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IS THE CASIMIR EFFECT RELEVANT TO SONOLUMINESCENCE?

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1. Sonoluminescence being observed during more than half century [1] has not received satisfactory explanation yet. As known this phenomenon represents the emission of visual light by spherical bubbles of air or other gas injected in water and subjected to an intense acoustic wave in such a way that the radius of bubbles changes periodically. In the last years of his life Schwinger proposed [2] that the bases of sonoluminescence is formed by the Casimir effect. While changing the size of bubbles the zero point energy of the vacuum electromagnetic field (the Casimir energy) of a cavity in a dielectric medium changes too. According to Schwinger, it is these changes of the electromagnetic energy that are emitted as a visual light in sonoluminescent flashes. In Schwinger's calculations the Casimir energy for the configuration in hand proves to be of the same order as the energy of the photons in an individual flash (~ 10 MeV). Other authors obtained results both consistent with Schwinger's calculation [3] and differing from it in 10 orders [4, 5]. This disagreement is basically due to different methods used for removing the divergences in the problem under consideration.

In the present note the calculation of the Casimir energy of a dielectric ball placed in an endless dielectric medium (or cavity in this medium) is carried out under following conditions. In the first place a realistic description of dielectric properties of media is used which takes into account dispersion [6]. On the other hand the most simple and reliable method for removing the divergences, the zeta function technique, is applied. Till now these conditions were not combined in studies of the problem in question.

2. When calculating the Casimir energy we shall use the mode-by-mode summation of the eigenfrequencies of the vacuum electromagnetic oscillations by applying the contour integration in a complex frequency plane [7, 5]. Consider a ball material of which is characterized by permittivity ε_1 and permeability μ_1. The ball is assumed to be placed in an infinite medium with permittivity ε_2 and permeability μ_2. For this configuration the frequencies of transverse-electric (TE) and transverse-magnetic (TM) modes are determined

by the equations [8]

$$\Delta_l^{\rm TE}(a\omega) \equiv \sqrt{\varepsilon_1\mu_2}\,\tilde{s}_l'(k_1a)\tilde{e}_l(k_2a) - \sqrt{\varepsilon_2\mu_1}\,\tilde{s}_l(k_1a)\tilde{e}_l'(k_2a) = 0, \quad (1)$$

$$\Delta_l^{\rm TM}(a\omega) \equiv \sqrt{\varepsilon_2\mu_1}\,\tilde{s}_l'(k_1a)\tilde{e}_l(k_2a) - \sqrt{\varepsilon_1\mu_2}\,\tilde{s}_l(k_1a)\tilde{e}_l'(k_2a) = 0, \quad (2)$$

where $\tilde{s}_l(x) = \sqrt{\pi x/2}\,J_{l+1/2}(x)$ and $\tilde{e}_l(x) = \sqrt{\pi x/2}\,H^{(1)}_{l+1/2}(x)$ are the Riccati-Bessel functions, $k_i = \sqrt{\varepsilon_i\mu_i}\,\omega$, $i = 1,2$ are the wave numbers inside and outside the ball, respectively; prime stands for the differentiation with respect to the argument (k_1a or k_2a) of the Riccati-Bessel functions.

As usual we define the Casimir energy by the formula

$$E = \frac{1}{2}\sum_s(\omega_s - \bar{\omega}_s), \quad (3)$$

where ω_s are the roots of Eqs. (1) and (2) and $\bar{\omega}_s$ are the same roots under condition $a \to \infty$. Here s is a collective index that stands for a complete set of indices specifying the roots of Eqs. (1) and (2): $s = \{l, m, n\}$ $l = 1, 2, \ldots;$ $m = -(l+1), -l, \ldots, l+1,$ $n = 1, 2, \ldots$. The roots of Eqs. (1) and (2) do not depend on the azimuthal quantum number m. Therefore the corresponding sum gives a multiplier $(2l+1)$. Further we use the principle of argument theorem from the complex analysis in order to present the sum over n in terms of the contour integral. As a result Eq. (3) can be rewritten as follows:

$$E = \sum_{l=1}^{\infty} E_l, \quad E_l = \frac{l+1/2}{2\pi i}\oint\limits_C dz\, z\,\frac{d}{dz}\ln\frac{\Delta_l^{\rm TE}(az)\Delta_l^{\rm TM}(az)}{\Delta_l^{\rm TE}(\infty)\Delta_l^{\rm TM}(\infty)}, \quad (4)$$

where the contour C surrounds, counterclockwise, the roots of the frequency equations (1) and (2) in the right half-plane. This contour can be deformed into a segment $(-i\Lambda, i\Lambda)$ of the imaginary axis and a semicircle of radius Λ with $\Lambda \to \infty$. In this limit the contribution of the semicircle into the integral (4) vanishes with the result [5]

$$E_l \;=\; \frac{l+1/2}{\pi a}\int\limits_0^{\infty} dy \ln\left\{\frac{4e^{-2(q_1-q_2)}}{(\sqrt{\varepsilon_1\mu_2}+\sqrt{\varepsilon_2\mu_1})^2}\right. \quad (5)$$

$$\times\left[\sqrt{\varepsilon_1\varepsilon_2\mu_1\mu_2}\left((s_l'(q_1)e_l(q_2))^2+(s_l(q_1)e_l'(q_2))^2\right)\right.$$
$$\left.\left.-(\varepsilon_1\mu_2+\varepsilon_2\mu_1)s_l(q_1)s_l'(q_2)e_l(q_2)e_l'(q_2)\right]\right\},$$

where $q_i=\sqrt{\varepsilon_i\mu_i}\,y,\ i=1,2$ and $s_l(z),\ e_l(z)$ are the modified Riccati-Bessel functions: $s_l(z)=(\pi z/2)^{1/2}I_\nu(z),\ e_l(z)=(2z/\pi)^{1/2}K_\nu(z)$, $\nu=l+1/2$.

Further we will content ourselves by examining the case when both the media are nonmagnetic $\mu_1=\mu_2=1$ and their permittivities $\varepsilon_1,\ \varepsilon_2$ differ slightly. In view of this we can put in Eq. (5) $q_1=q_2$ keeping in remain ε_1 and ε_2 exactly. It gives

$$E_l = \frac{l+1/2}{\pi a}\int_0^\infty dy\ln\left\{1-\xi^2\left[(s_l(y)e_l(y))'\right]^2\right\},\qquad (6)$$
$$\xi^2 = \left(\frac{\sqrt{\varepsilon_1}-\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}+\sqrt{\varepsilon_2}}\right)^2.$$

Now we are going to take into account the effect of dispersion considering the parameter ξ^2 in Eq. (6) as a function of $y=a\omega/i$. Justification of the mode-by-mode summation method in applying to dispersive and absorptive media has been considered in [9]. For definiteness we put $\varepsilon_1=1+\delta,\ \varepsilon_2=1,\ \delta\ll 1$, then $\xi^2\simeq\delta^2/16$. We substitute δ by

$$\delta(y)=\delta_0/[1+(y/\nu y_0)^2],\quad \nu=l+1/2,\qquad (7)$$

where δ_0 is a static value of $\delta(y)$ and the parameter y_0 is determined by a "plasma" frequency ω_0: $y_0=a\,\omega_0$. The function describing dispersion in Eq. (7) is a standard one [one-absorption-frequency Sellmeir dispersion relation] except for its dependence on l. We have introduced this dependence in order to be able to use the zeta function technique below. This complication does not contradict the main goal pursued by using this function, namely, it should simulate crudely the behaviour of $\delta(y)$ at large y. As known [10], the general theoretical principles lead to the following properties of the function $\varepsilon(\omega)$ in the upper half-plane ω. On the imaginary axis

$\omega=iy,\ y>0$ the function $\varepsilon(iy)$ acquires real values, and with increasing y it steadily decreases from the static value $1+\delta_0>0$ (for dielectrics) to 1. Obviously formula (7) meets these requirements.

Substituting (7) into (6) and making use of the uniform asymptotic expansion for the modified Bessel functions [11] when $l\to\infty$ one obtains

$$E_l\underset{l\to\infty}{\simeq} -\frac{3}{64a}\left(\frac{\delta_0}{4}\right)^2 f_1(a\omega_0)+ + \frac{9}{2^{14}\nu^2}\left(\frac{\delta_0}{4}\right)^2\left[6f_2(a\omega_0)-7f_3(a\omega_0)\left(\frac{\delta_0}{4}\right)^2\right]+\mathcal{O}(\nu^{-4}).\qquad (8)$$

where

$$f_1(z) = \frac{z}{(1+z)^4}\left(z^3+4z^2+\frac{16}{3}z+\frac{4}{3}\right),\qquad (9)$$
$$f_2(z) = \frac{z^3}{(1+z)^7}\left(\frac{521}{9}+\frac{1127}{27}z+\frac{593}{27}z^2+7z^3+z^4\right),\qquad (10)$$
$$f_3(z) = \frac{z}{(1+z)^9}\left(\frac{80}{63}+\frac{80}{7}z+\frac{928}{21}z^2+\frac{1952}{21}z^3+\frac{5960}{63}z^4 + 80z^5+\frac{320}{9}z^6+9z^7+z^8\right).\qquad (11)$$

We carry out the summation of the partial Casimir energies (6) with the help of the zeta function technique [12] taking into account asymptotics (8)

$$E=\sum_{l=1}^\infty E_l = \sum_{l=1}^\infty\left[E_l+\frac{3}{64a}\left(\frac{\delta_0}{4}\right)^2 f_1(a\omega_0)-\frac{3}{64a}\left(\frac{\delta_0}{4}\right)^2 f_1(a\omega_0)\right]$$
$$=\sum_{l=1}^\infty \bar{E}_l-\frac{3}{64a}\left(\frac{\delta_0}{4}\right)^2 f_1(a\omega_0)\sum_{l=1}^\infty(l+1/2)^0\qquad (12)$$
$$=\sum_{l=1}^\infty \bar{E}_l-\frac{3}{64a}\left(\frac{\delta_0}{4}\right)^2 f_1(a\omega_0)[\zeta(0,1/2)-1].$$

Here $\bar{E}_l = E_l + (3/64a)\,(\delta_0/4)^2\, f_1(a\omega_0)$ is the renormalized partial Casimir energy, $\zeta(s,q)$ is the Hurwitz zeta function. As $\zeta(0,1/2)=0$, we get for the Casimir energy (12)

$$E = \sum_{l=1}^{\infty} \bar{E}_l + \frac{3}{64a}\left(\frac{\delta_0}{4}\right)^2 f_1(a\omega_0). \tag{13}$$

With allowance for (8) one can obtain the estimation for the sum $\sum_{i=1}^{\infty} \bar{E}_l$

$$\begin{aligned} \sum_{i=1}^{\infty} \bar{E}_l &\simeq \frac{9}{2^{14}}\left(\frac{\delta_0}{4}\right)^2 \left[6f_2(a\omega_0) - 7f_3(a\omega_0)\left(\frac{\delta_0}{4}\right)^2\right] \sum_{l=1}^{\infty} \frac{1}{(l+1/2)^2} \\ &= \frac{9}{2^{14}}\left(\frac{\delta_0}{4}\right)^2 \left[6f_2(a\omega_0) - 7f_3(a\omega_0)\left(\frac{\delta_0}{4}\right)^2\right] \left(\frac{\pi^2}{2} - 4\right) \\ &= 5.135 \times 10^{-4}\left(\frac{\delta_0}{4}\right)^2 \left[6f_2(a\omega_0) - 7f_3(a\omega_0)\left(\frac{\delta_0}{4}\right)^2\right]. \end{aligned} \tag{14}$$

Thus the Casimir energy of a dielectric ball is

$$E \simeq \frac{3}{64a}\left(\frac{\delta_0}{4}\right)^2 \left\{f_1\,(a\omega_0) + 0.066\, f_2(a\omega_0) - 0.0048\,\delta_0^2\, f_3(a\omega_0)\right\}, \tag{15}$$

dispersion resulting only in the positive functions $f_i(a\omega_0)$, $i=1,2,3$. When z increases the functions $f_i(z)$ approach 1 (see Fig. 1), and (15) turns into the expression for the partial Casimir energy of a solid ball without dispersion [5].

Considering the behaviour of the functions $f_i(z)$ (see Fig. 1) one concludes that the main contribution, with a few percents accuracy, gives the first term in braces in Eq. (15) with the result

$$E \simeq \frac{3}{64a}\left(\frac{\delta_0}{4}\right)^2 f_1(a\omega_0). \tag{16}$$

Obviously the change of the energy sign or a considerable increasing its magnitude due to the dispersion effect [13] is out of the question.

Let us estimate the value of $f_1(a\omega_0)$. The parameter ω_0 can be determined by demanding that at this frequency the photons do not 'feel' the interface between two media. This condition will be certainly met when the wave length of photon is less than the interatomic distance in media $d \sim 10^{-8}$ cm. Actually it is the condition of applicability of the macroscopic description of dielectric media [10]. Sonoluminescence is observed with the air bubbles in water [1], the radius of bubbles being $a \sim 10^{-4}$ cm. Hence it follows that $a\,\omega_0 \simeq a/d = 10^4$ and $f_1(10^4) = 0.999\ldots$. Thus the allowance for the dispersion in calculating the Casimir energy of a dielectric ball (or spherical cavity in a slab of a dielectric) practically has no effect on the final result.

Certainly the real picture of dispersion in the whole frequency range $0 < \omega < \infty$ for any dielectric, including water, is exceedingly complicated and cannot be described by a simple equation (7) with a single parameter ω_0. As known absorption of the electromagnetic waves in water and, as a consequence, their dispersion takes place already in the radio frequency band. Putting in this case $\lambda \sim 10^4$ cm, we obtain $a\,\omega_0 \sim 1$ and $f_1(1) = 0.729\ldots$. From here one can infer that the effective value of $a\,\omega_0$ should be less than 10^4. In order for a more precise evaluation of this parameter to be done a more detailed consideration of the dispersion mechanism is needed. Obviously this may lead only to diminution of the absolute value of the Casimir energy. However this issue is beyond the scope of the present paper for the main conclusion (see below) does not depend on this point.

It is worth noting two peculiarities of the final formula (16). When the radius of the bubble decreases its Casimir energy increases. This behaviour is completely opposite to one needed for explanation of sonoluminescence (as known, emission of light takes place at the end of collapsing the bubbles in liquid). Besides, this energy is immensely smaller than the amount of energy emitted in a separate sonoluminescent flash ($\sim$ 10 MeV). Actually taking $a = 10^{-4}$ cm and $\delta_0 = 3/4$ (water) we arrive at $E \simeq 5 \cdot 10^{-3}$ eV.

Thus the results of this paper unambiguously testify that the

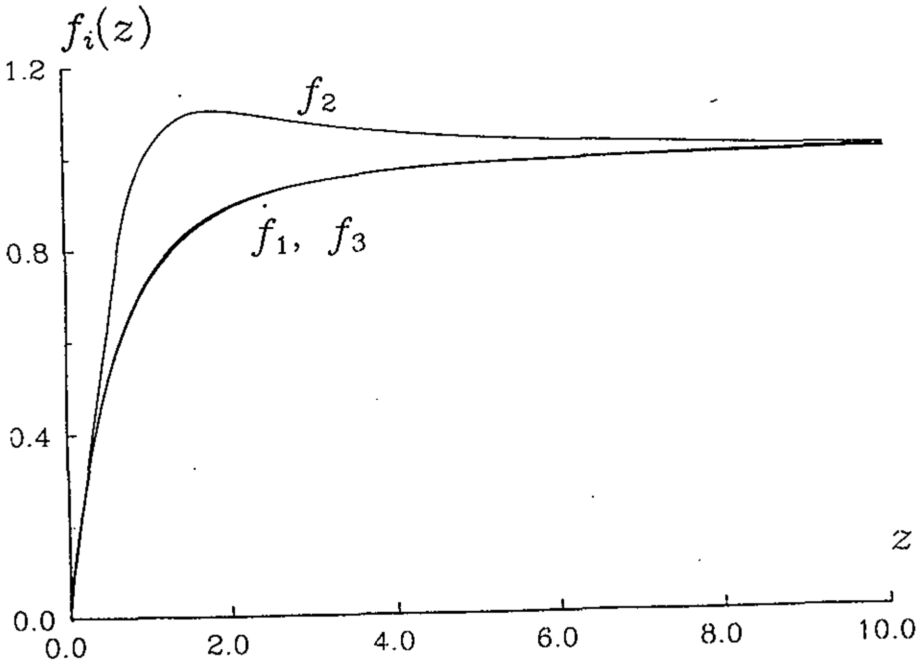

Fig.1. Functions $f_i(z)$, i=1,2,3 defining the expansion (8). When z increases the functions approach their limiting value equal to 1.

Casimir effect is irrelevant to sonoluminescence.

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