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HYPERBOLICAL INSTANTONS AND GAUGE FIELDS VACUUM STRUCTURE

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1 Introduction

Instantons are localized classical solutions of Euclidean field equations of any theory. They correspond to finite action integrals ([1]-[3]). It was expected that nonperturbative instanton effects will lead to quark confinement in QCD ([4], [5]). But this supposition did not justify hopes in (3+1) dimensions.

Vacuum conception is one of the main conceptions of quantum field theory. The instantons can drastically change vacuum state structure. In its turn any change of the vacuum structure can transform the theory properties.

Instanton solutions cannot be obtained in nonlinear theory, if nonlinear terms of fild equations are considered with respect to solutions of linear part of these equations by perturbation theory. Standard perturbation theory can be regarded as the particular case of quasiclassical method when fluctuations are quantized in neighbourhood of trivial classical solution. In nonperturbative theory we have to do the same in neighbourhood of nontrivial instanton solutions.

The main property of instanton solutions in field theory is self-duality of them. This fact was first noted in ([6]) and was being used by all authors when instanton solutions was being looking for. In spite of number of instanton properties repeat in different situations, a single general definition of instantons is absent now. Therefore it is proposed here that in Euclidean gauge field theory instanton is self- or antiself-dual solution of equation $F^a_{\mu\nu} = \mp {}^*F^a_{\mu\nu}$, where $F^a_{\mu\nu}$ - stress tensor of the gauge field A^a_μ, and ${}^*F^a_{\mu\nu}$ - dual stress tensor. Then the rest of the known instanton properties followed this definition. Such approach permits us to generalize our instanton definition to field equations of theories in pseudo-Euclidean and pseudo-Riemannian spaces. The correspondent solutions we shall name the hyperbolical instantons. Then absolutely new possibility is open. It is the possibility to obtain instantons in unified theory of all fundamental interactions equally gravity in real (3+1)-dimensional Riemannian space-time V_4 ([7]). Here we discuss the properties of such instantons and their role in construction of vacuum state in General Relativity, QCD and unified gauge field theory.

2 Different vacuum concepts

2.1 Vacuum definitions

In quantum and classical field theories there are used different vacuum concepts. The examples of such concepts are following:

1. Gauge field vector-potential A^a_μ is zero:

$$A^a_\mu = 0.$$

This is trivial vacuum definition.

2. Gauge field stress tensor $F^a_{\mu\nu}$ is zero:

$$F^a_{\mu\nu} = 0,$$

where $F^a_{\mu\nu} = \partial_{[\mu}A^a_{\nu]} - \frac{1}{2}f^a_{bc}A^b_{[\mu}A^c_{\nu]}$. This gauge field we name longitudinal one. Sometimes the gauge fields of such sort are named pure gauge, but this name do not correspond to its physical sense. Our definition means that this gauge field is experimentally invisible. Therefore we can not test existence of the gauge field by any experiment. Such gauge field is regarded as nonphysical one.

3. Motion of particles in external gauge field is force-free one. Then all components of energy-momentum tensor $T^{\mu\nu}$ are zero:

$$T^{\mu\nu} = 0,$$

where $T^{\mu\nu} = \frac{\delta L}{\delta g_{\mu\nu}}$ -Euler-Lagrange derivative of Lagrangian L relatively metrical tensor $g_{\mu\nu}$ of space-time V_4. This is vacuum definition in GR.

2.2 Force-free motion in detail

Let us consider Einstein's equation

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}, \qquad (1)$$

where $T^{\mu\nu}$-energy-momentum tensor of any nongravitational source.

When $T^{\mu\nu} = 0$, we have vacuum Einstein's equation:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 0, \quad \text{or } R_{\mu\nu} = 0. \qquad (2)$$

These equations describe force-free motion of probe particles in external gravity field. Such motion is only feasible along the geodetic lines of Riemannian space-time V_4 with metrics $g_{\mu\nu}$. In reality it is nontrivial vacuum, which structure is invisible until some particles or real physical fields will appear.

When $T^{\mu\nu} \neq 0$, we have Einstein's equation (1). This equation followed by covariant conservation law

$$T_{\mu\nu}{}^{;\nu} = 0. \qquad (3)$$

In ([8]) it was shown that conservation law (3) followed by equation of motion

$$\dot{p}_\mu = F^a_{\mu\nu}j^\nu_a, \qquad (4)$$

where p_μ is 4-momentum and j^ν_a is current of particles.

Equation (4) shows that if the gauge field is longitudinal, the motion of particles is as force-free one as under condition $T^{\mu\nu} = 0$. When $F^a_{\mu\nu} = 0$ in Minkowski space-time the particle trajectories are straight lines. They are the integral curves of equation

$$\dot{p}_\mu = 0. \qquad (5)$$

But in Riemannian V_4 the correspondent integral curves of equation (5) are geodetic lines of this Riemannian space-time, and can be regarded as the straight lines in small neighbourhood of each point only. So in Riemannian space the second vacuum definition lead to the definition of geodetic motion in external gravity field. It is naturally because the condition $F^a_{\mu\nu} = 0$ leads to $T^{\mu\nu} = 0$, but contrary assertion is wrong because $T^{\mu\nu} = 0$ do not lead to $F^a_{\mu\nu} = 0$.

So when $F^a_{\mu\nu} = 0$ or $T^{\mu\nu} = 0$ and gravity is absent, V_4 is flat space-time, and the gauge field is experimentally invisible by any particle motions.

Equation (5) can also appear when $F^a_{\mu\nu} \neq 0$, but $F^a_{\mu\nu}j^\nu_a = 0$. Then we have forceless configurations of the gauge field which are similar to forceless configurations of currents in external electromagnetic field. Such configurations are exrerimentally visible and known.

2.3 Force-free motion and stationary states

In quantum electrodynamics the definition of stationary state implies that wave function (state vector) Φ_r is eigenfunction of energy-momentum operator $\hat{P}_\mu$ ([9]):

$$\hat{P}_\mu\Phi_r = p_\mu\Phi_r,$$

where p_μ - eigenvalue of the operator $\hat{P}_\mu$.

Such definition of the stationary states followed by definition of vacuum state in the form:

$$p_\mu = 0.$$

In second quantized electrodynamics the operator $\hat{P}_\mu$ is the image of classical energy-momentum 4-vector P_μ, which is

$$P_\mu = \int T^0_\mu n_0 d^3x = \text{const}. \qquad (6)$$

This correspond to Schrödinger representation, where dynamical variables are characterized by the time independent operators ([10]).

The condition P_μ =const is integral conservation law of energy-momentum, which is obtained from differential conservation law in Minkowski V_4:

$$\partial_\mu T^{\mu\nu} = 0. \qquad (7)$$

But in Riemannian space-time we have not this conservation law. Instead of equation (7) we have equation (3). Integration of equation (3) does not lead to the energy-momentum vector P_μ in the form (6). Such P_μ in Riemannian V_4 does not exist!

Perhaps for quantization in Riemannian V_4 (and unification of all interactions equally gravity) it will be useful to take energy-momentum tensor T^ν_μ instead of energy-momentum vector P_μ. Then the vacuum definition can be taken in the form $T^\nu_\mu = 0$, i.e. the vacuum state has to correspond to force-free motion and vacuum in GR.

3 Gauge fields, vacuum structure and energy-momentum conservation: unified approach

3.1 General principles

Why GR must be a gauge theory? Modern theoretical physics goal is unification of all fundamental interactions: mechanics, electrodynamics, nuclear forces and gravity.

The ways of this problem decision being proposed now:

1. Single big symmetry group G_r generating all conservation laws for all interactions (for example, Grand Unification);
2. Single big wave function ψ which components correspond to each particle or field (for example, supersymmetry);
3. Single equation which components correspond to equations of each interaction (Kaluza-Klein theory and its extensions);
4. Single construction principle of each interaction theory under conservation individuality of each interaction.

In all cases the gauge invariance is used. Fourth way was proposed by me in 1967 and published in ([8],[11]). It does not use any compensating procedure. In this case each gauge field theory can be produced by choice:

- field variables;
- symmetry groups (space-time symmetry and internal symmetry);
- transformation properties of field variables under two types of symmetry groups;
- order of derivatives of the field variables in Lagrangian.

It is necessary and sufficiently for construction of variational and geometrical theory of any fundamental interaction. *Including GR!*

So the base of construction of unified gauge theory of all fundamental interaction consist of:

- Lagrangian formalism;
- Lie's groups (local and global) and its representations;
- Noether's theorems (first and second);
- fiber bundle spaces geometry.

Corresponding mathematical technique permits us to find equations of particle motion and fields, conservation laws, constraints and so on by the single method for all interactions.

3.2 Global symmetry G_r

The theory is invariant under transformations of Lie group G_r, where r is number of group parameters. First Noether's theorem is true.

If we have the Lagrangian L, field variables u and variations of the form u as

$$\bar{\delta}u = a_a(x,u,u',..)\epsilon^a, \quad \epsilon^a - \text{the group parameters},$$

we shall obtain following Noether's identities:

$$a_a(x,u,u',..)\frac{\delta L}{\delta u} \equiv \partial_\mu J^\mu_a. \tag{8}$$

Here $\frac{\delta L}{\delta u}$ - Euler-Lagrange derivative. Equations of motion correspond to the case $\frac{\delta L}{\delta u} = 0$.

If $\frac{\delta L}{\delta u} = 0$, we obtain the differential conservation law by (8):

$$\partial_\mu J^\mu_a = 0, \quad J^\mu_a - \text{conservative current}.$$

Thus conservation laws exist and are nontrivial, when G_r-symmetry is true. At the same time these conservation laws exist only on the solutions of field or particles motion equations (i.e. on extremals of action integral S).

If $\frac{\delta L}{\delta u} = \theta \neq 0$, we have the broken conservation law of the special form:

$$\partial_\mu J^\mu_a = \theta a_a(x,u,u',..). \tag{9}$$

Then it is spoken about broken symmetry G_r. In this case we are working beyond the extremals of S, but really G_r-symmetry of the Lagrangian L and action integral S can be conserve if (9) is fulfiled. Similar situation appears when θ corresponds to the new terms in L ([11]).

3.3 Local symmetry $G_{\infty r}$

The theory is invariant under the transformations of infinite Lie's group $G_{\infty r}$, which are determined by r functions $\epsilon^a(x)$ and its derivatives up to k^{th} order. Second Noether's theorem is true.

Variations of the form u are

$$\bar{\delta}u = a_a(x,u,u',..)\epsilon^a(x) + b^\mu_a(x,u,u',..)\epsilon^a_{,\mu}.$$

Second Noether's theorem identities are

$$a_a(x,u,u',..)\frac{\delta L}{\delta u} \equiv \partial_\mu(b^\mu_a \frac{\delta L}{\delta u}).$$

If $\frac{\delta L}{\delta u} = 0$, these identities produce the trivial result 0=0.

But if $G_{\infty r}$-symmetry is broken, i.e. $\frac{\delta L}{\delta u} = \theta \neq 0$, we have nontrivial covariant conservation laws in the form:

$$\partial_\mu(b^\mu_a \theta) - a_a \theta = 0,$$

which can be rewritten as $\nabla_\mu j^\mu_a = 0$, where $j^\mu_a = b^\mu_a \theta$- conservative current.

The covariant conservation law (3) in GR is namely of such nature. The conservative current in GR is energy-momentum tensor T^ν_μ.

3.4 Integral conservation laws

In Minkowski V_4 conservative charges (dynamical constants) are the integrals of form ([10]): $Q_a = \int J^0_a n_0 d^3x = \text{const}$. When $J^\mu_a = T^\mu_\nu$, the dynamical constants are the components of energy-momentum 4-vector P_ν:

$$Q_a = Q_\nu = \int T^0_\nu n_0 d^3x. \tag{10}$$

This number of dynamical constants correspondents to the invariance of theory under displacement transformations of Poincaré group. The displacements form abelian invariant subgroup of Poincaré group. Therefore the components of P_ν commute with each other and with the rest dynamical constants of Poincaré-symmetrical theory.

P_ν of form (10) is a result of differential conservation law (7) integration by the Gauss theorem in Minkowski space-time. But in Riemannian V_4 this procedure is impossible.

In Riemannian space-time V_4 the symmetry of Minkowski space-time becomes local one. It acts in tangential space in each point of Riemannian V_4. Instead of Poincaré group P_{10} we obtain two symmetry groups: 1) local Lorentz group $G_{\infty r}$ acting in flat tangential space in neighbourhood of each point of V_4, and 2) group of arbitrary continuous coordinate transformations $G_{\infty 4}$ of General Relativity. Second symmetry acts in the base of tangential fibre bundle space (i.e. in Riemannian V_4).

In Riemannian space-time integration procedure is only determinated for external forms. But $T^{\mu\nu}$ is symmetrical tensor of rank two and is not any external form. Therefore instead of integration of equation (7) it is necessary to integrate equation (3) by the modified procedure. Such procedure was proposed by J.L.Synge ([12]). It is integration along Killing's vector ξ^μ_a trajectories by formula

$$Q_a = \int T^0_\mu \xi^\mu_a n_0 d^3x.$$

Killing's vectors ξ^μ_a obey to equation:

$$\xi_{a\mu;\nu} + \xi_{a\nu;\mu} = 0. \tag{11}$$

The solutions of these equations determine the algebra of symmetry group (group of motions) of Riemannian V_4. This group conserves the metrical tensor $g_{\mu\nu} : \delta g_{\mu\nu} = 0$. Coordinate transformations can be written as $\delta x^\mu = \xi^\mu_a \epsilon^a(x)$.

When Riemannian curvature tensor $R_{\mu\nu\tau\lambda}$ is zero (V_4 is flat), the solutions of equation (11) generate Poincaré algebra. If $R_{\mu\nu\tau\lambda} \neq 0$, we have no any group of motions which has 4-dimentional abelian invariant subgroup. Therefore the energy-momentum 4-vector P_ν does not exist in Riemannian space-time.

4 Gauge field vacuum and GR

In the unified theory of all fundamental interactions the vacuum concept is closely connected with correct definition of the gauge field energy-momentum tensor $T^{(gf)}_{\mu\nu}$ and, consequently, with Einstein's equations.

In unified geometric theory of the gauge fields ([8]) the main equation system is analog of Hilbert-Wheeler-Misner equation system in geometrodynamics ([13]-[14]). This equation system was obtained in 1970 by me in ([15]). It has the form:

$$\hat{F}^{\mu\nu}_{a\ ;\nu} = 0 \tag{12}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T^{(gf)}_{\mu\nu} \tag{13}$$

where $T^{(gf)}_{\mu\nu}$-stress-energy tensor of any gauge field which has Maxwell-type form:

$$T^{(gf)}_{\mu\nu} = F^a_{\mu\tau} F^{\tau}_{a\ \nu} - \frac{1}{4} g_{\mu\nu} F^a_{\lambda\tau} F^{\lambda\tau}_a = (F^a_{\mu\tau} - i\, {}^*F^a_{\mu\tau})(F_a{}^\tau{}_\nu + i\, {}^*F_a{}^\tau{}_\nu) \tag{14}$$

Let us consider the gauge gravity theory with $g_{\mu\nu}$ and Ricci connections $\Delta_\mu(ik)$ as field variables ((GR+$SO(3,1)$)-gauge gravity). Its equation system is analogous to geometrodynamics one and equations (12)-(13) of any gauge field in Riemannian V_4. Such a theory takes into account the extend of real objects and describes the real gravity forces acting on them, i.e. tidal forces. In the case of (GR+$SO(3,1)$)-gauge gravity the equation system (12)-(13) transforms to following:

$$\hat{R}^{\mu\nu}(ik)_{;\nu} = 0 \tag{15}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T^{(g)}_{\mu\nu}, \tag{16}$$

where the energy-momentum tensor of tidal gravity forces $T^{(g)}_{\mu\nu}$ has the form:

$$T^{(g)}_{\mu\nu} = (R_{\mu\tau}(ik) R^\tau_\nu(ik) - \frac{1}{4} g_{\mu\nu} R_{\lambda\tau}(ik) R^{\lambda\tau}(ik)) =$$
$$(R^\sigma_\mu(ik) - {}^* R^{*\sigma}_\mu(ik))(R_{\nu\sigma}(ik) + {}^* R^*_{\nu\sigma}(ik)) \tag{17}$$

With respect to $g_{\mu\nu}$ we have here the gauge gravity theory of higher order ([16]). $R^{\mu\nu}(ik)$ is analogous to the stress tensor $F^a_{\mu\nu}$ of any gauge field. In this theory vacuum can be defined by condition $T_{\mu\nu} = 0$ as it is in GR. Then the gravity vacuum definiton is: $T^{(g)}_{\mu\nu} = 0$. Besides trivial solution $R_{\mu\nu}(ik) = 0$ this equation has nontrivial solutions. These solutions are analogous to the electrodinamics solutions which obey the duality equations:

$$F_{\mu\nu} = \pm i\ {}^*F_{\mu\nu}$$

Nontrivial solutions of duality equations are named the instantons. They minimize the action integral $S = \int F_{\mu\nu}F^{\mu\nu}dv$ and transform it into the topological constant.

In the case of the gauge field the instantons are nontrivial solutions of the equations $T^{(gf)}_{\mu\nu} = 0$ and duality equations

$$F^a_{\mu\nu} = \pm i\ {}^*F^a_{\mu\nu}$$

They minimize the action integral $S = \int F^a_{\mu\nu}F^{\mu\nu}_a dv$ and transform it into the topological constant (Pontryagin's index).

In the case of the gauge gravity the equation $T^{(g)}_{\mu\nu} = 0$ implies arising of the vacuum state of the real gravity and the transition to GR. All solutions of vacuum Einstein equations are the solutions of the gauge gravity equations. Let us show it.

Instead of duality equations

$$R^{\tau\lambda}_{\mu\nu} = \pm i\ {}^*R^{\tau\lambda}_{\mu\nu} \tag{18}$$

it is necessary take twice dual equations

$$R^{\tau\lambda}_{\mu\nu} = \pm\ {}^*R^{*\tau\lambda}_{\mu\nu} \tag{19}$$

which are followed by

$$R_{\tau\nu} = \pm\ {}^*R^*_{\tau\nu} \tag{20}$$

The duality equations (18) which are analog of electromagnetic conditions of duality have only trivial solutions in the case of gravity (Euclidean V_4).

Taking into account that equations (17) followed by $R = 0$ we can transform them to the form ([15])

$$R^\mu_\nu = -\kappa(R^{\mu\sigma\tau\lambda} - {}^*R^{*\mu\sigma\tau\lambda})(R_{\nu\sigma\tau\lambda} + {}^*R^*_{\nu\sigma\tau\lambda}) \tag{21}$$

Therefore $T^{(g)}_{\mu\nu} = 0$ if either

$$R^{\tau\lambda}_{\mu\nu} = +\ {}^*R^{*\tau\lambda}_{\mu\nu} \quad \text{and} \quad R_{\tau\nu} = +\ {}^*R^*_{\tau\nu} = R_{\tau\nu} - \frac{1}{2}g_{\tau\nu}R \longmapsto R = 0 \tag{22}$$

and we have not any new solution, or

$$R^{\tau\lambda}_{\mu\nu} = -\ {}^*R^{*\tau\lambda}_{\mu\nu} \quad \text{and} \quad R_{\tau\nu} = -\ {}^*R^*_{\tau\nu} \longmapsto R_{\tau\nu} = 0. \tag{23}$$

So all vacuum Einstein spaces are the hyperbolical instantons by definition in the frame of (GR+$SO(3,1)$)-gauge gravity theory ([16]). Therefore all solutions of vacuum GR-equations describe the vacuum structure of the gauge gravity theory and

Schwarzschild solution is one of them. The hyperbolical signature is not an obstacle to being instanton.

Thus it is shown that the gravity (including Einstein's GR) has to be consider the gauge field in the single scheme with other interactions and quantization procedure has to be analogous to that of any nonabelian gauge field. It is necessary to note that under condition $T_{\mu\nu} = 0$ we obtain always the Einstein gravity vacuum equation independently of the gauge field type. Thus all gauge field instantons can take part in creation of space-time vacuum structure.

5 Summary

If vacuum definition is $T_{\mu\nu} = 0$, then a) gauge field equation system followed by

$$F^a_{\mu\nu} = \mp i\ {}^*F^a_{\mu\nu},$$

and corresponding solutions are named the instantons (after transition to Euclidean signature of metrics); b) (GR+$SO(3,1)$)-gauge gravity equation system followed by

$$R_{\mu\sigma\tau\lambda} = \mp\ {}^*R^*_{\mu\sigma\tau\lambda},$$

and corresponding solutions are named the hyperbolical instantons. Nontrivial solutions of type a) exist in space of Euclidean signature only. Nontrivial solutions of type b) exist in Riemannian space of hyperbolical signature.

As a result of hyperbolical instantons definition

- All Einstein's equation vacuum solutions are the hyperbolical instantons;
- All gauge field instantons correspond to vacuum Einstein's equations, force-free motion and the vacuum space-time structure.

Instantons properties.

The main properties of instantons are following:

1. If $T_{\mu\nu} = 0$ and gravity is absent, the instantons determine nontrivial vacuum structure of Minkowski V_4 and force-free motion in each gauge field with $T^{(gf)}_{\mu\nu} = 0$.
2. If $T_{\mu\nu}$ is a source in Einstein's equation and gravity is present, then the instantons determine nontrivial vacuum structure of Riemannian space-time V_4 by Einstein's equation
$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = T_{\mu\nu}.$$
When $T_{\mu\nu} = 0$, we have force-free motion in gravity field.
3. The real gravity forces correspond to the tidal forces and $T^{(g)}_{\mu\nu} \neq 0$. When $T^{(g)}_{\mu\nu} = 0$, we have the gravity hyperbolical instantons.

4. Gauge field action integrals are minimized by instantons and hyperbolical instantons and turn into the topological constants (Pontryagin's index):

$$S^{(gf)}_{min} = \int F^a_{\mu\nu} F^{\mu\nu}_a dv = \int F^a_{\mu\nu} {}^*F^{\mu\nu}_a dv = \text{const},$$

$$S^{(g)}_{min} = \int R_{\mu\nu\tau\lambda} R^{\mu\nu\tau\lambda} dv = \int R_{\mu\nu\tau\lambda} {}^*R^{*\mu\nu\tau\lambda} dv = \text{const};$$

These integrals are absolute minimum of the corresponding action integrals independently on the field equations.

5. Instantons don't propagate in V_4 because all dimensions are occupied and integration must be done throughout the whole space-time V_4. Instantons and hyperbolical instantons are the special configuration of V_4;

6. Topological charge variations characterize creation and annihilation of holes in V_4, but not exchange between quasiparticles. Singularities of manifold enclose its contributions in the topological charge as well.

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