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ABOUT $\pi \leftrightarrow K$ MESON OSCILLATIONS

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1 Introduction

The vacuum oscillation of neutral K mesons is well investigated at the present time [1]. This oscillation is the result of d, s quark mixings and is described by Cabibbo-Kobayashi-Maskawa matrices [2]. The angle mixing θ of neutral K mesons is $\theta = 45^O$ since $K^o, \bar{K}^o$ masses are equal (see CPT theorem). Besides, since their masses are equal these oscillations are real ones, i.e. their transitions to each other go without suppression. The case when two particles with different masses having widths oscillate was discussed in works [3]. Now it is necessary to discuss $\pi \leftrightarrow K$ oscillations as an oscillation example of particles having large difference of masses. The $\pi \leftrightarrow K$ transitions (or oscillations) arise from the existence of quark mixings ($d' = cos\theta d + sin\theta s$). The same transitions (or oscillations) arise in the model of dynamical analogy of Cabibbo-Kobayashi-Maskawa matrices [4].

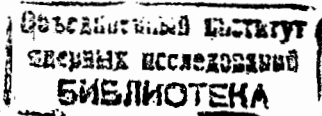Let us pass to detailed discussion of the $\pi \leftrightarrow K$ transitions.

2 The $\pi \overset{W sin\theta}{\leftrightarrow} K$ Transitions (Oscillations)

2.1 $\pi \overset{W}{\leftrightarrow} \pi$ Transitions

We begin the discussion of the problem of $\pi \overset{W}{\leftrightarrow} K$ transitions (or oscillations) with consideration of transitions (oscillations) of $\pi \overset{W}{\leftrightarrow} \pi$ through W bosons, i.e. through the following Feinmann diagram:
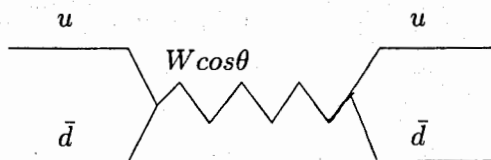
The amplitude M of this process (after using the standard proce-

dure [5]) has the form

$$M = \frac{G}{\sqrt{2}}cos\theta F_\alpha Q^\alpha = \frac{g_W^2}{8m_W^2}F_\alpha Q^\alpha, \qquad (1)$$

where $F_\alpha = f_\pi\phi_\pi p_\alpha$, $Q^\alpha = \bar{d}_L\gamma^\alpha u_L$, ϕ_π is π-meson wave function and $\mid \phi_\pi \mid_2 = 1$, f_π is π constant decay ($f_\pi \cong 125$ GeV), p_α is π four-momentum, Q^α is quark current, θ is Cabibbo's quark mixing angle.

If we use the expression $\hat{p}_\pi = \hat{p}_u + \hat{p}_{\bar{d}}$ and the Dirac equation

$$(\hat{p} - m)u = 0$$

(where u is quark wave function), then one can rewrite Eq.(1) in the following form:

$$M = \frac{G}{\sqrt{2}}cos\theta f_\pi\phi_\pi(m_u + m_{\bar{d}})\bar{d}_L\gamma_5 u_L. \qquad (2)$$

Using the standard procedure for $\overline{\mid M \mid}^2$, one obtains the following expression:

$$\overline{\mid M \mid}^2 = G^2cos^2\theta f_\pi^2(m_u + m_{\bar{d}})^2 4(p_{\bar{d}}p_u) \cong$$
$$\cong 4G^2cos^2\theta f_\pi^2(m_u + m_{\bar{d}})^2 m_\pi E_u, \qquad (3)$$

where $E_u \cong E_d \cong \frac{m_\pi}{2}$.

Then the transition $\pi \xrightarrow{W} \pi$ probability $W(...)$ is

$$W(\pi \xrightarrow{W} \pi) = \frac{\overline{\mid M \mid}^2}{2m_\pi} \int \frac{d^3p_u}{2E_u(2\pi)^3} \frac{d^3p_u}{2E_d(2\pi)^3}(2\pi)^4\delta(p_u + p_{\bar{d}} - p_\pi) =$$

$$= \frac{\overline{\mid M \mid}^2}{4\pi m_\pi} \int \delta(E_{\bar{d}} + E_u - m_\pi)\frac{E_{\bar{d}}dE_{\bar{d}}}{E_u} \cong$$

$$\cong \frac{\overline{\mid M \mid}^2}{4\pi}\frac{E}{m_\pi^2}, \qquad (4)$$

where $E \cong \frac{m_\pi}{2}$.

Then using the expression for G_W^2, one can rewrite equation (4) in the form

$$W(\pi \xrightarrow{W} \pi) \cong \frac{G^2 f_\pi^2 cos^2\theta(m_u + m_{\bar{d}})^2 m_\pi}{8\pi} = \qquad (5)$$

$$= \left(\frac{g_W^2}{4\sqrt{2}m_W^2}\right)^2 \frac{f_\pi^2 cos^2\theta(m_u + m_{\bar{d}})^2 m_\pi}{8\pi} =$$

$$= W_0(\pi \rightarrow \mu\nu_\mu)(\frac{m_u + m_{\bar{d}}}{m_\mu})^2,$$

where

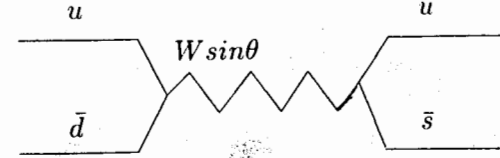$$W_0(\pi \rightarrow \mu\nu_\mu) = \frac{G^2 f_\pi^2 cos^2\theta(m_\mu)^2 m_\pi}{8\pi}$$

and

$$\tau_0' = \frac{1}{W_0(...)}.$$

Then the time $\tau(...)$ of $\pi \xrightarrow{W} \pi$ transition is

$$\tau(\pi \xrightarrow{W} \pi) = \frac{1}{W(\pi \xrightarrow{W} \pi)}. \qquad (6)$$

2.2 $\pi \overset{W sin\theta}{\rightarrow} K$ Transitions (Oscillations)

The diagram for $\pi \overset{W sin\theta}{\rightarrow} K$ transitions when one takes into account d, s quark mixings and W exchange has the form

It is clear that at d, s mixings the transition from the mass shell of π meson does not take place , i.e. K meson created from π meson remains on the mass shell of π meson.

Repeating the same calculations in (2)–(5) for $\pi \overset{W sin\theta}{\leftrightarrow} K$ transitions for the probability $W(...)$ of $\pi \overset{W sin\theta}{\rightarrow} K$ transitions, one obtains the following expression:

$$W(\pi \overset{W sin\theta}{\rightarrow} K) \cong \frac{G^2 f_\pi^2 sin^2\theta(m_u + m_{\bar{d}})^2 m_\pi}{8\pi} = \qquad (7)$$

$$= \left(\frac{g_W^2}{4\sqrt{2}m_W^2}\right)^2 \frac{f_\pi^2 cos^2\theta(m_u + m_{\bar{d}})^2 m_\pi}{8\pi}.$$

Then the time t of $\pi \overset{W sin\theta}{\rightarrow} K$ transition is

$$\tau(\pi \xrightarrow{W sin\theta} K) = \frac{1}{W(\pi \xrightarrow{W sin\theta} K)}. \tag{8}$$

The relation between the time $\tau(\pi, \pi)$ of $\pi \xrightarrow{W cos\theta} \pi$ transition and the time $\tau(\pi, K)$ of $\pi \xrightarrow{W sin\theta} K$ transition is

$$\frac{\tau(\pi \xrightarrow{W sin\theta} K)}{\tau(\pi \xrightarrow{W cos\theta} \pi)} = \frac{1}{tg^2\theta}. \tag{9}$$

So, at the $\pi \leftrightarrow K$ transitions, $\bar{s}$ remains on the mass shell of $\bar{d}$ quark (i.e. K is on π mass shell) and then K mesons transit back into π mesons and this process goes on the background of π decays. It is clear that these oscillations (transitions) are virtual ones and can be seen through K meson decays if virtual K mesons transit to their own mass shell. Since K mesons take part in the strong interactions, one can do it through their quasiinelastic strong interactions. This problem will be considered in the next work.

Now we pass to computation of the probability of $\pi \leftrightarrow K$ oscillations.

3 Probability of $\pi \xrightarrow{W sin\theta} K$ (Virtual) Oscillations

The mass matrix of π and K mesons has the form

$$\begin{pmatrix} m_\pi & 0 \\ 0 & m_K \end{pmatrix}. \tag{10}$$

Due to the presence of strangeness violation in the weak interactions, a nondiagonal term appears in this matrix and then this mass matrix is transformed in the following nondiagonal matrix:

$$\begin{pmatrix} m_\pi & m_{\pi K} \\ m_{\pi K} & m_K \end{pmatrix}, \tag{11}$$

which is diagonalized by turning through the angle β and

$$tg2\beta = \frac{2m_{\pi K}}{\mid m_\pi - m_K \mid},$$

$$sin2\beta = \frac{2m_{\pi K}}{\sqrt{(m_\pi - m_K)^2 + (2m_{\pi K})^2}}. \tag{12}$$

It is interesting to remark that expression (12) can be obtained from the Breit-Wigner distribution [3]

$$P \sim \frac{\Gamma/2)^2}{(E - E_0)^2 + (\Gamma/2)^2} \tag{13}$$

by using the following substitutions:

$$E = m_K, \ E_0 = m_\pi, \ \Gamma/2 = 2m_{\pi K}, \tag{14}$$

where $\Gamma \equiv W(...)$.

Now we can consider two cases of π, K oscillations:

1. If we consider the real transition of π into K mesons then

$$sin^2 2\beta \cong \frac{4m_{\pi K}^2}{(m_\pi - m_K)^2} \cong 0, \tag{15}$$

i.e. the probability of the real transition of π mesons into K mesons through weak interaction is very small since $m_{\pi K}$ is very small.

How can we understand this real $\pi \to K$ transition?

If $2m_{\pi K} = \frac{\Gamma}{2}$ is not zero, then it means that the mean mass of π meson is m_π and this mass is **distributed** by $sin^2 2\beta$ (or by the Breit-Wigner formula) and the probability of the $\pi \to K$ transition differs from zero. So, this is a solution of the problem of origin of mixing angle in the theory of vacuum oscillation.

2. If we consider the virtual transition of π into K meson then, since $m_\pi = m_K$,

$$tg2\beta = \infty,$$

i.e. $\beta = \pi/4$, then

$$sin^2 2\beta = 1. \tag{16}$$

We will consider the second case since it is of real interest.

If at $t = 0$ we have the flow $N(\pi, 0)$ of π mesons then at $t \neq 0$ this flow will decrease since π mesons decay and then we have the following flow $N(\pi, t)$ of π mesons:

$$N(\pi, t) = exp(-\frac{t}{\tau_0})N(\pi, 0), \tag{17}$$

where $\tau_0 = \tau_0' \frac{E_\pi}{m_\pi}$.

One can express the time $\tau(\pi \leftrightarrow K)$ through the time of τ_0, then

$$\tau(\pi \xrightarrow{W \sin\theta} K) = \tau_0 (\frac{m_\mu}{m_u + m\bar{d}})^2 \frac{1}{tg^2\theta}. \tag{18}$$

The expression for the flow $N(\pi \to K, t)$, i.e. probability of π to K meson transitions at time t, has the form

$$N(\pi \to K, t) = N(\pi, t) \sin^2 \left[\frac{\pi t}{\tau(\pi \xrightarrow{W} K)}\right] =$$

$$= N(\pi, 0) exp(-\frac{t}{\tau_0}) \sin^2 \left[\frac{\pi t}{\tau_0} \frac{tg^2\theta}{(\frac{m_\mu}{m_u + m_{\bar{d}}})^2}\right]. \tag{19}$$

Since $\tau(\pi \to K) >> \tau_0$ at $t = \tau(\pi \to K)$ nearly all π mesons will decay, therefore to determine a more effective time (or distance) for observation of $\pi \to K$ transitions it is necessary to find the extremum of $N(\pi \to K, t)$, i.e. Eq. (19):

$$\frac{dN(\pi \to K, t)}{dt} = 0. \tag{20}$$

From Eqs. (19) and (20) one obtain the following equation:

$$\frac{2\pi tg^2\theta}{(\frac{m_\mu}{m_u + m_{\bar{d}}})^2} = tg \left[\frac{t\pi tg^2\theta}{\tau_0 (\frac{m_\mu}{m_u + m_{\bar{d}}})^2}\right]. \tag{21}$$

If one takes into account that the argument of the right part of (21) is a very small value, one can rewrite the right part of (21) in the form

$$tg \left[\frac{t\pi tg^2\theta}{\tau_0(\frac{m_\mu}{m_u + m_{\bar{d}}})^2}\right] \cong \frac{t\pi tg^2\theta}{\tau_0(\frac{m_\mu}{m_u + m_{\bar{d}}})^2}. \tag{22}$$

Using (21) and (22) one obtains that the extremum of $N(...)$ takes place at

$$\frac{t}{\tau_0} \cong 2 \quad or \quad t \cong 2\tau_0. \tag{23}$$

And the extremal distance R for observation of $\pi \to K$ oscillations is

$$R = t v_\pi \cong 2\tau_0 v_\pi, \tag{24}$$

and the equation for $N(\pi \to K, 2\tau_0)$ has the following form:

$$N(\pi \to K, 2\tau_0') = N(\pi, 0) exp(-2) \sin^2 \left[2\pi \frac{tg^2\theta}{(\frac{m_\mu}{m_u + m_d})^2}\right] \cong \tag{25}$$

$$\cong N(\pi, 0) 5.1 \bullet 10^{-6}.$$

where $m_u + m_d \cong 15$ MeV, $tg^2\theta \cong 0.048$.

Let us pass to discussion of kinematics of K meson creation processes.

4 Kinematics Processes of K Meson Creation

So, if one has π mesons, then with the probability determined by Eq. (19) they virtually transit into K mesons and if these virtual K mesons participate in quasielastic strong interactions then they become real K mesons. Then through K meson decays one can verify this process.

The energy threshold $E_{thre,\pi}$ of the quasielastic reaction $\pi^+ + p \to K^+ + p$ is

$$E_{thre,\pi} = 0.61 \ GeV.$$

Besides, this quasielastic reaction of π mesons can create K mesons in inelastic reactions. An example of this inelastic reaction is the following reaction:

$$\pi^+ + n \to K^+ + \Lambda. \tag{26}$$

The energy threshold $E_{thre,\pi}^{inel}$ is

$$E_{thre,\pi}^{inel} = 0.91 \ GeV.$$

To avoid the problem with K mesons created in inelastic reaction, one must take energies E_π of π mesons less than 0.91 GeV, i.e. E_π must be

$$0.61 \leq E_\pi \leq 0.91 \ GeV. \tag{27}$$

The optimal distances for observation of $\pi \leftrightarrow K$ oscillations can be computed using Eqs.(24) and (27).

5 Conclusion

In the framework of the Standard Model the probability (and time) (see Eq.(19)) of the $\pi \leftrightarrow K$ transitions (oscillations) were computed. These transitions are virtual ones since masses of π and K mesons differ considerably. These transitions (oscillations) can be registered through K decays after transitions of virtual K mesons to their own mass shell by using their quasielastic strong interactions. But for avoiding the background of inelastic K mesons the energies E_π of π mesons must be less than the threshold energy of their creation, i.e. $E_\pi < 0.91$ GeV. The optimal distances for obsevation of these oscillations were computed (see Eq. (24)).

The computation of $\pi \leftrightarrow K$ oscillations can also be performed in the framework of the model of dynamical analogy of Cabibbo-Kobayashi-Maskawa matrices [4] through $B\pm$ boson exchanges. At low energetic limit the results (time and length of oscillations) are the same.

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Об осцилляциях $\pi \leftrightarrow K$ мезонов

В рамках стандартной модели производится расчет вероятности (и времени) $\pi \leftrightarrow K$-переходов. Эти переходы являются виртуальными, т.к. массы π- и K-мезонов сильно различаются. Такие переходы можно наблюдать через распады K-мезонов, если через квазиупругое сильное взаимодействие перевести эти виртуальные K-мезоны на собственную массовую поверхность. Чтобы избежать фона от неупруго родившихся K-мезонов, энергии E_π π-мезонов должны быть меньше порога рождения, т.е. $E_\pi < 0{,}91$ ГэВ. Подсчитываются оптимальные расстояния для наблюдения этих осцилляций.

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About $\pi \leftrightarrow K$ Meson Oscillations

In the framework of the Standard Model the probability (and time) of $\pi \leftrightarrow K$ transitions (oscillations) are computed. These transitions are virtual ones since masses of π and K mesons differ considerably. These transitions (oscillations) can be registered through K decays after transitions of virtual K mesons to their own mass shell by using their quasielastic strong interactions. But for avoiding the background from inelastic K mesons, the energies E_π of π mesons must be less than the threshold energy of their creation, i.e. $E_\pi < 0.91$ GeV. The optimal distances for observation of these oscillations are computed.

The investigation has been performed at the Laboratory of Particle Physics, JINR.