4 and *5* illustrate the agreement between emulated data for gamma quanta and protons and the model function (9) with the obtained values of parameters from *Table 1a* and *1b*.

4.Results for the CELESTE experiment

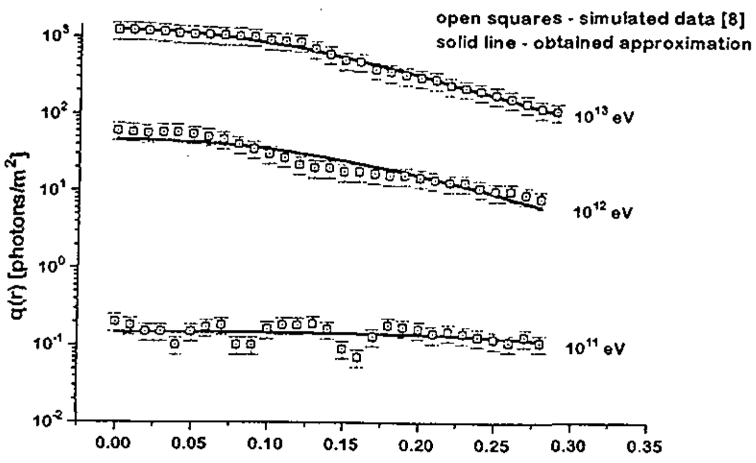
The possibilities of our method were examined by investigation of "real" data emulated like a x,y- distribution for three detector arrays: two sets of CELESTE and a new, more effective, spiral array. Using the formulae (6-8) and *Tables 1a* and *1b* for different energies of the initiated primary proton or gamma quantum and different parameters $x_0, y_0, (\theta = 0, \varphi = 0)$, the expected average Cherenkov flux densities q_i^{smooth} for each detector were calculated. The quasi-experimental data were prepared stochastically-like according to the formula

$$q_i^{expt} = \left(1 + (-1)^i \delta\right) q_i^{smooth}, \qquad (10)$$

where δ is a fluctuation parameter.

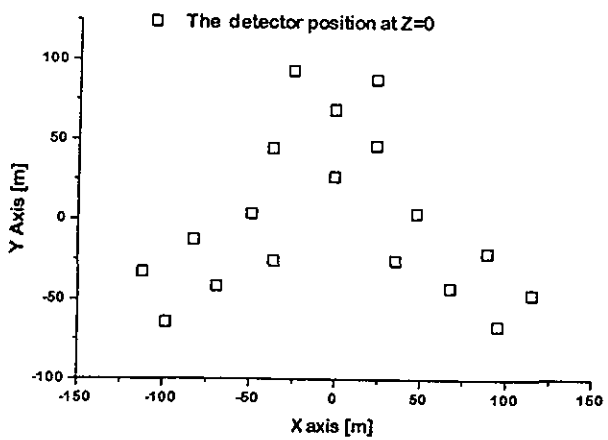
Solving the inverse problem the relative density measurement precision σ was assumed to be 0.3. Solution was obtained with initial approximations for the unknown parameters, which are "natural" for the real "Hotovo" problem. The results for the estimated energy of the initiated particle or gamma quantum and the coordinates of axis position in detectors plane are shown in the next tables.

In *Figure 6* The first 18 realized detectors of CELESTE array are shown.



Proton showers r-distribution, E=10^{11}-10^{13} eV

Figure 5



18 CELESTE detectors

Figure 6

Table3 :The difference between proposed and obtained parameters for 18 detectors of CELESTE (fluctuations $\delta = 0.1$)

	Energy	X,Y[m]	0	50	150	300	500
Gamma quanta	10GeV	$\Delta X, m$	-11.20	-11.60	-17.00	-31.00	-77.00
		$\Delta Y, m$	-10.20	-10.90	-18.00	-31.00	-79.00
		$\Delta E/E$	-0.64	-0.65	-0.69	-0.88	-0.99
	100GeV	$\Delta X, m$	-8.48	-11.20	-14.00	-14.00	-8.00
		$\Delta Y, m$	-8.21	-12.30	-11.00	-11.00	-5.00
		$\Delta E/E$	-0.43	-0.46	-0.50	-0.59	-0.61
	1TeV	$\Delta X, m$	-3.38	-9.30	-20.00	-14.00	2.00
		$\Delta Y, m$	-4.13	-10.50	-20.00	-10.00	6.00
		$\Delta E/E$	-0.33	-0.37	-0.47	-0.55	-0.57
	10TeV	$\Delta X, m$	-1.82	-5.80	-20.00	-15.00	-3.00
		$\Delta Y, m$	-2.28	-6.40	-20.00	-12,00	-7.00
		$\Delta E/E$	-0.08	-0.13	-0.38	-0.57	-0.61
Protons	100GeV	$\Delta X, m$	-84.90	-108.90	-27.00	-12.00	-66.00
		$\Delta Y, m$	-71.70	-53.16	-4.00	-7.00	-64.00
		$\Delta E/E$	-0.98	-0.98	-0.98	-0.98	-1.00
	1TeV	$\Delta X, m$	-4.77	-9.40	-14.00	-10.00	1.00
		$\Delta Y, m$	-4.72	-9.70	-13.00	-6.00	4.00
		$\Delta E/E$	-0.82	-0.83	-0.85	-0.86	-0.87
	10TeV	$\Delta X, m$	-3.22	-9.20	-25.00	-37.00	-3.00
		$\Delta Y, m$	-4.25	-10.40	-25.00	-35.00	3.00
		$\Delta E/E$	-0.63	-0.64	-0.72	-0.83	-0.82

In *Table3* $\Delta X = X_{obt} - X_{giv}$, $\Delta Y = Y_{obt} - Y_{giv}$ and $\Delta E/E = (E_{obt} - E_{giv})/E_{giv}$,

X,Y = 0;50;150;300 and 500 m are the axis positions of emulated showers (the point X=Y=0 is the detector array centre). The energies E = 0.01, 0.1, 1 and 10 TeV are for the primary gamma quanta and E= 0.1 , 1 and 10 TeV for primary protons. The presented results are for fluctuations of density $\delta = 0.1$, which is the maximum value the problem (4) can be solved acceptably for this configuration.

One can see that the determination of the axis coordinates is accurate for large axis distances from the array centre (up to X,Y=500m), with exception of the lowest energies: 10 GeV for gamma quanta and 100 GeV for protons. Some tendency to an increase of the accuracies with the growth of energy is shown, especially for the axis in the array centre .

The shower coordinates at long distances can be accurately obtained because of the specific method of solving the nonlinear systems.The emulated data for low energies ($<$ 1 TeV) at distances more than 300 m from the detector array centre have only illustrative character. Of course, with usual experimental technique it is not possible to register 10^{-3} or 10^{-4} photons per m^2.

Figure 7 presents the complete CELESTE array [2] with 186 detectors.

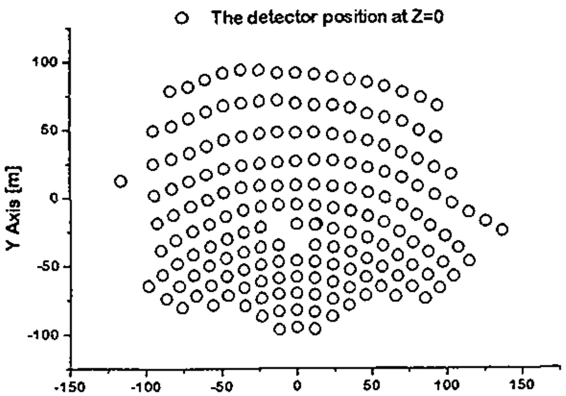


Figure 7

Table 4 :The difference between assumed and obtained parameters for 186 detectors of CELESTE (fluctuations $\delta = 0.5$)

	Energy	X,Y[m]	0	50	150	300	500
Gamma quanta	10GeV	$\Delta X, m$	-4.17	-9.80	-12.00	-19.00	-26.00
		$\Delta Y, m$	2.52	-7.00	-10.00	-18.00	-25.00
		$\Delta E/E$	-0.82	-0.71	-0.72	-0.83	-0.97
	100GeV	$\Delta X, m$	-3.10	-7.80	-12.00	-28.00	-41.00
		$\Delta Y, m$	2.17	-5.80	-11.00	-25.00	-39.00
		$\Delta E/E$	-0.68	-0.53	-0.56	-0.75	-0.93
	1TeV	$\Delta X, m$	-0.85	-3.80	-11.00	-32.00	-48.00
		$\Delta Y, m$	0.76	-2.60	-9.00	-29.00	-45.00
		$\Delta E/E$	-0.54	-0.42	-0.47	-0.69	-0.91
	10TeV	$\Delta X, m$	-0.31	-2.20	-11.00	-32.00	-50.00
		$\Delta Y, m$	0.08	-1.40	-10.00	-30.00	-47.00
		$\Delta E/E$	-0.20	-0.21	-0.33	-0.69	-0.93
Protons	100GeV	$\Delta X, m$	0.00	-117.80	-44.00	-28.00	-59.00
		$\Delta Y, m$	0.00	-67.50	-39.00	-25.00	-56.00
		$\Delta E/E$	-1.00	-0.99	-0.99	-0.98	-1.00
	1TeV	$\Delta X, m$	-1.36	-4.90	-13.00	-30.00	-43.00
		$\Delta Y, m$	1.12	-3.30	-11.00	-27.00	-40.00
		$\Delta E/E$	-0.90	-0.85	-0.86	-0.91	-0.97
	10TeV	$\Delta X, m$	-0.81	-3.80	-13.00	-41.00	-71.00
		$\Delta Y, m$	0.74	-2.80	-10.00	-38.00	-67.00
		$\Delta E/E$	-0.74	-0.67	-0.70	-0.84	-0.95

The results for the complete CELESTE array and density fluctuations $\delta = 0.5$ are presented in *Table 4*. It is shown that the obtained axis positions are determined better for the central events and the differences increased for large axis distances.

In *Figure 8* the proposed detector array which permits an optimal estimation of ACL parameters is shown.

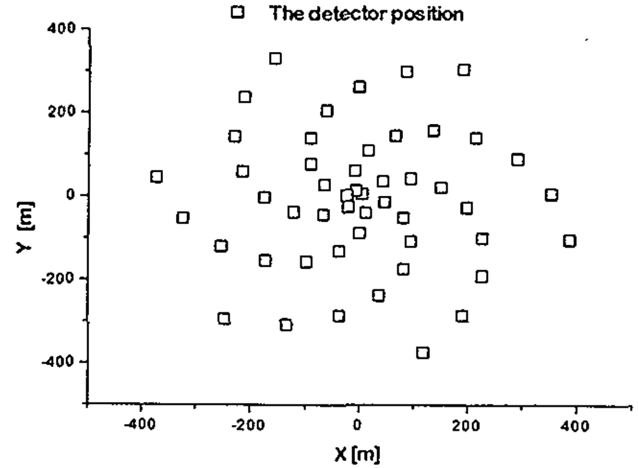


Figure 8

Table5: The difference between assumed and obtained parameters for the proposed new configuration (fluctuations $\delta = 0.5$)

	Energy	X,Y[m]	0	50	150	300	500
Gamma quanta	10GeV	$\Delta X, m$	-1.60	-2.10	-1.00	-3.00	-12.00
		$\Delta Y, m$	-0.75	-1.70	-2.00	-3.00	-10.00
		$\Delta E/E$	-0.67	-0.68	-1.00	-0.71	-0.73
	100GeV	$\Delta X, m$	-1.63	-0.49	1.00	-2.00	-16.00
		$\Delta Y, m$	0.67	-2.30	1.00	1.00	-13.00
		$\Delta E/E$	-0.48	-0.70	-0.49	-0.52	-0.63
	1TeV	$\Delta X, m$	-1.38	-2.30	1.00	-6.00	-28.00
		$\Delta Y, m$	-0.74	-0.70	3.00	0.00	-20.00
		$\Delta E/E$	-0.57	-0.37	-0.39	-0.41	0.18
	10TeV	$\Delta X, m$	-0.94	-1.90	-4.00	-4.00	-25.00
		$\Delta Y, m$	-0.47	-1.10	-1.00	-5.00	-22.00
		$\Delta E/E$	-0.16	-0.17	-0.22	-0.15	0.18
Protons	100GeV	$\Delta X, m$	-6.96	-9.30	-13.00	1.00	2.00
		$\Delta Y, m$	-4.26	-8.30	-18.00	-8.00	2.00
		$\Delta E/E$	-0.98	-0.98	-0.98	-0.98	-0.98
	1TeV	$\Delta X, m$	-1.66	-2.30	-2.00	-8.00	-20.00
		$\Delta Y, m$	-0.89	-0.80	1.00	-5.00	-19.00
		$\Delta E/E$	-0.84	-0.84	-0.84	-0.85	-0.83
	10TeV	$\Delta X, m$	-1.36	-2.60	-1.00	-6.00	-31.00
		$\Delta Y, m$	-0.75	-0.90	4.00	5.00	-25.00
		$\Delta E/E$	-0.65	-0.65	-0.65	-0.65	-0.75

In this case the accuracies in the determination of axis positions are 5-10 times better than in the other configurations, especially for energies 10-100 GeV.

5. Conclusion

A new method for estimation of ACL parameters is proposed. A new approximation of Cherenkov light lateral distribution function was obtained. The indeterminacy and errors of obtained shower parameters and the energy of initiated primary particle or gamma quantum, related to the stochastic nature of the shower development at different depths shall be investigated in the future.

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